Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.7-hyper^m-a+b-cosh^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [85]. This is test number [170].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (85)	% 0.00 (0)
Mathematica	% 98.82 (84)	% 1.18 (1)
Maple	% 100.00 (85)	% 0.00 (0)
Maxima	% 40.00 (34)	% 60.00 (51)
Fricas	% 78.82 (67)	% 21.18 (18)
Sympy	% 20.00 (17)	% 80.00 (68)
Giac	% 54.12 (46)	% 45.88 (39)
Mupad	% 64.71 (55)	% 35.29 (30)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

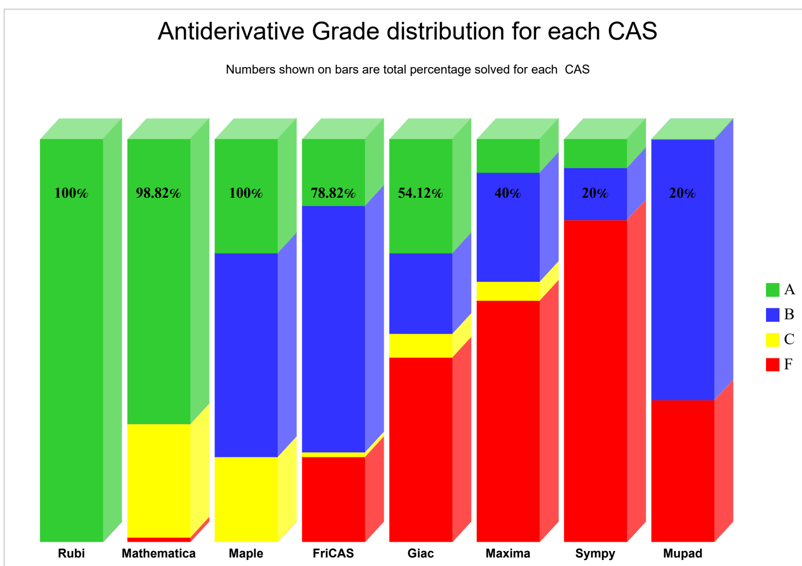
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

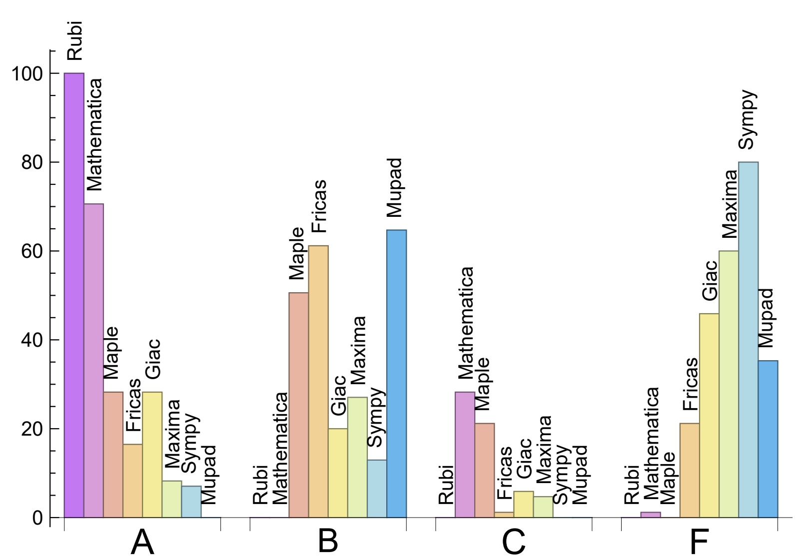
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.59	0.00	28.24	1.18
Maple	28.24	50.59	21.18	0.00
Maxima	8.24	27.06	4.71	60.00
Fricas	16.47	61.18	1.18	21.18
Sympy	7.06	12.94	0.00	80.00
Giac	28.24	20.00	5.88	45.88
Mupad	0.00	64.71	0.00	35.29

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Maxima	51	98.04 %	1.96 %	0.00 %
Fricas	18	50.00 %	33.33 %	16.67 %
Sympy	68	55.88 %	44.12 %	0.00 %
Giac	39	56.41 %	2.56 %	41.03 %
Mupad	30	76.67 %	23.33 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

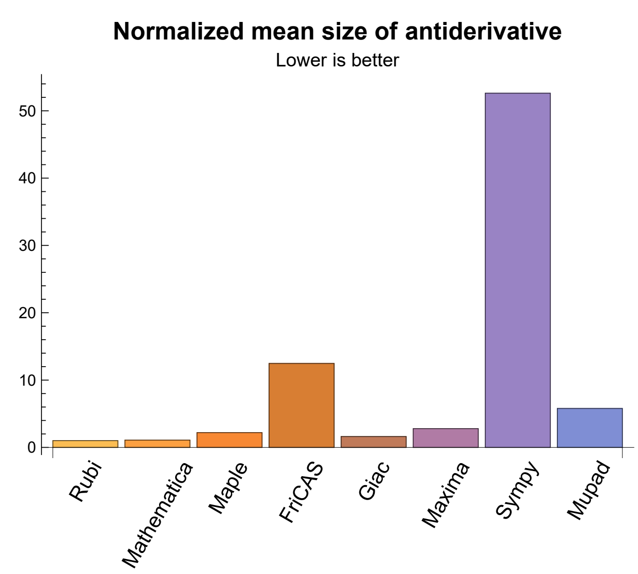
1.3 Performance

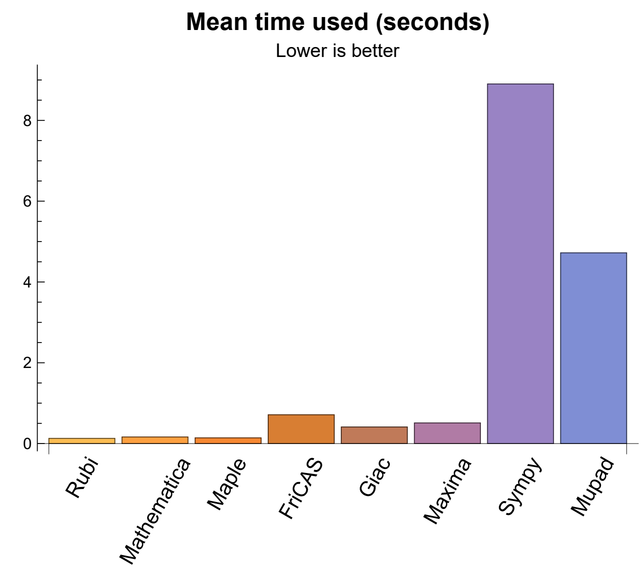
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	83.32	1.00	49.00	1.00
Mathematica	0.16	74.51	1.08	52.00	1.00
Maple	0.14	126.16	2.19	96.00	1.96
Maxima	0.51	124.91	2.79	56.00	2.11
Fricas	0.71	911.19	12.47	360.00	9.54
Sympy	8.90	1566.65	52.61	87.00	4.00
Giac	0.41	74.11	1.62	41.50	1.61
Mupad	4.72	443.51	5.79	243.00	3.33

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

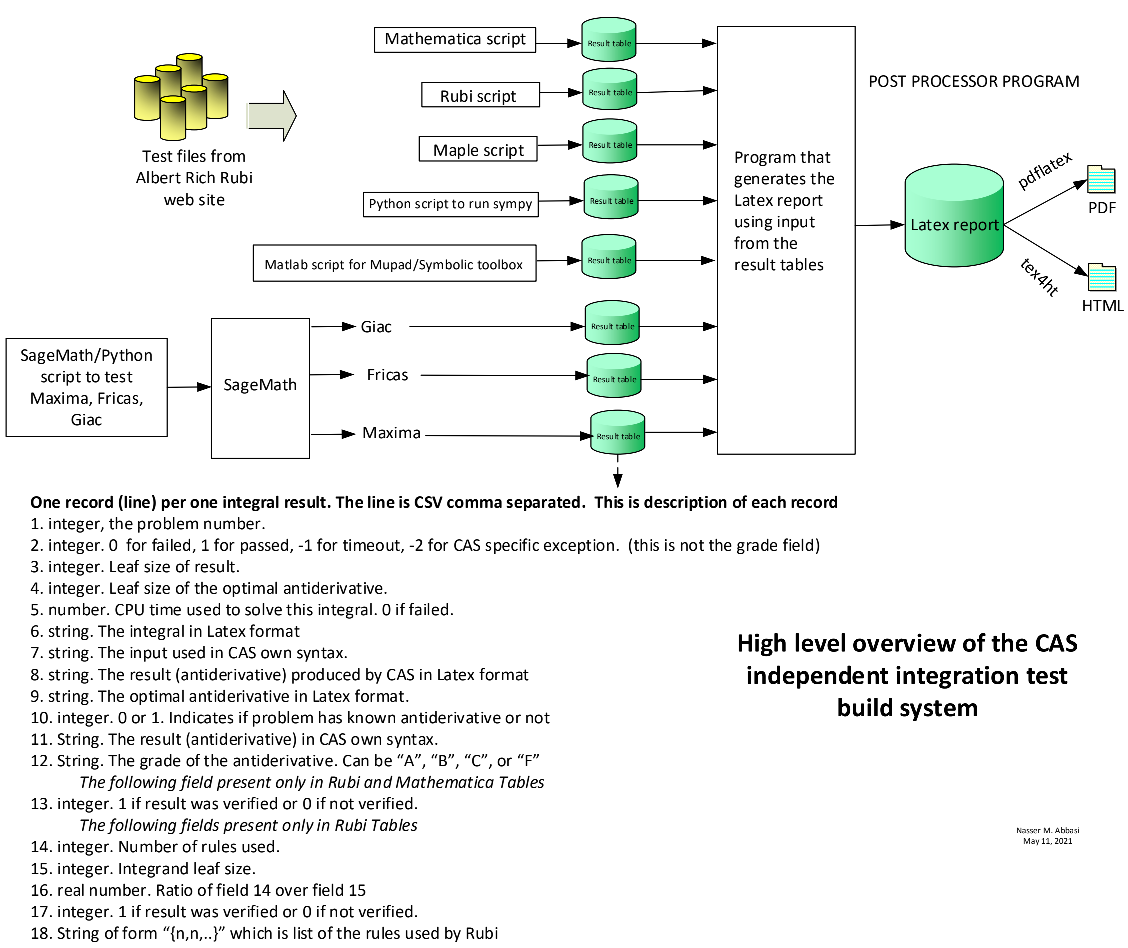
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 61, 63, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85 }

B grade: { }

C grade: { 6, 7, 8, 10, 11, 12, 56, 57, 58, 59, 60, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 81 }

F grade: { 35 }

2.1.3 Maple

A grade: { 2, 9, 10, 11, 19, 43, 44, 45, 47, 48, 49, 50, 51, 54, 55, 76, 77, 78, 79, 80, 82, 83, 84, 85 }

B grade: { 1, 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 52, 53, 58, 59, 63, 74 }

C grade: { 3, 56, 57, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 81 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 3, 37, 44, 49, 54, 76 }

B grade: { 2, 4, 5, 13, 14, 15, 16, 17, 18, 21, 23, 25, 27, 29, 31, 33, 34, 35, 36, 38, 39, 40, 63 }

C grade: { 43, 48, 53, 80 }

F grade: { 6, 7, 8, 9, 10, 11, 12, 19, 20, 22, 24, 26, 28, 30, 32, 41, 42, 45, 46, 47, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 81, 82, 83, 84, 85 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 15, 25, 43, 44, 48, 49, 53, 54, 80, 84, 85 }

B grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 58, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83 }

C grade: { 81 }

F grade: { 41, 42, 45, 46, 47, 50, 51, 52, 55, 56, 57, 64, 65, 66, 67, 68, 69, 82 }

2.1.6 Sympy

A grade: { 2, 3, 9, 16, 19, 27 }

B grade: { 1, 35, 36, 37, 38, 39, 40, 58, 59, 63, 74 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 15, 16, 19, 25, 27, 29, 31, 33, 37, 39, 40, 65, 66, 68, 69, 72, 74, 76, 81 }

B grade: { 13, 14, 17, 18, 21, 23, 34, 35, 36, 38, 44, 49, 54, 58, 59, 63, 71 }

C grade: { 43, 48, 53, 62, 80 }

F grade: { 6, 7, 8, 9, 10, 11, 12, 20, 22, 24, 26, 28, 30, 32, 41, 42, 45, 46, 47, 50, 51, 52, 55, 56, 57, 60, 61, 64, 67, 70, 73, 75, 77, 78, 79, 82, 83, 84, 85 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 56, 57, 58, 59, 60, 61, 62, 63, 65, 68, 71, 74, 75, 76, 81 }

C grade: { }

F grade: { 33, 34, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 66, 67, 69, 70, 72, 73, 77, 78, 79, 80, 82, 83, 84, 85 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	78	25	14	153	26	25
normalized size	1	1.00	0.95	3.90	1.25	0.70	7.65	1.30	1.25
time (sec)	N/A	0.046	0.004	0.086	0.313	0.506	2.263	0.133	0.944

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	17	7	10	12	7
normalized size	1	1.00	1.00	1.14	2.43	1.00	1.43	1.71	1.00
time (sec)	N/A	0.044	0.003	0.061	0.312	0.497	1.301	0.114	0.905

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	11	6	6	3	6	6
normalized size	1	1.00	1.00	1.83	1.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.039	0.000	0.077	0.322	0.450	0.760	0.135	0.027

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	22	37	61	100	0	21	21
normalized size	1	1.00	1.16	1.95	3.21	5.26	0.00	1.11	1.11
time (sec)	N/A	0.048	0.004	0.107	0.311	0.428	0.000	0.132	0.908

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	32	53	135	216	0	27	27
normalized size	1	1.00	1.10	1.83	4.66	7.45	0.00	0.93	0.93
time (sec)	N/A	0.051	0.004	0.107	0.318	0.389	0.000	0.135	0.919

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	148	395	0	2346	0	0	805
normalized size	1	1.00	1.90	5.06	0.00	30.08	0.00	0.00	10.32
time (sec)	N/A	0.105	0.246	0.089	0.000	0.453	0.000	0.000	1.546

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	120	214	0	1064	0	0	548
normalized size	1	1.00	2.22	3.96	0.00	19.70	0.00	0.00	10.15
time (sec)	N/A	0.087	0.187	0.085	0.000	0.544	0.000	0.000	1.304

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	83	97	0	416	0	0	257
normalized size	1	1.00	2.31	2.69	0.00	11.56	0.00	0.00	7.14
time (sec)	N/A	0.065	0.188	0.085	0.000	0.488	0.000	0.000	1.366

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	0	300	87	0	16
normalized size	1	1.00	1.00	0.68	0.00	12.00	3.48	0.00	0.64
time (sec)	N/A	0.036	0.022	0.053	0.000	0.516	1.062	0.000	0.971

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	99	52	0	349	0	0	462
normalized size	1	1.00	2.36	1.24	0.00	8.31	0.00	0.00	11.00
time (sec)	N/A	0.062	0.144	0.096	0.000	0.637	0.000	0.000	1.389

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	154	94	0	1332	0	0	2225
normalized size	1	1.00	2.52	1.54	0.00	21.84	0.00	0.00	36.48
time (sec)	N/A	0.104	0.291	0.110	0.000	0.489	0.000	0.000	6.894

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	219	184	0	5326	0	0	5056
normalized size	1	1.00	2.33	1.96	0.00	56.66	0.00	0.00	53.79
time (sec)	N/A	0.172	0.607	0.118	0.000	0.599	0.000	0.000	14.741

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	76	575	651	1308	0	166	248
normalized size	1	1.00	0.86	6.53	7.40	14.86	0.00	1.89	2.82
time (sec)	N/A	0.168	0.173	0.132	0.455	0.514	0.000	0.128	1.720

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	351	348	568	0	103	146
normalized size	1	1.00	0.88	5.95	5.90	9.63	0.00	1.75	2.47
time (sec)	N/A	0.116	0.099	0.114	0.427	0.484	0.000	0.141	1.266

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	183	120	300	0	52	79
normalized size	1	1.00	0.92	4.69	3.08	7.69	0.00	1.33	2.03
time (sec)	N/A	0.067	0.060	0.104	0.423	0.551	0.000	0.131	0.219

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	78	53	293	12026	39	267
normalized size	1	1.00	1.00	2.69	1.83	10.10	414.69	1.34	9.21
time (sec)	N/A	0.022	0.067	0.093	0.422	0.459	46.720	0.133	0.346

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	177	161	1875	0	107	245
normalized size	1	1.00	1.00	3.00	2.73	31.78	0.00	1.81	4.15
time (sec)	N/A	0.086	0.219	0.143	0.427	0.544	0.000	0.394	1.448

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	307	307	4977	0	189	333
normalized size	1	1.00	1.03	3.45	3.45	55.92	0.00	2.12	3.74
time (sec)	N/A	0.108	0.374	0.148	0.479	0.529	0.000	0.589	1.546

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	77	80	0	305	85	80	205
normalized size	1	1.00	0.79	0.82	0.00	3.11	0.87	0.82	2.09
time (sec)	N/A	0.117	0.092	0.051	0.000	0.520	1.541	0.147	3.512

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	86	317	0	2508	0	0	293
normalized size	1	1.00	1.10	4.06	0.00	32.15	0.00	0.00	3.76
time (sec)	N/A	0.090	0.290	0.124	0.000	0.519	0.000	0.000	1.394

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	76	326	651	1245	0	150	178
normalized size	1	1.00	0.86	3.70	7.40	14.15	0.00	1.70	2.02
time (sec)	N/A	0.210	0.224	0.109	0.501	0.482	0.000	0.135	1.330

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	176	0	1184	0	0	243
normalized size	1	1.00	1.09	3.14	0.00	21.14	0.00	0.00	4.34
time (sec)	N/A	0.073	0.171	0.101	0.000	0.526	0.000	0.000	1.255

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	188	347	573	0	95	142
normalized size	1	1.00	0.88	3.19	5.88	9.71	0.00	1.61	2.41
time (sec)	N/A	0.106	0.133	0.098	0.561	0.584	0.000	0.126	1.168

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	96	0	498	0	0	204
normalized size	1	1.00	1.00	2.53	0.00	13.11	0.00	0.00	5.37
time (sec)	N/A	0.059	0.031	0.091	0.000	0.453	0.000	0.000	1.162

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	110	120	317	0	50	376
normalized size	1	1.00	0.92	2.82	3.08	8.13	0.00	1.28	9.64
time (sec)	N/A	0.070	0.084	0.096	0.514	0.491	0.000	0.126	1.383

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	66	0	337	0	0	87
normalized size	1	1.00	1.00	2.28	0.00	11.62	0.00	0.00	3.00
time (sec)	N/A	0.032	0.012	0.073	0.000	0.457	0.000	0.000	1.189

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	78	53	293	12026	39	267
normalized size	1	1.00	1.00	2.69	1.83	10.10	414.69	1.34	9.21
time (sec)	N/A	0.022	0.050	0.082	0.414	0.560	44.772	0.132	0.002

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	45	84	0	360	0	0	208
normalized size	1	1.00	1.10	2.05	0.00	8.78	0.00	0.00	5.07
time (sec)	N/A	0.054	0.100	0.127	0.000	0.471	0.000	0.000	1.303

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	102	70	457	0	58	108
normalized size	1	1.00	1.00	2.68	1.84	12.03	0.00	1.53	2.84
time (sec)	N/A	0.079	0.087	0.142	0.557	0.452	0.000	0.121	0.279

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	131	0	963	0	0	447
normalized size	1	1.00	0.98	2.22	0.00	16.32	0.00	0.00	7.58
time (sec)	N/A	0.091	0.175	0.144	0.000	0.528	0.000	0.000	1.555

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	139	119	1377	0	87	239
normalized size	1	1.00	1.00	2.53	2.16	25.04	0.00	1.58	4.35
time (sec)	N/A	0.099	0.154	0.165	0.420	0.445	0.000	0.419	1.317

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	274	0	3239	0	0	1305
normalized size	1	1.00	0.96	3.04	0.00	35.99	0.00	0.00	14.50
time (sec)	N/A	0.138	0.309	0.148	0.000	0.583	0.000	0.000	36.098

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	236	134	1239	0	104	-1
normalized size	1	1.00	1.05	3.63	2.06	19.06	0.00	1.60	-0.02
time (sec)	N/A	0.057	0.232	0.111	0.525	0.466	0.000	0.144	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	106	477	344	5117	0	228	-1
normalized size	1	1.00	0.99	4.46	3.21	47.82	0.00	2.13	-0.01
time (sec)	N/A	0.123	0.628	0.125	0.592	0.601	0.000	0.682	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	B	B	B	B	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	0	86	34	66	60	34	50
normalized size	1	1.00	0.00	5.73	2.27	4.40	4.00	2.27	3.33
time (sec)	N/A	0.012	0.023	0.054	0.591	0.400	0.678	0.143	0.136

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	113	59	214	211	59	76
normalized size	1	1.00	1.00	3.23	1.69	6.11	6.03	1.69	2.17
time (sec)	N/A	0.025	0.131	0.058	0.417	0.534	3.467	0.133	1.049

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	129	83	575	428	71	112
normalized size	1	1.00	1.00	2.53	1.63	11.27	8.39	1.39	2.20
time (sec)	N/A	0.051	0.179	0.059	0.424	0.556	13.574	0.119	0.995

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	16	10	20	14	10	10
normalized size	1	1.00	1.00	8.00	5.00	10.00	7.00	5.00	5.00
time (sec)	N/A	0.016	0.003	0.068	0.310	0.503	0.413	0.109	0.063

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	32	49	84	34	18	18
normalized size	1	1.00	1.55	2.91	4.45	7.64	3.09	1.64	1.64
time (sec)	N/A	0.018	0.003	0.066	0.346	0.425	1.093	0.137	0.980

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	48	111	185	54	24	24
normalized size	1	1.00	1.42	2.53	5.84	9.74	2.84	1.26	1.26
time (sec)	N/A	0.020	0.004	0.071	0.353	0.482	2.785	0.113	0.064

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	114	0	0	0	0	-1
normalized size	1	1.00	1.08	2.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.080	0.282	0.000	0.419	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	58	0	0	0	0	-1
normalized size	1	1.00	1.06	3.41	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.023	0.354	0.000	0.461	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	11	1	0	31	13
normalized size	1	1.00	1.00	1.15	0.85	0.08	0.00	2.38	1.00
time (sec)	N/A	0.022	0.005	0.144	0.588	0.573	0.000	0.128	1.016

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	2	0	31	11
normalized size	1	1.00	1.00	1.27	1.00	0.18	0.00	2.82	1.00
time (sec)	N/A	0.018	0.007	0.160	1.416	0.392	0.000	0.131	0.953

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	62	0	0	0	0	-1
normalized size	1	1.00	1.03	1.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.042	0.328	0.000	0.684	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	135	321	0	0	0	0	-1
normalized size	1	1.00	1.02	2.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.475	0.324	0.000	0.581	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	99	0	0	0	0	-1
normalized size	1	1.00	0.93	1.80	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.052	0.377	0.000	0.475	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	23	1	0	66	-1
normalized size	1	1.00	0.76	0.64	0.70	0.03	0.00	2.00	-0.03
time (sec)	N/A	0.028	0.031	0.201	0.879	0.571	0.000	0.148	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	23	19	0	66	-1
normalized size	1	1.00	0.79	0.72	0.79	0.66	0.00	2.28	-0.03
time (sec)	N/A	0.026	0.023	0.239	0.431	0.483	0.000	0.136	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	78	96	0	0	0	0	-1
normalized size	1	1.00	0.77	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.074	0.423	0.000	0.806	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	66	0	0	0	0	-1
normalized size	1	1.00	1.08	1.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.077	0.192	0.000	0.504	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	45	0	0	0	0	-1
normalized size	1	1.00	1.06	2.65	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.039	0.291	0.000	0.475	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	34	19	1	0	40	-1
normalized size	1	1.00	1.18	2.00	1.12	0.06	0.00	2.35	-0.06
time (sec)	N/A	0.022	0.009	0.145	0.824	0.486	0.000	0.154	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	16	17	17	0	39	-1
normalized size	1	1.00	1.20	1.07	1.13	1.13	0.00	2.60	-0.07
time (sec)	N/A	0.019	0.009	0.173	1.342	0.778	0.000	0.116	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	61	0	0	0	0	-1
normalized size	1	1.00	1.03	1.56	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.049	0.341	0.000	0.654	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	105	100	0	0	0	0	633
normalized size	1	1.00	0.36	0.35	0.00	0.00	0.00	0.00	2.20
time (sec)	N/A	0.478	0.129	0.786	0.000	0.000	0.000	0.000	5.157

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	105	94	0	0	0	0	633
normalized size	1	1.00	0.36	0.33	0.00	0.00	0.00	0.00	2.20
time (sec)	N/A	0.233	0.093	0.702	0.000	0.000	0.000	0.000	5.894

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	133	216	0	602	330	275	291
normalized size	1	1.00	1.46	2.37	0.00	6.62	3.63	3.02	3.20
time (sec)	N/A	0.130	0.995	0.054	0.000	0.538	3.261	0.140	3.431

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	147	212	0	602	405	275	295
normalized size	1	1.00	1.55	2.23	0.00	6.34	4.26	2.89	3.11
time (sec)	N/A	0.121	0.614	0.066	0.000	0.569	3.719	0.165	3.416

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	485	121	121	0	771	0	0	1563
normalized size	1	1.34	0.34	0.34	0.00	2.14	0.00	0.00	4.33
time (sec)	N/A	1.037	0.240	0.090	0.000	0.913	0.000	0.000	7.553

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	109	127	0	779	0	0	1487
normalized size	1	1.00	1.08	1.26	0.00	7.71	0.00	0.00	14.72
time (sec)	N/A	0.125	0.199	0.088	0.000	0.506	0.000	0.000	8.901

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	45	37	0	590	0	281	205
normalized size	1	1.00	0.26	0.21	0.00	3.35	0.00	1.60	1.16
time (sec)	N/A	0.155	0.076	0.073	0.000	0.477	0.000	0.215	1.010

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	100	45	115	75	43	61
normalized size	1	1.00	0.96	4.00	1.80	4.60	3.00	1.72	2.44
time (sec)	N/A	0.017	0.107	0.077	0.444	0.629	1.874	0.119	0.118

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	139	156	0	0	0	0	-1
normalized size	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.904	0.287	0.111	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	132	177	0	0	0	1	844
normalized size	1	1.00	0.77	1.04	0.00	0.00	0.00	0.01	4.94
time (sec)	N/A	0.226	0.218	0.095	0.000	0.000	0.000	0.320	58.392

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	158	233	0	0	0	1	-1
normalized size	1	1.00	0.64	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	0.282	0.102	0.000	0.000	0.000	0.839	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	139	150	0	0	0	0	-1
normalized size	1	1.00	0.28	0.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.625	0.269	0.090	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	132	183	0	0	0	1	855
normalized size	1	1.00	0.75	1.05	0.00	0.00	0.00	0.01	4.89
time (sec)	N/A	0.231	0.174	0.097	0.000	0.000	0.000	0.354	57.402

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	158	239	0	0	0	1	-1
normalized size	1	1.00	0.74	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.218	0.095	0.000	0.000	0.000	0.833	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	445	62	0	3228	0	0	-1
normalized size	1	1.00	2.00	0.28	0.00	14.48	0.00	0.00	-0.00
time (sec)	N/A	0.562	0.104	0.074	0.000	1.045	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	208	0	158	0	140	337
normalized size	1	1.00	0.82	2.51	0.00	1.90	0.00	1.69	4.06
time (sec)	N/A	0.105	0.464	0.087	0.000	1.593	0.000	0.140	2.555

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	127	47	0	3773	0	1	-1
normalized size	1	1.00	0.98	0.36	0.00	29.25	0.00	0.01	-0.01
time (sec)	N/A	0.185	0.138	0.090	0.000	1.281	0.000	0.139	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	445	64	0	3260	0	0	-1
normalized size	1	1.00	2.17	0.31	0.00	15.90	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.096	0.073	0.000	2.123	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	111	426	0	694	632	10	329
normalized size	1	1.00	1.56	6.00	0.00	9.77	8.90	0.14	4.63
time (sec)	N/A	0.122	0.234	0.089	0.000	1.666	22.047	0.141	4.524

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	136	0	713	0	0	271
normalized size	1	1.00	0.93	1.97	0.00	10.33	0.00	0.00	3.93
time (sec)	N/A	0.077	0.455	0.125	0.000	1.874	0.000	0.000	2.628

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	47	0	23	27
normalized size	1	1.00	1.00	0.93	1.53	3.13	0.00	1.53	1.80
time (sec)	N/A	0.033	0.009	0.093	0.300	1.143	0.000	1.001	1.112

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	42	0	357	0	0	-1
normalized size	1	1.00	1.00	1.08	0.00	9.15	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.026	0.082	0.000	1.205	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	31	0	248	0	0	-1
normalized size	1	1.00	1.00	1.19	0.00	9.54	0.00	0.00	-0.04
time (sec)	N/A	0.064	0.017	0.100	0.000	3.996	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	63	0	0	-1
normalized size	1	1.00	1.00	0.92	0.00	4.85	0.00	0.00	-0.08
time (sec)	N/A	0.041	0.011	0.094	0.000	0.486	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	21	12	7	1	0	38	-1
normalized size	1	1.00	1.62	0.92	0.54	0.08	0.00	2.92	-0.08
time (sec)	N/A	0.061	0.012	0.174	0.428	0.457	0.000	0.722	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	145	150	0	1138	0	191	1173
normalized size	1	1.00	0.95	0.98	0.00	7.44	0.00	1.25	7.67
time (sec)	N/A	0.235	1.419	0.140	0.000	1.445	0.000	8.022	0.915

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	0	-1
normalized size	1	1.00	1.00	0.75	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	0.021	0.111	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	0	1648	0	0	-1
normalized size	1	1.00	1.00	0.76	0.00	36.62	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.022	0.087	0.000	1.428	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	113	0	0	-1
normalized size	1	1.00	1.00	0.83	0.00	3.90	0.00	0.00	-0.03
time (sec)	N/A	0.087	0.023	0.072	0.000	0.480	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	0	156	0	0	-1
normalized size	1	1.00	0.96	0.81	0.00	3.32	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.022	0.046	0.000	0.474	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [62] had the largest ratio of [.7500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	16	0.188
2	A	2	2	1.00	16	0.125
3	A	2	2	1.00	16	0.125
4	A	3	2	1.00	16	0.125
5	A	3	2	1.00	16	0.125
6	A	4	3	1.00	15	0.200
7	A	4	3	1.00	15	0.200
8	A	3	3	1.00	15	0.200
9	A	2	2	1.00	13	0.154
10	A	4	4	1.00	13	0.308
11	A	5	5	1.00	15	0.333
12	A	6	6	1.00	15	0.400
13	A	6	6	1.00	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	5	5	1.00	15	0.333
15	A	4	4	1.00	15	0.267
16	A	2	2	1.00	10	0.200
17	A	4	3	1.00	15	0.200
18	A	4	3	1.00	15	0.200
19	A	7	7	1.00	13	0.538
20	A	4	3	1.00	15	0.200
21	A	6	6	1.00	15	0.400
22	A	4	3	1.00	15	0.200
23	A	5	5	1.00	15	0.333
24	A	3	3	1.00	15	0.200
25	A	3	3	1.00	15	0.200
26	A	2	2	1.00	13	0.154
27	A	2	2	1.00	10	0.200
28	A	4	4	1.00	13	0.308
29	A	3	3	1.00	15	0.200
30	A	5	5	1.00	15	0.333
31	A	4	3	1.00	15	0.200
32	A	6	6	1.00	15	0.400
33	A	4	4	1.00	10	0.400
34	A	5	5	1.00	10	0.500
35	A	2	2	1.00	8	0.250
36	A	4	4	1.00	8	0.500
37	A	5	5	1.00	8	0.625
38	A	3	3	1.00	10	0.300
39	A	3	2	1.00	10	0.200
40	A	3	2	1.00	10	0.200
41	A	2	2	1.00	12	0.167
42	A	1	1	1.00	10	0.100
43	A	3	3	1.00	12	0.250
44	A	3	3	1.00	10	0.300
45	A	2	2	1.00	12	0.167
46	A	6	6	1.00	12	0.500
47	A	4	4	1.00	10	0.400
48	A	4	4	1.00	12	0.333
49	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	6	6	1.00	12	0.500
51	A	2	2	1.00	12	0.167
52	A	1	1	1.00	10	0.100
53	A	3	3	1.00	12	0.250
54	A	3	3	1.00	10	0.300
55	A	2	2	1.00	12	0.167
56	A	8	3	1.00	10	0.300
57	A	8	3	1.00	11	0.273
58	A	7	5	1.00	8	0.625
59	A	7	5	1.00	10	0.500
60	A	10	6	1.34	10	0.600
61	A	4	3	1.00	11	0.273
62	A	10	6	1.00	8	0.750
63	A	3	3	1.00	10	0.300
64	A	12	3	1.00	10	0.300
65	A	7	3	1.00	10	0.300
66	A	9	3	1.00	10	0.300
67	A	12	3	1.00	11	0.273
68	A	7	3	1.00	11	0.273
69	A	9	3	1.00	11	0.273
70	A	11	5	1.00	8	0.625
71	A	7	3	1.00	8	0.375
72	A	9	3	1.00	8	0.375
73	A	11	5	1.00	10	0.500
74	A	8	6	1.00	10	0.600
75	A	10	6	1.00	10	0.600
76	A	4	4	1.00	11	0.364
77	A	4	4	1.00	15	0.267
78	A	3	3	1.00	15	0.200
79	A	3	3	1.00	13	0.231
80	A	4	4	1.00	15	0.267
81	A	11	10	1.00	15	0.667
82	A	4	4	1.00	15	0.267
83	A	5	5	1.00	15	0.333
84	A	4	4	1.00	15	0.267
85	A	5	5	1.00	15	0.333

Chapter 3

Listing of integrals

3.1 $\int \frac{\sinh^4(x)}{a-a\cosh^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}-\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] 1/2*x/a-1/2*cosh(x)*sinh(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{x}{2a}-\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a - a*Cosh[x]^2),x]
```

```
[Out] x/(2*a) - (Cosh[x]*Sinh[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a - a\cosh^2(x)}\,dx = -\frac{\int \sinh^2(x)\,dx}{a}$$
$$= -\frac{\cosh(x)\sinh(x)}{2a} + \frac{\int 1\,dx}{2a}$$
$$= \frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{\frac{1}{4}\sinh(2x) - \frac{x}{2}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a - a*Cosh[x]^2),x]
[Out] -((-1/2*x + Sinh[2*x]/4)/a)
```

fricas [A] time = 0.51, size = 14, normalized size = 0.70

$$-\frac{\cosh(x)\sinh(x) - x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a-a*cosh(x)^2),x, algorithm="fricas")
[Out] -1/2*(cosh(x)*sinh(x) - x)/a
```

giac [A] time = 0.13, size = 26, normalized size = 1.30

$$-\frac{\left(2\,e^{(2\,x)} - 1\right)e^{(-2\,x)} - 4\,x + e^{(2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a-a*cosh(x)^2),x, algorithm="giac")
[Out] -1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))/a
```

maple [B] time = 0.09, size = 78, normalized size = 3.90

$$-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a} + \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a-a*cosh(x)^2),x)
[Out] -1/2/a/(tanh(1/2*x)-1)^2-1/2/a/(tanh(1/2*x)-1)-1/2/a*ln(tanh(1/2*x)-1)+1/2/
a/(tanh(1/2*x)+1)^2-1/2/a/(tanh(1/2*x)+1)+1/2/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 25, normalized size = 1.25

$$\frac{x}{2\,a} - \frac{e^{(2\,x)}}{8\,a} + \frac{e^{(-2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a-a*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/a - 1/8*e^(2*x)/a + 1/8*e^(-2*x)/a
```

mupad [B] time = 0.94, size = 25, normalized size = 1.25

$$\frac{e^{-2x}}{8a} - \frac{e^{2x}}{8a} + \frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a - a*cosh(x)^2),x)
```

```
[Out] exp(-2*x)/(8*a) - exp(2*x)/(8*a) + x/(2*a)
```

sympy [B] time = 2.26, size = 153, normalized size = 7.65

$$\frac{x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{2x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{x}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{}{2a\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a-a*cosh(x)**2),x)
```

```
[Out] x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 2*x*tanh(x/2)*
*2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + x/(2*a*tanh(x/2)**4 - 4*a*
tanh(x/2)**2 + 2*a) - 2*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 +
 2*a) - 2*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

3.2 $\int \frac{\sinh^3(x)}{a-a\cosh^2(x)}\,dx$

Optimal. Leaf size=7

$$-\frac{\cosh(x)}{a}$$

[Out] -cosh(x)/a

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2638}

$$-\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^3/(a - a*Cosh[x]^2),x]

[Out] -(Cosh[x]/a)

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a-a\cosh^2(x)}\,dx &= -\frac{\int \sinh(x)\,dx}{a}\\ &= -\frac{\cosh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^3/(a - a*Cosh[x]^2),x]

[Out] -(Cosh[x]/a)

fricas [A] time = 0.50, size = 7, normalized size = 1.00

$$-\frac{\cosh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a-a*cosh(x)^2),x, algorithm="fricas")

```
[Out] -cosh(x)/a
```

giac [A] time = 0.11, size = 12, normalized size = 1.71

$$-\frac{e^{(-x)} + e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a-a*cosh(x)^2),x, algorithm="giac")
[Out] -1/2*(e^(-x) + e^x)/a
```

maple [A] time = 0.06, size = 8, normalized size = 1.14

$$-\frac{\cosh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a-a*cosh(x)^2),x)
[Out] -cosh(x)/a
```

maxima [B] time = 0.31, size = 17, normalized size = 2.43

$$-\frac{e^{(-x)}}{2\,a} - \frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a-a*cosh(x)^2),x, algorithm="maxima")
[Out] -1/2*e^(-x)/a - 1/2*e^x/a
```

mupad [B] time = 0.91, size = 7, normalized size = 1.00

$$-\frac{\cosh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a - a*cosh(x)^2),x)
[Out] -cosh(x)/a
```

sympy [A] time = 1.30, size = 10, normalized size = 1.43

$$\frac{2}{a\tanh^{2}\left(\frac{x}{2}\right) - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a-a*cosh(x)**2),x)
[Out] 2/(a*tanh(x/2)**2 - a)
```

3.3 $\int \frac{\sinh^2(x)}{a-a\cosh^2(x)}\,dx$

Optimal. Leaf size=6

$$-\frac{x}{a}$$

[Out] -x/a

Rubi [A] time = 0.04, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 8}

$$-\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^2/(a - a*Cosh[x]^2),x]

[Out] -(x/a)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a-a\cosh^2(x)}\,dx &= -\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$-\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^2/(a - a*Cosh[x]^2),x]

[Out] -(x/a)

fricas [A] time = 0.45, size = 6, normalized size = 1.00

$$-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a-a*cosh(x)^2),x, algorithm="fricas")

[Out] -x/a

giac [A] time = 0.13, size = 6, normalized size = 1.00

$$-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a-a*cosh(x)^2),x, algorithm="giac")
[Out] -x/a
```

maple [C] time = 0.08, size = 11, normalized size = 1.83

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a-a*cosh(x)^2),x)
[Out] -2/a*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.32, size = 6, normalized size = 1.00

$$-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a-a*cosh(x)^2),x, algorithm="maxima")
[Out] -x/a
```

mupad [B] time = 0.03, size = 6, normalized size = 1.00

$$-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a - a*cosh(x)^2),x)
[Out] -x/a
```

sympy [A] time = 0.76, size = 3, normalized size = 0.50

$$-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a-a*cosh(x)**2),x)
[Out] -x/a
```

3.4 $\int \frac{\operatorname{csch}^2(x)}{a-a\cosh^2(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\coth^3(x)}{3a}-\frac{\coth(x)}{a}$$

```
[Out] -coth(x)/a+1/3*coth(x)^3/a
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\coth^3(x)}{3a}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a - a*Cosh[x]^2),x]
```

```
[Out] -(Coth[x]/a) + Coth[x]^3/(3*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a-a\cosh^2(x)}\,dx &= -\frac{\int \operatorname{csch}^4(x)\,dx}{a}\\ &= -\frac{i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth(x)\right)}{a}\\ &= -\frac{\coth(x)}{a}+\frac{\coth^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.16

$$-\frac{\frac{2\coth(x)}{3}-\frac{1}{3}\coth(x)\operatorname{csch}^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a - a*Cosh[x]^2),x]
```

```
[Out] -(((2*Coth[x])/3 - (Coth[x]*Csch[x]^2)/3)/a)
```

fricas [B] time = 0.43, size = 100, normalized size = 5.26

$$\frac{8\,(\cosh(x)+2\,\sinh(x))}{3\left(a\cosh(x)^5+5\,a\cosh(x)\sinh(x)^4+a\sinh(x)^5-3\,a\cosh(x)^3+\left(10\,a\cosh(x)^2-3\,a\right)\sinh(x)^3+\left(10\,a\cosh(x)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a-a*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 8/3*(cosh(x) + 2*sinh(x))/(a*cosh(x)^5 + 5*a*cosh(x)*sinh(x)^4 + a*sinh(x)^
5 - 3*a*cosh(x)^3 + (10*a*cosh(x)^2 - 3*a)*sinh(x)^3 + (10*a*cosh(x)^3 - 9*
a*cosh(x))*sinh(x)^2 + 2*a*cosh(x) + (5*a*cosh(x)^4 - 9*a*cosh(x)^2 + 4*a)*
sinh(x))
```

giac [A] time = 0.13, size = 21, normalized size = 1.11

$$\frac{4\left(3e^{(2x)}-1\right)}{3a\left(e^{(2x)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a-a*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 4/3*(3*e^(2*x) - 1)/(a*(e^(2*x) - 1)^3)
```

maple [B] time = 0.11, size = 37, normalized size = 1.95

$$\frac{\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-3\tanh\left(\frac{x}{2}\right)+\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}-\frac{3}{\tanh\left(\frac{x}{2}\right)}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a-a*cosh(x)^2),x)
```

```
[Out] 1/8/a*(1/3*tanh(1/2*x)^3-3*tanh(1/2*x)+1/3/tanh(1/2*x)^3-3/tanh(1/2*x))
```

maxima [B] time = 0.31, size = 61, normalized size = 3.21

$$-\frac{4e^{(-2x)}}{3ae^{(-2x)}-3ae^{(-4x)}+ae^{(-6x)}-a}+\frac{4}{3\left(3ae^{(-2x)}-3ae^{(-4x)}+ae^{(-6x)}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a-a*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -4*e^(-2*x)/(3*a*e^(-2*x) - 3*a*e^(-4*x) + a*e^(-6*x) - a) + 4/3/(3*a*e^(-2
*x) - 3*a*e^(-4*x) + a*e^(-6*x) - a)
```

mupad [B] time = 0.91, size = 21, normalized size = 1.11

$$\frac{4\left(3e^{2x}-1\right)}{3a\left(e^{2x}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a - a*cosh(x)^2)),x)
```

```
[Out] (4*(3*exp(2*x) - 1))/(3*a*(exp(2*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{csch}^{2}(x)}{\cosh^{2}(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a-a*cosh(x)**2),x)
```

```
[Out] -Integral(csch(x)**2/(cosh(x)**2 - 1), x)/a
```

3.5 $\int \frac{\operatorname{csch}^4(x)}{a-a\cosh^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\coth^5(x)}{5a}-\frac{2\coth^3(x)}{3a}+\frac{\coth(x)}{a}$$

```
[Out] coth(x)/a-2/3*coth(x)^3/a+1/5*coth(x)^5/a
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\coth^5(x)}{5a}-\frac{2\coth^3(x)}{3a}+\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a - a*Cosh[x]^2),x]
```

```
[Out] Coth[x]/a - (2*Coth[x]^3)/(3*a) + Coth[x]^5/(5*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{a-a\cosh^2(x)}\,dx &= -\frac{\int \operatorname{csch}^6(x)\,dx}{a}\\ &= \frac{i\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,-i\coth(x)\right)}{a}\\ &= \frac{\coth(x)}{a}-\frac{2\coth^3(x)}{3a}+\frac{\coth^5(x)}{5a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.10

$$-\frac{-\frac{8\coth(x)}{15}-\frac{1}{5}\coth(x)\operatorname{csch}^4(x)+\frac{4}{15}\coth(x)\operatorname{csch}^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a - a*Cosh[x]^2),x]
```

```
[Out] -(((-8*Coth[x])/15 + (4*Coth[x]*Csch[x]^2)/15 - (Coth[x]*Csch[x]^4)/5)/a)
```

fricas [B] time = 0.39, size = 216, normalized size = 7.45

$$15\left(a\cosh(x)^8+8\,a\cosh(x)\sinh(x)^7+a\sinh(x)^8-5\,a\cosh(x)^6+\left(28\,a\cosh(x)^2-5\,a\right)\sinh(x)^6+2\left(28\,a\cosh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a-a*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 16/15*(11*cosh(x)^2 + 18*cosh(x)*sinh(x) + 11*sinh(x)^2 - 5)/(a*cosh(x)^8 +
 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 - 5*a*cosh(x)^6 + (28*a*cosh(x)^2 - 5*
a)*sinh(x)^6 + 2*(28*a*cosh(x)^3 - 15*a*cosh(x))*sinh(x)^5 + 10*a*cosh(x)^4
 + 5*(14*a*cosh(x)^4 - 15*a*cosh(x)^2 + 2*a)*sinh(x)^4 + 4*(14*a*cosh(x)^5
- 25*a*cosh(x)^3 + 10*a*cosh(x))*sinh(x)^3 - 11*a*cosh(x)^2 + (28*a*cosh(x)
^6 - 75*a*cosh(x)^4 + 60*a*cosh(x)^2 - 11*a)*sinh(x)^2 + 2*(4*a*cosh(x)^7 -
 15*a*cosh(x)^5 + 20*a*cosh(x)^3 - 9*a*cosh(x))*sinh(x) + 5*a)
```

giac [A] time = 0.14, size = 27, normalized size = 0.93

$$\frac{16\left(10\,e^{(4\,x)} - 5\,e^{(2\,x)} + 1\right)}{15\,a\left(e^{(2\,x)} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a-a*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(a*(e^(2*x) - 1)^5)
```

maple [B] time = 0.11, size = 53, normalized size = 1.83

$$\frac{\frac{\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{5} - \frac{5\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3} + 10\tanh\left(\frac{x}{2}\right) - \frac{5}{3\tanh\left(\frac{x}{2}\right)^{3}} + \frac{10}{\tanh\left(\frac{x}{2}\right)} + \frac{1}{5\tanh\left(\frac{x}{2}\right)^{5}}}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a-a*cosh(x)^2),x)
```

```
[Out] 1/32/a*(1/5*tanh(1/2*x)^5-5/3*tanh(1/2*x)^3+10*tanh(1/2*x)-5/3/tanh(1/2*x)^
3+10/tanh(1/2*x)+1/5/tanh(1/2*x)^5)
```

maxima [B] time = 0.32, size = 135, normalized size = 4.66

$$\frac{16\,e^{(-2\,x)}}{3\left(5\,ae^{(-2\,x)} - 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} - 5\,ae^{(-8\,x)} + ae^{(-10\,x)} - a\right)} - \frac{32\,e^{(-4\,x)}}{3\left(5\,ae^{(-2\,x)} - 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} - 5\,ae^{(-8\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a-a*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*a*e^(-2*x) - 10*a*e^(-4*x) + 10*a*e^(-6*x) - 5*a*e^(-8*x)
+ a*e^(-10*x) - a) - 32/3*e^(-4*x)/(5*a*e^(-2*x) - 10*a*e^(-4*x) + 10*a*e^(
-6*x) - 5*a*e^(-8*x) + a*e^(-10*x) - a) - 16/15/(5*a*e^(-2*x) - 10*a*e^(-4*
x) + 10*a*e^(-6*x) - 5*a*e^(-8*x) + a*e^(-10*x) - a)
```

mupad [B] time = 0.92, size = 27, normalized size = 0.93

$$\frac{16\,\left(10\,e^{4\,x} - 5\,e^{2\,x} + 1\right)}{15\,a\,\left(e^{2\,x} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a - a*cosh(x)^2)),x)
```

```
[Out] (16*(10*exp(4*x) - 5*exp(2*x) + 1))/(15*a*(exp(2*x) - 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{csch}^4(x)}{\cosh^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a-a*cosh(x)**2),x)
```

```
[Out] -Integral(csch(x)**4/(cosh(x)**2 - 1), x)/a
```

3.6 $\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=78

$$\frac{(a^2+3ab+3b^2)\cosh(x)}{b^3}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(a+3b)\cosh^3(x)}{3b^2}+\frac{\cosh^5(x)}{5b}$$

```
[Out] (a^2+3*a*b+3*b^2)*cosh(x)/b^3-1/3*(a+3*b)*cosh(x)^3/b^2+1/5*cosh(x)^5/b-(a+
b)^3*arctan(cosh(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 390, 205}

$$\frac{(a^2+3ab+3b^2)\cosh(x)}{b^3}-\frac{(a+3b)\cosh^3(x)}{3b^2}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{\cosh^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^7/(a + b*Cosh[x]^2),x]
```

```
[Out] -(((a + b)^3*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2))) + ((a^2
+ 3*a*b + 3*b^2)*Cosh[x])/b^3 - ((a + 3*b)*Cosh[x]^3)/(3*b^2) + Cosh[x]^5/(
5*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx = -\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a+bx^2}\,dx, x, \cosh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{a^2+3ab+3b^2}{b^3}+\frac{(a+3b)x^2}{b^2}-\frac{x^4}{b}+\frac{a^3+3a^2b+3ab^2+b^3}{b^3\left(a+bx^2\right)}\right)dx, x, \cosh(x\right.$$
$$= \frac{\left(a^2+3ab+3b^2\right)\cosh(x)}{b^3}-\frac{(a+3b)\cosh^3(x)}{3b^2}+\frac{\cosh^5(x)}{5b}-\frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x\right.}{b^3}$$
$$= -\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{\left(a^2+3ab+3b^2\right)\cosh(x)}{b^3}-\frac{(a+3b)\cosh^3(x)}{3b^2}+\frac{\cosh^5(x)}{5b}$$

Mathematica [C] time = 0.25, size = 148, normalized size = 1.90

$$\frac{\left(8a^2+22ab+19b^2\right)\cosh(x)}{8b^3}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(4a+9b)\cos}{48b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^7/(a + b*Cosh[x]^2),x]
```

```
[Out] -(((a + b)^3*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/(Sqrt[a]*
b^(7/2))) - ((a + b)^3*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])
/(Sqrt[a]*b^(7/2)) + ((8*a^2 + 22*a*b + 19*b^2)*Cosh[x])/(8*b^3) - ((4*a +
9*b)*Cosh[3*x])/(48*b^2) + Cosh[5*x]/(80*b)
```

fricas [B] time = 0.45, size = 2346, normalized size = 30.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/480*(3*a*b^3*cosh(x)^10 + 30*a*b^3*cosh(x)*sinh(x)^9 + 3*a*b^3*sinh(x)^1
0 - 5*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^8 + 5*(27*a*b^3*cosh(x)^2 - 4*a^2*b^2 -
 9*a*b^3)*sinh(x)^8 + 40*(9*a*b^3*cosh(x)^3 - (4*a^2*b^2 + 9*a*b^3)*cosh(x)
)*sinh(x)^7 + 30*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^6 + 10*(63*a*b^3
*cosh(x)^4 + 24*a^3*b + 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 + 9*a*b^3)*co
sh(x)^2)*sinh(x)^6 + 4*(189*a*b^3*cosh(x)^5 - 70*(4*a^2*b^2 + 9*a*b^3)*cosh
(x)^3 + 45*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x))*sinh(x)^5 + 30*(8*a^3
*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^4 + 10*(63*a*b^3*cosh(x)^6 - 35*(4*a^2*
b^2 + 9*a*b^3)*cosh(x)^4 + 24*a^3*b + 66*a^2*b^2 + 57*a*b^3 + 45*(8*a^3*b +
 22*a^2*b^2 + 19*a*b^3)*cosh(x)^2)*sinh(x)^4 + 3*a*b^3 + 40*(9*a*b^3*cosh(x
)^7 - 7*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^5 + 15*(8*a^3*b + 22*a^2*b^2 + 19*a*b
^3)*cosh(x)^3 + 3*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x))*sinh(x)^3 - 5*
(4*a^2*b^2 + 9*a*b^3)*cosh(x)^2 + 5*(27*a*b^3*cosh(x)^8 - 28*(4*a^2*b^2 + 9
*a*b^3)*cosh(x)^6 + 90*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^4 - 4*a^2*
b^2 - 9*a*b^3 + 36*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^2)*sinh(x)^2 -
 240*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(x)^4*sinh(x) + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3*s
inh(x)^2 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2*sinh(x)^3 + 5*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*sinh(x)^5)*sqrt(-a*b)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh
(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(
b*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 4*(cosh(x)^3 + 3*cosh(x)*sinh(x)
```

```
^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))*sqrt(-a*b) + b)/(b*co
sh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(
3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*si
nh(x) + b)) + 10*(3*a*b^3*cosh(x)^9 - 4*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^7 + 1
8*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^5 + 12*(8*a^3*b + 22*a^2*b^2 +
19*a*b^3)*cosh(x)^3 - (4*a^2*b^2 + 9*a*b^3)*cosh(x))*sinh(x))/(a*b^4*cosh(x
)^5 + 5*a*b^4*cosh(x)^4*sinh(x) + 10*a*b^4*cosh(x)^3*sinh(x)^2 + 10*a*b^4*c
osh(x)^2*sinh(x)^3 + 5*a*b^4*cosh(x)*sinh(x)^4 + a*b^4*sinh(x)^5), 1/480*(3
*a*b^3*cosh(x)^10 + 30*a*b^3*cosh(x)*sinh(x)^9 + 3*a*b^3*sinh(x)^10 - 5*(4*
a^2*b^2 + 9*a*b^3)*cosh(x)^8 + 5*(27*a*b^3*cosh(x)^2 - 4*a^2*b^2 - 9*a*b^3)
*sinh(x)^8 + 40*(9*a*b^3*cosh(x)^3 - (4*a^2*b^2 + 9*a*b^3)*cosh(x))*sinh(x)
^7 + 30*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^6 + 10*(63*a*b^3*cosh(x)^
4 + 24*a^3*b + 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^2)*
sinh(x)^6 + 4*(189*a*b^3*cosh(x)^5 - 70*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^3 + 4
5*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x))*sinh(x)^5 + 30*(8*a^3*b + 22*a
^2*b^2 + 19*a*b^3)*cosh(x)^4 + 10*(63*a*b^3*cosh(x)^6 - 35*(4*a^2*b^2 + 9*a
*b^3)*cosh(x)^4 + 24*a^3*b + 66*a^2*b^2 + 57*a*b^3 + 45*(8*a^3*b + 22*a^2*b
^2 + 19*a*b^3)*cosh(x)^2)*sinh(x)^4 + 3*a*b^3 + 40*(9*a*b^3*cosh(x)^7 - 7*(
4*a^2*b^2 + 9*a*b^3)*cosh(x)^5 + 15*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(
x)^3 + 3*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x))*sinh(x)^3 - 5*(4*a^2*b^
2 + 9*a*b^3)*cosh(x)^2 + 5*(27*a*b^3*cosh(x)^8 - 28*(4*a^2*b^2 + 9*a*b^3)*c
osh(x)^6 + 90*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^4 - 4*a^2*b^2 - 9*a
*b^3 + 36*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^2)*sinh(x)^2 - 480*((a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(x)^4*sinh(x) + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3*sinh(x)^2
+ 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2*sinh(x)^3 + 5*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(
x)^5)*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(x) + sinh(x))/a) + 480*((a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(x)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
x)^4*sinh(x) + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3*sinh(x)^2 + 10*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2*sinh(x)^3 + 5*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*cosh(x)*sinh(x)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^5)
*sqrt(a*b)*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 +
(4*a + b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(a*b)/(a*b)) + 1
0*(3*a*b^3*cosh(x)^9 - 4*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^7 + 18*(8*a^3*b + 22
*a^2*b^2 + 19*a*b^3)*cosh(x)^5 + 12*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(
x)^3 - (4*a^2*b^2 + 9*a*b^3)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^5 + 5*a*b^4*c
osh(x)^4*sinh(x) + 10*a*b^4*cosh(x)^3*sinh(x)^2 + 10*a*b^4*cosh(x)^2*sinh(x
)^3 + 5*a*b^4*cosh(x)*sinh(x)^4 + a*b^4*sinh(x)^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-54,60]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-64,24]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.09, size = 395, normalized size = 5.06

$$-\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{a}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{7}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{a}{3b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{}{4b(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a+b*cosh(x)^2),x)
```

```
[Out] -1/5/b/(tanh(1/2*x)-1)^5-1/2/b/(tanh(1/2*x)-1)^4+1/2/b^2/(tanh(1/2*x)-1)^2*
a+7/8/b/(tanh(1/2*x)-1)^2+1/3/b^2/(tanh(1/2*x)-1)^3*a+1/4/b/(tanh(1/2*x)-1)
^3-1/b^3/(tanh(1/2*x)-1)*a^2-5/2/b^2/(tanh(1/2*x)-1)*a-15/8/b/(tanh(1/2*x)-
1)+1/5/b/(tanh(1/2*x)+1)^5-1/2/b/(tanh(1/2*x)+1)^4+1/2/b^2/(tanh(1/2*x)+1)^
2*a+7/8/b/(tanh(1/2*x)+1)^2-1/3/b^2/(tanh(1/2*x)+1)^3*a-1/4/b/(tanh(1/2*x)+
1)^3+1/b^3/(tanh(1/2*x)+1)*a^2+5/2/b^2/(tanh(1/2*x)+1)*a+15/8/b/(tanh(1/2*x
)+1)-1/b^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a+2*b)/(a*b)^(1/
2))*a^3-3/b^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a+2*b)/(a*b)^
(1/2))*a^2-3/b/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a+2*b)/(a*b)
^(1/2))*a-1/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a+2*b)/(a*b)^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,b^{2}e^{(10\,x)}+3\,b^{2}-5\left(4\,ab+9\,b^{2}\right)e^{(8\,x)}+30\left(8\,a^{2}+22\,ab+19\,b^{2}\right)e^{(6\,x)}+30\left(8\,a^{2}+22\,ab+19\,b^{2}\right)e^{(4\,x)}-5\left(4\,ab+\right.\right.}{480\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*x) + 3*b^2 - 5*(4*a*b + 9*b^2)*e^(8*x) + 30*(8*a^2 + 22*
a*b + 19*b^2)*e^(6*x) + 30*(8*a^2 + 22*a*b + 19*b^2)*e^(4*x) - 5*(4*a*b + 9
*b^2)*e^(2*x))*e^(-5*x)/b^3 - 1/128*integrate(256*((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*e^(3*x) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^x)/(b^4*e^(4*x) + b^4 +
 2*(2*a*b^3 + b^4)*e^(2*x)), x)
```

mupad [B] time = 1.55, size = 805, normalized size = 10.32

$$\frac{e^{-5\,x}}{160\,b}+\frac{e^{5\,x}}{160\,b}+\frac{e^{-x}\,\left(8\,a^{2}+22\,a\,b+19\,b^{2}\right)}{16\,b^{3}}-\frac{\left(2\,\mathrm{atan}\left(\frac{e^{x}\,(a+b)^{3}\,\sqrt{a\,b^{7}}}{2\,a\,b^{3}\,\sqrt{(a+b)^{6}}}\right)-2\,\mathrm{atan}\left(\frac{2\,e^{3\,x}\left(a^{7}\,\sqrt{a\,b^{7}}+b^{7}\,\sqrt{a\,b^{7}}+7\,a\,b^{6}\,\sqrt{a\,b^{7}}+7\,a^{6}\,b\,\sqrt{\right.}{a\,b^{3}\,\sqrt{(a+b)^{6}}\,\left(\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a + b*cosh(x)^2),x)
```

```
[Out] exp(-5*x)/(160*b) + exp(5*x)/(160*b) + (exp(-x)*(22*a*b + 8*a^2 + 19*b^2))/
(16*b^3) - ((2*atan((exp(x)*(a + b)^3*(a*b^7)^(1/2))/(2*a*b^3*((a + b)^6)^(
1/2))) - 2*atan((2*exp(3*x)*(a^7*(a*b^7)^(1/2) + b^7*(a*b^7)^(1/2) + 7*a*b^
6*(a*b^7)^(1/2) + 7*a^6*b*(a*b^7)^(1/2) + 21*a^2*b^5*(a*b^7)^(1/2) + 35*a^3
*b^4*(a*b^7)^(1/2) + 35*a^4*b^3*(a*b^7)^(1/2) + 21*a^5*b^2*(a*b^7)^(1/2)))/
(a*b^3*((a + b)^6)^(1/2)*(16*a*b^3 + 16*a^3*b + 4*a^4 + 4*b^4 + 24*a^2*b^2)
) + (a*b^8*exp(x)*(a*b^7)^(1/2)*((4*(2*a*b^7*(6*a*b^5 + 6*a^5*b + a^6 + b^6
 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2) + 8*a^2*b^6*(6*a*b^5 + 6*a^5
*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2) + 12*a^3*b^5*(
6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2)
 + 8*a^4*b^4*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*
a^4*b^2)^(1/2) + 2*a^5*b^3*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20
*a^3*b^3 + 15*a^4*b^2)^(1/2)))/(a^2*b^15*(a + b)^3) + (2*(a^7*(a*b^7)^(1/2)
 + b^7*(a*b^7)^(1/2) + 7*a*b^6*(a*b^7)^(1/2) + 7*a^6*b*(a*b^7)^(1/2) + 21*a
^2*b^5*(a*b^7)^(1/2) + 35*a^3*b^4*(a*b^7)^(1/2) + 35*a^4*b^3*(a*b^7)^(1/2)
+ 21*a^5*b^2*(a*b^7)^(1/2)))/(a^2*b^11*(a*b^7)^(1/2)*((a + b)^6)^(1/2))))/(
16*a*b^3 + 16*a^3*b + 4*a^4 + 4*b^4 + 24*a^2*b^2))*(6*a*b^5 + 6*a^5*b + a^
6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2))/(2*(a*b^7)^(1/2)) -
```

```
(exp(-3*x)*(4*a + 9*b))/(96*b^2) - (exp(3*x)*(4*a + 9*b))/(96*b^2) + (exp(x
)*(22*a*b + 8*a^2 + 19*b^2))/(16*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**7/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.7 $\int \frac{\sinh^5(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=54

$$\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\cosh(x)}{b^2}+\frac{\cosh^3(x)}{3b}$$

```
[Out] -(a+2*b)*cosh(x)/b^2+1/3*cosh(x)^3/b+(a+b)^2*arctan(cosh(x)*b^(1/2)/a^(1/2)
)/b^(5/2)/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 390, 205}

$$-\frac{(a+2b)\cosh(x)}{b^2}+\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{\cosh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((a + 2*b
)*Cosh[x])/b^2 + Cosh[x]^3/(3*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5(x)}{a+b\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a+bx^2}\,dx,x,\cosh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2\left(a+bx^2\right)}\right)dx,x,\cosh(x)\right)\\
&= -\frac{(a+2b)\cosh(x)}{b^2}+\frac{\cosh^3(x)}{3b}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cosh(x)\right)}{b^2}\\
&= \frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\cosh(x)}{b^2}+\frac{\cosh^3(x)}{3b}
\end{aligned}$$

Mathematica [C] time = 0.19, size = 120, normalized size = 2.22

$$\frac{-3\sqrt{b}\,(4a+7b)\cosh(x)+\frac{12(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{12(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}+b^{3/2}\cosh(3x)}{12b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] ((12*(a + b)^2*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/Sqrt[a]
 + (12*(a + b)^2*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/Sqrt[
a] - 3*Sqrt[b]*(4*a + 7*b)*Cosh[x] + b^(3/2)*Cosh[3*x])/(12*b^(5/2))
```

fricas [B] time = 0.54, size = 1064, normalized size = 19.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/24*(a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 - 3*(4
*a^2*b + 7*a*b^2)*cosh(x)^4 + 3*(5*a*b^2*cosh(x)^2 - 4*a^2*b - 7*a*b^2)*sin
h(x)^4 + 4*(5*a*b^2*cosh(x)^3 - 3*(4*a^2*b + 7*a*b^2)*cosh(x))*sinh(x)^3 +
a*b^2 - 3*(4*a^2*b + 7*a*b^2)*cosh(x)^2 + 3*(5*a*b^2*cosh(x)^4 - 4*a^2*b -
7*a*b^2 - 6*(4*a^2*b + 7*a*b^2)*cosh(x)^2)*sinh(x)^2 - 12*((a^2 + 2*a*b + b
^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^2*sinh(x) + 3*(a^2 + 2*a*b +
b^2)*cosh(x)*sinh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^3)*sqrt(-a*b)*log((b*c
osh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*
(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (2*a - b)*cosh(x))*s
inh(x) - 4*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)
*sinh(x) + cosh(x))*sqrt(-a*b) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 +
b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2
 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 6*(a*b^2*cosh(x)^5 -
 2*(4*a^2*b + 7*a*b^2)*cosh(x)^3 - (4*a^2*b + 7*a*b^2)*cosh(x))*sinh(x))/(a
*b^3*cosh(x)^3 + 3*a*b^3*cosh(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*
b^3*sinh(x)^3), 1/24*(a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*s
inh(x)^6 - 3*(4*a^2*b + 7*a*b^2)*cosh(x)^4 + 3*(5*a*b^2*cosh(x)^2 - 4*a^2*b
 - 7*a*b^2)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 - 3*(4*a^2*b + 7*a*b^2)*cosh(x
))*sinh(x)^3 + a*b^2 - 3*(4*a^2*b + 7*a*b^2)*cosh(x)^2 + 3*(5*a*b^2*cosh(x)
^4 - 4*a^2*b - 7*a*b^2 - 6*(4*a^2*b + 7*a*b^2)*cosh(x)^2)*sinh(x)^2 + 24*((
a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^2*sinh(x) + 3*
(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^3)*sqrt
(a*b)*arctan(1/2*sqrt(a*b)*(cosh(x) + sinh(x))/a) - 24*((a^2 + 2*a*b + b^2)
*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^2*sinh(x) + 3*(a^2 + 2*a*b + b^2
)*cosh(x)*sinh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^3)*sqrt(a*b)*arctan(1/2*(
b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + b)*cosh(x) + (3*
b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(a*b)/(a*b)) + 6*(a*b^2*cosh(x)^5 - 2*(
4*a^2*b + 7*a*b^2)*cosh(x)^3 - (4*a^2*b + 7*a*b^2)*cosh(x))*sinh(x))/(a*b^3
*cosh(x)^3 + 3*a*b^3*cosh(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*b^3*
sinh(x)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-97,37]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-81,22]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.08, size = 214, normalized size = 3.96

$$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{a}{b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a+b*cosh(x)^2),x)
```

```
[Out] -1/3/b/(tanh(1/2*x)-1)^3-1/2/b/(tanh(1/2*x)-1)^2+1/b^2/(tanh(1/2*x)-1)*a+3/
2/b/(tanh(1/2*x)-1)+1/3/b/(tanh(1/2*x)+1)^3-1/2/b/(tanh(1/2*x)+1)^2-1/b^2/(
tanh(1/2*x)+1)*a-3/2/b/(tanh(1/2*x)+1)+1/b^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b
)*tanh(1/2*x)^2-2*a+2*b)/(a*b)^(1/2))*a^2+2/b/(a*b)^(1/2)*arctan(1/4*(2*(a+
b)*tanh(1/2*x)^2-2*a+2*b)/(a*b)^(1/2))*a+1/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*
tanh(1/2*x)^2-2*a+2*b)/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(be^{(6x)}-3(4a+7b)e^{(4x)}-3(4a+7b)e^{(2x)}+b\right)e^{(-3x)}}{24b^2}+\frac{1}{32}\int\frac{64\left(\left(a^2+2ab+b^2\right)e^{(3x)}-\left(a^2+2ab+b^2\right)e^{x}\right)}{b^3e^{(4x)}+b^3+2\left(2ab^2+b^3\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(b*e^(6*x) - 3*(4*a + 7*b)*e^(4*x) - 3*(4*a + 7*b)*e^(2*x) + b)*e^(-3*
x)/b^2 + 1/32*integrate(64*((a^2 + 2*a*b + b^2)*e^(3*x) - (a^2 + 2*a*b + b^
2)*e^x)/(b^3*e^(4*x) + b^3 + 2*(2*a*b^2 + b^3)*e^(2*x)), x)
```

mupad [B] time = 1.30, size = 548, normalized size = 10.15

$$\frac{e^{-3x}}{24b}+\frac{e^{3x}}{24b}-\frac{e^{-x}\,(4a+7b)}{8b^2}+\frac{\left(2\,\mathrm{atan}\left(\frac{e^x\,(a+b)^2\,\sqrt{a\,b^5}}{2\,a\,b^2\,\sqrt{(a+b)^4}}\right)-2\,\mathrm{atan}\left(\frac{a\,b^6\,e^x\left(\frac{4\left(6\,a^2\,b^4\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}+6\,a^3\,b^3\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a + b*cosh(x)^2),x)
```

```
[Out] exp(-3*x)/(24*b) + exp(3*x)/(24*b) - (exp(-x)*(4*a + 7*b))/(8*b^2) + ((2*at
an((exp(x)*(a + b)^2*(a*b^5)^(1/2))/(2*a*b^2*((a + b)^4)^(1/2))) - 2*atan((
a*b^6*exp(x)*((4*(6*a^2*b^4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/
2) + 6*a^3*b^3*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2) + 2*a^4*b^
2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2) + 2*a*b^5*(4*a*b^3 + 4*
a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2)))/(a^2*b^11*(a + b)^2) + (2*(a^5*(a*b^
5)^(1/2) + b^5*(a*b^5)^(1/2) + 5*a*b^4*(a*b^5)^(1/2) + 5*a^4*b*(a*b^5)^(1/2
) + 10*a^2*b^3*(a*b^5)^(1/2) + 10*a^3*b^2*(a*b^5)^(1/2)))/(a^2*b^8*(a*b^5)^
(1/2)*((a + b)^4)^(1/2)))*(a*b^5)^(1/2))/(12*a*b^2 + 12*a^2*b + 4*a^3 + 4*b
^3) + (2*exp(3*x)*(a^5*(a*b^5)^(1/2) + b^5*(a*b^5)^(1/2) + 5*a*b^4*(a*b^5)^
```

```
(1/2) + 5*a^4*b*(a*b^5)^(1/2) + 10*a^2*b^3*(a*b^5)^(1/2) + 10*a^3*b^2*(a*b^
5)^(1/2)))/(a*b^2*((a + b)^4)^(1/2)*(12*a*b^2 + 12*a^2*b + 4*a^3 + 4*b^3)))
)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2))/(2*(a*b^5)^(1/2)) - (e
xp(x)*(4*a + 7*b))/(8*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.8 $\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=36

$$\frac{\cosh(x)}{b}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] cosh(x)/b-(a+b)*arctan(cosh(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 388, 205}

$$\frac{\cosh(x)}{b}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] -(((a + b)*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + Cosh[x]/
b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx,x,\cosh(x)\right)\\
&=\frac{\cosh(x)}{b}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cosh(x)\right)}{b}\\
&=-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{\cosh(x)}{b}
\end{aligned}$$

Mathematica [C] time = 0.19, size = 83, normalized size = 2.31

$$\frac{\cosh(x)}{b}-\frac{(a+b)\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] -(((a + b)*(ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] + ArcTan[(S
qrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]]))/(Sqrt[a]*b^(3/2))) + Cosh[x]/b
```

fricas [B] time = 0.49, size = 416, normalized size = 11.56

$$\left[\frac{ab\cosh(x)^2+2\,ab\cosh(x)\sinh(x)+ab\sinh(x)^2-\sqrt{-ab}\,((a+b)\cosh(x)+(a+b)\sinh(x))\log\left(\frac{b\cosh(x)^4+4\,b}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - sqrt(-a*b)*((
a + b)*cosh(x) + (a + b)*sinh(x))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
+ b*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)
^2 + 4*(b*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 4*(cosh(x)^3 + 3*cosh(x)
*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))*sqrt(-a*b) +
b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)
^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cos
h(x))*sinh(x) + b)) + a*b)/(a*b^2*cosh(x) + a*b^2*sinh(x)), 1/2*(a*b*cosh(x
)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - 2*sqrt(a*b)*((a + b)*cosh(x)
+ (a + b)*sinh(x))*arctan(1/2*sqrt(a*b)*(cosh(x) + sinh(x))/a) + 2*sqrt(a*b
)*((a + b)*cosh(x) + (a + b)*sinh(x))*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)
*sinh(x)^2 + b*sinh(x)^3 + (4*a + b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + b)*si
nh(x))*sqrt(a*b)/(a*b)) + a*b)/(a*b^2*cosh(x) + a*b^2*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[81,-22]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[55,-12]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.08, size = 97, normalized size = 2.69

$$\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{\arctan\left(\frac{2(a+b)\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2a+2b}{4\sqrt{ab}}\right)a}{b\sqrt{ab}}-\frac{\arctan\left(\frac{2(a+b)\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2a+2b}{4\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*cosh(x)^2),x)
```

```
[Out] 1/b/(tanh(1/2*x)+1)-1/b/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a+2
*b)/(a*b)^(1/2))*a-1/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a+2*b)
/(a*b)^(1/2))-1/b/(tanh(1/2*x)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2x)}+1\right)e^{(-x)}}{2\,b}-\frac{1}{8}\int\frac{16\left((a+b)e^{(3x)}-(a+b)e^{x}\right)}{b^{2}e^{(4x)}+b^{2}+2\left(2\,ab+b^{2}\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x)/b - 1/8*integrate(16*((a + b)*e^(3*x) - (a + b)*e^
x)/(b^2*e^(4*x) + b^2 + 2*(2*a*b + b^2)*e^(2*x)), x)
```

mupad [B] time = 1.37, size = 257, normalized size = 7.14

$$\frac{e^{-x}}{2\,b}+\frac{e^{x}}{2\,b}+\frac{\left(2\,\mathrm{atan}\left(\frac{e^{3\,x}\left(a^{3}\,\sqrt{a\,b^{3}}+b^{3}\,\sqrt{a\,b^{3}}+3\,a\,b^{2}\,\sqrt{a\,b^{3}}+3\,a^{2}\,b\,\sqrt{a\,b^{3}}\right)}{2\,a\,b\,\left((a+b)^{2}\right)^{3/2}}+\frac{a\,b^{4}\,e^{x}\,\sqrt{a\,b^{3}}\left(\frac{8\left(a^{2}+2\,a\,b+b^{2}\right)^{3/2}}{a\,b^{6}\,(a+b)^{3}}+\frac{2\left(a^{3}\,\sqrt{a\,b^{3}}+b^{3}\,\sqrt{a\,b^{3}}+3\,a\,b^{2}\,\sqrt{a\,b^{3}}+3\,a \cdots\right.}{a^{2}\,b^{5}\,\sqrt{a\,b^{3}}\,\left((a+b)^{2}\right)^{3/2}}\right.}{4}\right.\right.}{2\,\sqrt{a\,b^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*cosh(x)^2),x)
```

```
[Out] exp(-x)/(2*b) + exp(x)/(2*b) + ((2*atan((exp(3*x)*(a^3*(a*b^3)^(1/2) + b^3*
(a*b^3)^(1/2) + 3*a*b^2*(a*b^3)^(1/2) + 3*a^2*b*(a*b^3)^(1/2)))/(2*a*b*((a
+ b)^2)^(3/2)) + (a*b^4*exp(x)*(a*b^3)^(1/2)*((8*(2*a*b + a^2 + b^2)^(3/2))
/(a*b^6*(a + b)^3) + (2*(a^3*(a*b^3)^(1/2) + b^3*(a*b^3)^(1/2) + 3*a*b^2*(a
*b^3)^(1/2) + 3*a^2*b*(a*b^3)^(1/2)))/(a^2*b^5*(a*b^3)^(1/2)*((a + b)^2)^(3
/2))))/4) - 2*atan((exp(x)*(a + b)^3*(a*b^3)^(1/2))/(2*a*b*((a + b)^2)^(3/2
))))*(2*a*b + a^2 + b^2)^(1/2))/(2*(a*b^3)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.9 $\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] arctan(cosh(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cosh(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.52, size = 300, normalized size = 12.00

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2\,(2\,a-b)\cosh(x)^2+2\left(3\,b\cosh(x)^2-2\,a+b\right)\sinh(x)^2+4\left(b\cosh(x)^3-(2\,a-b)\cosh(x)\right)\sin}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+2\,a+b\right)\mathrm{si}}\right.}{2\,ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2
*(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)
^3 - (2*a - b)*cosh(x))*sinh(x) - 4*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh
(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))*sqrt(-a*b) + b)/(b*cosh(x)^4 +
 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(
x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b
))/(a*b), (sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(x) + sinh(x))/a) - sqrt(a*b
)*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + b)
*cosh(x) + (3*b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(a*b)/(a*b)))/(a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-58,31]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-85,-18]Undef/Unsigned Inf encountered in limitLimit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.05, size = 17, normalized size = 0.68

$$\frac{\arctan\left(\frac{\cosh(x)b}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*cosh(x)^2),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(cosh(x)*b/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/(b*cosh(x)^2 + a), x)
```

mupad [B] time = 0.97, size = 16, normalized size = 0.64

$$\frac{\mathrm{atan}\left(\frac{b\cosh(x)}{\sqrt{a\,b}}\right)}{\sqrt{a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*cosh(x)^2),x)
```

```
[Out] atan((b*cosh(x))/(a*b)^(1/2))/(a*b)^(1/2)
```

sympy [A] time = 1.06, size = 87, normalized size = 3.48

$$\begin{cases} \frac{\tilde{\infty}}{\cosh(x)} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b\cosh(x)} & \text{for } a = 0 \\ \frac{\cosh(x)}{a} & \text{for } b = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\cosh(x)\right)}{2\sqrt{a}\,b\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\cosh(x)\right)}{2\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo/cosh(x), Eq(a, 0) & Eq(b, 0)), (-1/(b*cosh(x)), Eq(a, 0)), (
cosh(x)/a, Eq(b, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + cosh(x))/(2*sqrt(a)*b*
sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + cosh(x))/(2*sqrt(a)*b*sqrt(1/b)),
True))
```

3.10 $\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}-\frac{\tanh^{-1}(\cosh(x))}{a+b}$$

```
[Out] -arctanh(cosh(x))/(a+b)-arctan(cosh(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1/
2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3190, 391, 206, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}-\frac{\tanh^{-1}(\cosh(x))}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*(a + b))) - ArcTanh[
Cosh[x]]/(a + b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)}\,dx, x, \cosh(x)\right)$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(x)\right)}{a+b} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cosh(x)\right)}{a+b}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)} - \frac{\tanh^{-1}(\cosh(x))}{a+b}$$

Mathematica [C] time = 0.14, size = 99, normalized size = 2.36

$$\frac{-\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right) - \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right) + \sqrt{a}\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a}(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]]) - Sqrt[b]*A
rcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] + Sqrt[a]*Log[Tanh[x/2]]
)/(Sqrt[a]*(a + b))
```

fricas [B] time = 0.64, size = 349, normalized size = 8.31

$$\left[\sqrt{-\frac{b}{a}}\log\left(\frac{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2(2a-b)\cosh(x)^2+2\left(3b\cosh(x)^2-2a+b\right)\sinh(x)^2+4\left(b\cosh(x)^3-(2a-b)\cosh(x)\right)\sin}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2(2a+b)\cosh(x)^2+2\left(3b\cosh(x)^2+2a+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2
*(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)
^3 - (2*a - b)*cosh(x))*sinh(x) - 4*(a*cosh(x)^3 + 3*a*cosh(x)*sinh(x)^2 +
a*sinh(x)^3 + a*cosh(x) + (3*a*cosh(x)^2 + a)*sinh(x))*sqrt(-b/a) + b)/(b*c
osh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*
(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*s
inh(x) + b)) - 2*log(cosh(x) + sinh(x) + 1) + 2*log(cosh(x) + sinh(x) - 1))
/(a + b), -(sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(x) + sinh(x))) - sqrt(b/a)
*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + b)*
cosh(x) + (3*b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(b/a)/b) + log(cosh(x) + s
inh(x) + 1) - log(cosh(x) + sinh(x) - 1))/(a + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, need to choose a branch for the roo
```

```
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-26,-97]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[-71,-13]Undef/Unsigned Inf encountered in limitLimit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.10, size = 52, normalized size = 1.24

$$-\frac{b \arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{x}{2}\right)\right)-2a+2b}{4\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*cosh(x)^2),x)
```

```
[Out] -b/(a+b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a+2*b)/(a*b)^(1/2)
)+1/(a+b)*ln(tanh(1/2*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(e^x+1\right)}{a+b}+\frac{\log\left(e^x-1\right)}{a+b}-2\int\frac{be^{(3x)}-be^x}{ab+b^2+\left(ab+b^2\right)e^{(4x)}+2\left(2a^2+3ab+b^2\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -log(e^x + 1)/(a + b) + log(e^x - 1)/(a + b) - 2*integrate((b*e^(3*x) - b*e
^x)/(a*b + b^2 + (a*b + b^2)*e^(4*x) + 2*(2*a^2 + 3*a*b + b^2)*e^(2*x)), x)
```

mupad [B] time = 1.39, size = 462, normalized size = 11.00

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\left(16\,a^2\,\sqrt{-a^2-2\,a\,b-b^2}+b^2\,\sqrt{-a^2-2\,a\,b-b^2}+8\,a\,b\,\sqrt{-a^2-2\,a\,b-b^2}\right)}{16\,a^3+24\,a^2\,b+9\,a\,b^2+b^3}\right)}{\sqrt{-a^2-2\,a\,b-b^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\mathrm{e}^x\,\sqrt{a\,{\left(a+b\right)}^2}}{2\,a\,\left(a+b\right)}\right)-2\,\mathrm{atan}\left(\frac{\left(a^3\,b^{5/2}\,\sqrt{a^3+\dots}\right.}{\dots}\right.\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*cosh(x)^2)),x)
```

```
[Out] - (2*atan((exp(x)*(16*a^2*(- 2*a*b - a^2 - b^2)^(1/2) + b^2*(- 2*a*b - a^2
- b^2)^(1/2) + 8*a*b*(- 2*a*b - a^2 - b^2)^(1/2)))/(9*a*b^2 + 24*a^2*b + 16
*a^3 + b^3)))/(- 2*a*b - a^2 - b^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp(x
)*(a*(a + b)^2)^(1/2))/(2*a*(a + b))) - 2*atan(((a^3*b^(5/2)*(a*b^2 + 2*a^2
*b + a^3)^(1/2) + a^2*b^(7/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))*(exp(x)*((64*(
2*a*b^2 + 10*a^2*b + 8*a^3))/(a*b^3*(a*(a + b)^2)^(1/2)*(a*b + a^2)*(a*b^2
+ 2*a^2*b + a^3)^(1/2)) + (32*(b^(3/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2) + 4*a*
b^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2)))/(a^2*b^(5/2)*(a + b)*(a*b + a^2)*(a
*b^2 + 2*a^2*b + a^3)^(1/2))) + (32*exp(3*x)*(b^(3/2)*(a*b^2 + 2*a^2*b + a^
3)^(1/2) + 4*a*b^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2)))/(a^2*b^(5/2)*(a + b)
*(a*b + a^2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))))/(256*a + 64*b))))/(2*(a*b^2 +
 2*a^2*b + a^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(csch(x)/(a + b*cosh(x)**2), x)
```

3.11 $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=61

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\cosh(x))}{2(a+b)^2}-\frac{\coth(x)\operatorname{csch}(x)}{2(a+b)}$$

```
[Out] 1/2*(a+3*b)*arctanh(cosh(x))/(a+b)^2-1/2*coth(x)*csch(x)/(a+b)+b^(3/2)*arct
an(cosh(x)*b^(1/2)/a^(1/2))/(a+b)^2/a^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3190, 414, 522, 206, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\cosh(x))}{2(a+b)^2}-\frac{\coth(x)\operatorname{csch}(x)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2) + ((a + 3*b
)*ArcTanh[Cosh[x]])/(2*(a + b)^2) - (Coth[x]*Csch[x])/(2*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx, x, \cosh(x)\right)$$
$$= -\frac{\coth(x)\operatorname{csch}(x)}{2(a+b)} + \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \cosh(x)\right)}{2(a+b)}$$
$$= -\frac{\coth(x)\operatorname{csch}(x)}{2(a+b)} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cosh(x)\right)}{(a+b)^2} + \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \text{c}\right.}{2(a+b)^2}$$
$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2} + \frac{(a+3b)\tanh^{-1}(\cosh(x))}{2(a+b)^2} - \frac{\coth(x)\operatorname{csch}(x)}{2(a+b)}$$

Mathematica [C] time = 0.29, size = 154, normalized size = 2.52

$$\frac{-4a^{3/2}\log\left(\tanh\left(\frac{x}{2}\right)\right)+8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)-\sqrt{a}\,(a+b)\operatorname{csch}^2\left(\frac{x}{2}\right)}{8\sqrt{a}\,(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] (8*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] + 8*b^(3/2)*
ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] - Sqrt[a]*(a + b)*Csch[
x/2]^2 - 4*a^(3/2)*Log[Tanh[x/2]] - 12*Sqrt[a]*b*Log[Tanh[x/2]] - Sqrt[a]*(
a + b)*Sech[x/2]^2)/(8*Sqrt[a]*(a + b)^2)
```

fricas [B] time = 0.49, size = 1332, normalized size = 21.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a + b)*cosh(x)^3 + 6*(a + b)*cosh(x)*sinh(x)^2 + 2*(a + b)*sinh(x
)^3 - (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*b*cosh(x)^2 +
2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*cosh(x))*sinh(x) + b)*
sqrt(-b/a)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a
- b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (
2*a - b)*cosh(x))*sinh(x) + 4*(a*cosh(x)^3 + 3*a*cosh(x)*sinh(x)^2 + a*sinh
(x)^3 + a*cosh(x) + (3*a*cosh(x)^2 + a)*sinh(x))*sqrt(-b/a) + b)/(b*cosh(x)
^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*c
osh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x)
 + b)) + 2*(a + b)*cosh(x) - ((a + 3*b)*cosh(x)^4 + 4*(a + 3*b)*cosh(x)*sin
h(x)^3 + (a + 3*b)*sinh(x)^4 - 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a + 3*b)*cosh(
x)^2 - a - 3*b)*sinh(x)^2 + 4*((a + 3*b)*cosh(x)^3 - (a + 3*b)*cosh(x))*sin
h(x) + a + 3*b)*log(cosh(x) + sinh(x) + 1) + ((a + 3*b)*cosh(x)^4 + 4*(a +
3*b)*cosh(x)*sinh(x)^3 + (a + 3*b)*sinh(x)^4 - 2*(a + 3*b)*cosh(x)^2 + 2*(3
*(a + 3*b)*cosh(x)^2 - a - 3*b)*sinh(x)^2 + 4*((a + 3*b)*cosh(x)^3 - (a + 3
*b)*cosh(x))*sinh(x) + a + 3*b)*log(cosh(x) + sinh(x) - 1) + 2*(3*(a + b)*c
osh(x)^2 + a + b)*sinh(x))/((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b
```

```
+ b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b +
 b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*s
inh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*
a*b + b^2)*cosh(x))*sinh(x)), -1/2*(2*(a + b)*cosh(x)^3 + 6*(a + b)*cosh(x)
*sinh(x)^2 + 2*(a + b)*sinh(x)^3 - 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 +
 b*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(
x)^3 - b*cosh(x))*sinh(x) + b)*sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(x) + si
nh(x))) + 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*b*cosh(x
)^2 + 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*cosh(x))*sinh(x)
 + b)*sqrt(b/a)*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)
^3 + (4*a + b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(b/a)/b) +
2*(a + b)*cosh(x) - ((a + 3*b)*cosh(x)^4 + 4*(a + 3*b)*cosh(x)*sinh(x)^3 +
(a + 3*b)*sinh(x)^4 - 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a + 3*b)*cosh(x)^2 - a
- 3*b)*sinh(x)^2 + 4*((a + 3*b)*cosh(x)^3 - (a + 3*b)*cosh(x))*sinh(x) + a
+ 3*b)*log(cosh(x) + sinh(x) + 1) + ((a + 3*b)*cosh(x)^4 + 4*(a + 3*b)*cosh
(x)*sinh(x)^3 + (a + 3*b)*sinh(x)^4 - 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a + 3*b
)*cosh(x)^2 - a - 3*b)*sinh(x)^2 + 4*((a + 3*b)*cosh(x)^3 - (a + 3*b)*cosh(
x))*sinh(x) + a + 3*b)*log(cosh(x) + sinh(x) - 1) + 2*(3*(a + b)*cosh(x)^2
+ a + b)*sinh(x))/((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*co
sh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cos
h(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2
+ a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2
)*cosh(x))*sinh(x))]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[89,77]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Undef/Unsigned Inf encountered in limitLimit: Max order reach
ed or unable to make series expansion Error: Bad Argument Value
```

maple [**A**] time = 0.11, size = 94, normalized size = 1.54

$$\frac{\tanh^{2}\left(\frac{x}{2}\right)}{8a+8b}+\frac{b^{2}\arctan\left(\frac{2(a+b)\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2a+2b}{4\sqrt{ab}}\right)}{(a+b)^{2}\sqrt{ab}}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)a}{2(a+b)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)b}{2(a+b)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*cosh(x)^2),x)
```

```
[Out] 1/8*tanh(1/2*x)^2/(a+b)+b^2/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/
2*x)^2-2*a+2*b)/(a*b)^(1/2))-1/8/(a+b)/tanh(1/2*x)^2-1/2/(a+b)^2*ln(tanh(1/
2*x))*a-3/2/(a+b)^2*ln(tanh(1/2*x))*b
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a+3\,b)\log\left(e^{x}+1\right)}{2\left(a^{2}+2\,ab+b^{2}\right)}-\frac{(a+3\,b)\log\left(e^{x}-1\right)}{2\left(a^{2}+2\,ab+b^{2}\right)}-\frac{e^{(3\,x)}+e^{x}}{(a+b)e^{(4\,x)}-2\,(a+b)e^{(2\,x)}+a+b}+8\int\frac{}{4\left(a^{2}b+2\,ab^{2}+b^{3}+\left(a^{2}b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a + 3*b)*log(e^x + 1)/(a^2 + 2*a*b + b^2) - 1/2*(a + 3*b)*log(e^x - 1)
/(a^2 + 2*a*b + b^2) - (e^(3*x) + e^x)/((a + b)*e^(4*x) - 2*(a + b)*e^(2*x)
 + a + b) + 8*integrate(1/4*(b^2*e^(3*x) - b^2*e^x)/(a^2*b + 2*a*b^2 + b^3
+ (a^2*b + 2*a*b^2 + b^3)*e^(4*x) + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*e^(
2*x)), x)
```

mupad [B] time = 6.89, size = 2225, normalized size = 36.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*cosh(x)^2)),x)
```

```
[Out] ((2*atan((b^2*exp(x)*(a*(a + b)^4)^(1/2))/(2*a*(a + b)^2*(b^3)^(1/2))) - 2*
atan((exp(x)*((32*(b^8*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2
) + 36*a^2*b^6*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 47*a
^3*b^5*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 30*a^4*b^4*(
a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 9*a^5*b^3*(a*b^4 + 4
*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + a^6*b^2*(a*b^4 + 4*a^4*b + a^
5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 12*a*b^7*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*
b^3 + 6*a^3*b^2)^(1/2)))/(a^2*b^2*(a + b)^7*(a*b + a^2)*(b^3)^(1/2)*(2*a*b
+ a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(9*a*b^2 + 6*a^2*b + a^3 + b^3
)*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2)) + (64*(20*a^3*(b^3
)^(5/2) + 232*a^6*(b^3)^(3/2) + 2*a^9*(b^3)^(1/2) + 10*a^2*b^4*(b^3)^(3/2)
+ 20*a^4*b^2*(b^3)^(3/2) + 18*a^2*b^7*(b^3)^(1/2) + 102*a^3*b^6*(b^3)^(1/2)
 + 242*a^4*b^5*(b^3)^(1/2) + 310*a^5*b^4*(b^3)^(1/2) + 98*a^7*b^2*(b^3)^(1/
2) + 2*a*b^5*(b^3)^(3/2) + 10*a^5*b*(b^3)^(3/2) + 22*a^8*b*(b^3)^(1/2)))/(a
*b^4*(a + b)^5*(a*b + a^2)*(a*(a + b)^4)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2
 + 3*a^2*b + a^3 + b^3)*(9*a*b^2 + 6*a^2*b + a^3 + b^3)*(a*b^4 + 4*a^4*b +
a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))) + (32*exp(3*x)*(b^8*(a*b^4 + 4*a^4*b +
 a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 36*a^2*b^6*(a*b^4 + 4*a^4*b + a^5 + 4
*a^2*b^3 + 6*a^3*b^2)^(1/2) + 47*a^3*b^5*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3
 + 6*a^3*b^2)^(1/2) + 30*a^4*b^4*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3
*b^2)^(1/2) + 9*a^5*b^3*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/
2) + a^6*b^2*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 12*a*b
^7*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2)))/(a^2*b^2*(a + b)
^7*(a*b + a^2)*(b^3)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b
^3)*(9*a*b^2 + 6*a^2*b + a^3 + b^3)*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*
a^3*b^2)^(1/2)))*((a^2*b^10*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)
^(1/2))/64 + (a^3*b^9*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2)
)/8 + (7*a^4*b^8*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))/16
+ (7*a^5*b^7*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))/8 + (35
*a^6*b^6*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))/32 + (7*a^7
*b^5*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))/8 + (7*a^8*b^4*
(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))/16 + (a^9*b^3*(a*b^4
 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))/8 + (a^10*b^2*(a*b^4 + 4*a
^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))/64)))*(b^3)^(1/2))/(2*(a*b^4 + 4
*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2)) - (2*exp(x))/((a + b)*(exp(4*x
) - 2*exp(2*x) + 1)) - exp(x)/((a + b)*(exp(2*x) - 1)) - (atan((exp(x)*(a^7
*(- 4*a*b^3 - 4*a^3*b - a^4 - b^4 - 6*a^2*b^2)^(3/2) + 3*b^7*(- 4*a*b^3 - 4
*a^3*b - a^4 - b^4 - 6*a^2*b^2)^(3/2) + 55*a*b^6*(- 4*a*b^3 - 4*a^3*b - a^4
 - b^4 - 6*a^2*b^2)^(3/2) + 15*a^6*b*(- 4*a*b^3 - 4*a^3*b - a^4 - b^4 - 6*a
^2*b^2)^(3/2) + 297*a^2*b^5*(- 4*a*b^3 - 4*a^3*b - a^4 - b^4 - 6*a^2*b^2)^(
3/2) + 423*a^3*b^4*(- 4*a*b^3 - 4*a^3*b - a^4 - b^4 - 6*a^2*b^2)^(3/2) + 27
2*a^4*b^3*(- 4*a*b^3 - 4*a^3*b - a^4 - b^4 - 6*a^2*b^2)^(3/2) + 90*a^5*b^2*
(- 4*a*b^3 - 4*a^3*b - a^4 - b^4 - 6*a^2*b^2)^(3/2)))/(a^12*(6*a*b + a^2 +
9*b^2)^(1/2) + b^12*(6*a*b + a^2 + 9*b^2)^(1/2) + 24*a*b^11*(6*a*b + a^2 +
9*b^2)^(1/2) + 18*a^11*b*(6*a*b + a^2 + 9*b^2)^(1/2) + 216*a^2*b^10*(6*a*b
```

```
+ a^2 + 9*b^2)^(1/2) + 958*a^3*b^9*(6*a*b + a^2 + 9*b^2)^(1/2) + 2484*a^4*b
^8*(6*a*b + a^2 + 9*b^2)^(1/2) + 4122*a^5*b^7*(6*a*b + a^2 + 9*b^2)^(1/2) +
 4587*a^6*b^6*(6*a*b + a^2 + 9*b^2)^(1/2) + 3492*a^7*b^5*(6*a*b + a^2 + 9*b
^2)^(1/2) + 1818*a^8*b^4*(6*a*b + a^2 + 9*b^2)^(1/2) + 634*a^9*b^3*(6*a*b +
 a^2 + 9*b^2)^(1/2) + 141*a^10*b^2*(6*a*b + a^2 + 9*b^2)^(1/2)))*(6*a*b + a
^2 + 9*b^2)^(1/2))/(- 4*a*b^3 - 4*a^3*b - a^4 - b^4 - 6*a^2*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(csch(x)**3/(a + b*cosh(x)**2), x)
```

3.12 $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\cosh(x))}{8(a+b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}-\frac{\coth(x)\operatorname{csch}^3(x)}{4(a+b)}+\frac{(3a+7b)\coth(x)\operatorname{csch}(x)}{8(a+b)^2}$$

```
[Out] -1/8*(3*a^2+10*a*b+15*b^2)*arctanh(cosh(x))/(a+b)^3+1/8*(3*a+7*b)*coth(x)*c
sch(x)/(a+b)^2-1/4*coth(x)*csch(x)^3/(a+b)-b^(5/2)*arctan(cosh(x)*b^(1/2)/a
^(1/2))/(a+b)^3/a^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3190, 414, 527, 522, 206, 205}

$$-\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\cosh(x))}{8(a+b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}-\frac{\coth(x)\operatorname{csch}^3(x)}{4(a+b)}+\frac{(3a+7b)\coth(x)\operatorname{csch}(x)}{8(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3)) - ((3*a^
2 + 10*a*b + 15*b^2)*ArcTanh[Cosh[x]])/(8*(a + b)^3) + ((3*a + 7*b)*Coth[x]
*Csch[x])/(8*(a + b)^2) - (Coth[x]*Csch[x]^3)/(4*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a+bx^2\right)}\,dx, x, \cosh(x)\right)$$

$$= -\frac{\coth(x)\operatorname{csch}^3(x)}{4(a+b)} - \frac{\operatorname{Subst}\left(\int \frac{3a+4b+3bx^2}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx, x, \cosh(x)\right)}{4(a+b)}$$

$$= \frac{(3a+7b)\coth(x)\operatorname{csch}(x)}{8(a+b)^2} - \frac{\coth(x)\operatorname{csch}^3(x)}{4(a+b)} - \frac{\operatorname{Subst}\left(\int \frac{3a^2+7ab+8b^2+b(3a+7b)x^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \cosh\right.}{8(a+b)^2}$$

$$= \frac{(3a+7b)\coth(x)\operatorname{csch}(x)}{8(a+b)^2} - \frac{\coth(x)\operatorname{csch}^3(x)}{4(a+b)} - \frac{b^3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cosh(x)\right)}{(a+b)^3} - \frac{\left(3a^2\right.}{}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3} - \frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\cosh(x))}{8(a+b)^3} + \frac{(3a+7b)\coth(x)\operatorname{csch}}{8(a+b)^2}$$

Mathematica [C] time = 0.61, size = 219, normalized size = 2.33

$$\frac{2\sqrt{a}\left(3a^2+10ab+7b^2\right)\operatorname{csch}^2\left(\frac{x}{2}\right)+2\sqrt{a}\left(3a^2+10ab+7b^2\right)\operatorname{sech}^2\left(\frac{x}{2}\right)+8\left(\sqrt{a}\left(3a^2+10ab+15b^2\right)\log\left(\tanh\right.\right.}{64\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] (2*Sqrt[a]*(3*a^2 + 10*a*b + 7*b^2)*Csch[x/2]^2 - Sqrt[a]*(a + b)^2*Csch[x/
2]^4 + 8*(-8*b^(5/2)*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] -
8*b^(5/2)*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] + Sqrt[a]*(3*
a^2 + 10*a*b + 15*b^2)*Log[Tanh[x/2]]) + 2*Sqrt[a]*(3*a^2 + 10*a*b + 7*b^2)
*Sech[x/2]^2 + Sqrt[a]*(a + b)^2*Sech[x/2]^4)/(64*Sqrt[a]*(a + b)^3)
```

fricas [B] time = 0.60, size = 5326, normalized size = 56.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)^7 + 14*(3*a^2 + 10*a*b + 7*b^2)*co
sh(x)*sinh(x)^6 + 2*(3*a^2 + 10*a*b + 7*b^2)*sinh(x)^7 - 2*(11*a^2 + 26*a*b
```

```
 + 15*b^2)*cosh(x)^5 + 2*(21*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)^2 - 11*a^2 -
26*a*b - 15*b^2)*sinh(x)^5 + 10*(7*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)^3 - (11
*a^2 + 26*a*b + 15*b^2)*cosh(x))*sinh(x)^4 - 2*(11*a^2 + 26*a*b + 15*b^2)*c
osh(x)^3 + 2*(35*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)^4 - 10*(11*a^2 + 26*a*b +
 15*b^2)*cosh(x)^2 - 11*a^2 - 26*a*b - 15*b^2)*sinh(x)^3 + 2*(21*(3*a^2 + 1
0*a*b + 7*b^2)*cosh(x)^5 - 10*(11*a^2 + 26*a*b + 15*b^2)*cosh(x)^3 - 3*(11*
a^2 + 26*a*b + 15*b^2)*cosh(x))*sinh(x)^2 + 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x
)*sinh(x)^7 + b^2*sinh(x)^8 - 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - b^2)*s
inh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^5
+ 2*(35*b^2*cosh(x)^4 - 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 - 4*b^2*cosh(x)
^2 + 8*(7*b^2*cosh(x)^5 - 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(
7*b^2*cosh(x)^6 - 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 - b^2)*sinh(x)^2 + b^2
 + 8*(b^2*cosh(x)^7 - 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 - b^2*cosh(x))*sinh
(x))*sqrt(-b/a)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*
(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^
3 - (2*a - b)*cosh(x))*sinh(x) - 4*(a*cosh(x)^3 + 3*a*cosh(x)*sinh(x)^2 + a
*sinh(x)^3 + a*cosh(x) + (3*a*cosh(x)^2 + a)*sinh(x))*sqrt(-b/a) + b)/(b*co
sh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(
3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*si
nh(x) + b)) + 2*(3*a^2 + 10*a*b + 7*b^2)*cosh(x) - ((3*a^2 + 10*a*b + 15*b^
2)*cosh(x)^8 + 8*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 10*
a*b + 15*b^2)*sinh(x)^8 - 4*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^6 + 4*(7*(3*a
^2 + 10*a*b + 15*b^2)*cosh(x)^2 - 3*a^2 - 10*a*b - 15*b^2)*sinh(x)^6 + 8*(7
*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^3 - 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x))
*sinh(x)^5 + 6*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 10*a*b
+ 15*b^2)*cosh(x)^4 - 30*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 + 9*a^2 + 30*a
*b + 45*b^2)*sinh(x)^4 + 8*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^5 - 10*(3*a
^2 + 10*a*b + 15*b^2)*cosh(x)^3 + 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x))*sinh
(x)^3 - 4*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 10*a*b + 15*b
^2)*cosh(x)^6 - 15*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^4 + 9*(3*a^2 + 10*a*b
+ 15*b^2)*cosh(x)^2 - 3*a^2 - 10*a*b - 15*b^2)*sinh(x)^2 + 3*a^2 + 10*a*b +
 15*b^2 + 8*((3*a^2 + 10*a*b + 15*b^2)*cosh(x)^7 - 3*(3*a^2 + 10*a*b + 15*b
^2)*cosh(x)^5 + 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^3 - (3*a^2 + 10*a*b + 1
5*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((3*a^2 + 10*a*b + 15
*b^2)*cosh(x)^8 + 8*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 +
10*a*b + 15*b^2)*sinh(x)^8 - 4*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^6 + 4*(7*(
3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 - 3*a^2 - 10*a*b - 15*b^2)*sinh(x)^6 + 8
*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^3 - 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(
x))*sinh(x)^5 + 6*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 10*a
*b + 15*b^2)*cosh(x)^4 - 30*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 + 9*a^2 + 3
0*a*b + 45*b^2)*sinh(x)^4 + 8*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^5 - 10*(
3*a^2 + 10*a*b + 15*b^2)*cosh(x)^3 + 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x))*s
inh(x)^3 - 4*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 10*a*b + 1
5*b^2)*cosh(x)^6 - 15*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^4 + 9*(3*a^2 + 10*a
*b + 15*b^2)*cosh(x)^2 - 3*a^2 - 10*a*b - 15*b^2)*sinh(x)^2 + 3*a^2 + 10*a*
b + 15*b^2 + 8*((3*a^2 + 10*a*b + 15*b^2)*cosh(x)^7 - 3*(3*a^2 + 10*a*b + 1
5*b^2)*cosh(x)^5 + 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^3 - (3*a^2 + 10*a*b
+ 15*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(7*(3*a^2 + 10*a
*b + 7*b^2)*cosh(x)^6 - 5*(11*a^2 + 26*a*b + 15*b^2)*cosh(x)^4 - 3*(11*a^2
+ 26*a*b + 15*b^2)*cosh(x)^2 + 3*a^2 + 10*a*b + 7*b^2)*sinh(x))/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(x)^8 + 8*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)
*sinh(x)^7 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^8 - 4*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(x)^6 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 7*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(x)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + 6
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(x)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2 + 3*b^3 - 30*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*cosh(x)^5 - 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + 3*(a^3 + 3*a^2*
```

```
b + 3*a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 4*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2 + 4*(7*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(x)^6 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4 - a^3 - 3*a^2
*b - 3*a*b^2 - b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2
 + 8*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^7 - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(x)^5 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 - (a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(x))*sinh(x)), 1/8*(2*(3*a^2 + 10*a*b + 7*b^2)*
cosh(x)^7 + 14*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)*sinh(x)^6 + 2*(3*a^2 + 10*a
*b + 7*b^2)*sinh(x)^7 - 2*(11*a^2 + 26*a*b + 15*b^2)*cosh(x)^5 + 2*(21*(3*a
^2 + 10*a*b + 7*b^2)*cosh(x)^2 - 11*a^2 - 26*a*b - 15*b^2)*sinh(x)^5 + 10*(
7*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)^3 - (11*a^2 + 26*a*b + 15*b^2)*cosh(x))*
sinh(x)^4 - 2*(11*a^2 + 26*a*b + 15*b^2)*cosh(x)^3 + 2*(35*(3*a^2 + 10*a*b
 + 7*b^2)*cosh(x)^4 - 10*(11*a^2 + 26*a*b + 15*b^2)*cosh(x)^2 - 11*a^2 - 26*
a*b - 15*b^2)*sinh(x)^3 + 2*(21*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)^5 - 10*(11
*a^2 + 26*a*b + 15*b^2)*cosh(x)^3 - 3*(11*a^2 + 26*a*b + 15*b^2)*cosh(x))*s
inh(x)^2 - 8*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 - 4*b
^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7
*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 - 30*b^2*co
sh(x)^2 + 3*b^2)*sinh(x)^4 - 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 - 10*b^2*
cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 - 15*b^2*cosh(x)^
4 + 9*b^2*cosh(x)^2 - b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 - 3*b^2*cosh(
x)^5 + 3*b^2*cosh(x)^3 - b^2*cosh(x))*sinh(x))*sqrt(b/a)*arctan(1/2*sqrt(b/
a)*(cosh(x) + sinh(x))) + 8*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*
sinh(x)^8 - 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - b^2)*sinh(x)^6 + 6*b^2*c
osh(x)^4 + 8*(7*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x
)^4 - 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 - 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh
(x)^5 - 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 -
15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 - b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^
7 - 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 - b^2*cosh(x))*sinh(x))*sqrt(b/a)*arc
tan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + b)*cosh
(x) + (3*b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(b/a)/b) + 2*(3*a^2 + 10*a*b +
 7*b^2)*cosh(x) - ((3*a^2 + 10*a*b + 15*b^2)*cosh(x)^8 + 8*(3*a^2 + 10*a*b
+ 15*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 10*a*b + 15*b^2)*sinh(x)^8 - 4*(3*a^
2 + 10*a*b + 15*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 -
 3*a^2 - 10*a*b - 15*b^2)*sinh(x)^6 + 8*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x
)^3 - 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2 + 10*a*b +
15*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^4 - 30*(3*a^2 +
 10*a*b + 15*b^2)*cosh(x)^2 + 9*a^2 + 30*a*b + 45*b^2)*sinh(x)^4 + 8*(7*(3*
a^2 + 10*a*b + 15*b^2)*cosh(x)^5 - 10*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^3 +
 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x))*sinh(x)^3 - 4*(3*a^2 + 10*a*b + 15*b^
2)*cosh(x)^2 + 4*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^6 - 15*(3*a^2 + 10*a*
b + 15*b^2)*cosh(x)^4 + 9*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 - 3*a^2 - 10*
a*b - 15*b^2)*sinh(x)^2 + 3*a^2 + 10*a*b + 15*b^2 + 8*((3*a^2 + 10*a*b + 15
*b^2)*cosh(x)^7 - 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^5 + 3*(3*a^2 + 10*a*b
 + 15*b^2)*cosh(x)^3 - (3*a^2 + 10*a*b + 15*b^2)*cosh(x))*sinh(x))*log(cosh
(x) + sinh(x) + 1) + ((3*a^2 + 10*a*b + 15*b^2)*cosh(x)^8 + 8*(3*a^2 + 10*a
*b + 15*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 10*a*b + 15*b^2)*sinh(x)^8 - 4*(3
*a^2 + 10*a*b + 15*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^
2 - 3*a^2 - 10*a*b - 15*b^2)*sinh(x)^6 + 8*(7*(3*a^2 + 10*a*b + 15*b^2)*cos
h(x)^3 - 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2 + 10*a*b
 + 15*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^4 - 30*(3*a^
2 + 10*a*b + 15*b^2)*cosh(x)^2 + 9*a^2 + 30*a*b + 45*b^2)*sinh(x)^4 + 8*(7*
(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^5 - 10*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^
3 + 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x))*sinh(x)^3 - 4*(3*a^2 + 10*a*b + 15
*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^6 - 15*(3*a^2 + 10
*a*b + 15*b^2)*cosh(x)^4 + 9*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^2 - 3*a^2 -
10*a*b - 15*b^2)*sinh(x)^2 + 3*a^2 + 10*a*b + 15*b^2 + 8*((3*a^2 + 10*a*b +
 15*b^2)*cosh(x)^7 - 3*(3*a^2 + 10*a*b + 15*b^2)*cosh(x)^5 + 3*(3*a^2 + 10*
a*b + 15*b^2)*cosh(x)^3 - (3*a^2 + 10*a*b + 15*b^2)*cosh(x))*sinh(x))*log(c
```

```
osh(x) + sinh(x) - 1) + 2*(7*(3*a^2 + 10*a*b + 7*b^2)*cosh(x)^6 - 5*(11*a^2
 + 26*a*b + 15*b^2)*cosh(x)^4 - 3*(11*a^2 + 26*a*b + 15*b^2)*cosh(x)^2 + 3*
a^2 + 10*a*b + 7*b^2)*sinh(x))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^8 +
 8*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*sinh(x)^8 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^6 - 4*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*
sinh(x)^6 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 - 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(x)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4 + 3*a^3 + 9*a^2
*b + 9*a*b^2 + 3*b^3 - 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x
)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^5 - 10*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x))*sinh(
x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
sh(x)^2 + 4*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^6 - 15*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(x)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 9*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(x)^7 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^5 + 3*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(x)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x))*
sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-5,64]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[8,-81]Undef/Unsigned Inf encountered in limitEvaluation time: 0.57Li
mit: Max order reached or unable to make series expansion Error: Bad Argume
nt Value
```

maple [B] time = 0.12, size = 184, normalized size = 1.96

$$\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)a}{64\,(a+b)^2}+\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)b}{64\,(a+b)^2}-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)a}{8\,(a+b)^2}-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)b}{4\,(a+b)^2}-\frac{b^3\arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{x}{2}\right)\right)-2a+2b}{4\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}-\frac{1}{64\,(a+b)\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+b*cosh(x)^2),x)
```

```
[Out] 1/64/(a+b)^2*tanh(1/2*x)^4*a+1/64/(a+b)^2*tanh(1/2*x)^4*b-1/8/(a+b)^2*tanh(
1/2*x)^2*a-1/4/(a+b)^2*tanh(1/2*x)^2*b-b^3/(a+b)^3/(a*b)^(1/2)*arctan(1/4*(
2*(a+b)*tanh(1/2*x)^2-2*a+2*b)/(a*b)^(1/2))-1/64/(a+b)/tanh(1/2*x)^4+1/8/(a
+b)^2/tanh(1/2*x)^2*a+1/4/(a+b)^2/tanh(1/2*x)^2*b+3/8/(a+b)^3*ln(tanh(1/2*x
))*a^2+5/4/(a+b)^3*ln(tanh(1/2*x))*a*b+15/8/(a+b)^3*ln(tanh(1/2*x))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\,a^2+10\,ab+15\,b^2\right)\log\left(e^x+1\right)}{8\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{\left(3\,a^2+10\,ab+15\,b^2\right)\log\left(e^x-1\right)}{8\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{(3\,a+7\,b)e^{(7\,x}}{4\left(a^2+2\,ab+b^2+\left(a^2+2\,ab+b^2\right)e^{(8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*a^2 + 10*a*b + 15*b^2)*log(e^x + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
 + 1/8*(3*a^2 + 10*a*b + 15*b^2)*log(e^x - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^
3) + 1/4*((3*a + 7*b)*e^(7*x) - (11*a + 15*b)*e^(5*x) - (11*a + 15*b)*e^(3*
x) + (3*a + 7*b)*e^x)/(a^2 + 2*a*b + b^2 + (a^2 + 2*a*b + b^2)*e^(8*x) - 4*
(a^2 + 2*a*b + b^2)*e^(6*x) + 6*(a^2 + 2*a*b + b^2)*e^(4*x) - 4*(a^2 + 2*a*
b + b^2)*e^(2*x)) - 32*integrate(1/16*(b^3*e^(3*x) - b^3*e^x)/(a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*e^(4*x) + 2*(2*
a^4 + 7*a^3*b + 9*a^2*b^2 + 5*a*b^3 + b^4)*e^(2*x)), x)
```

mupad [B] time = 14.74, size = 5056, normalized size = 53.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^5*(a + b*cosh(x)^2)),x)
```

```
[Out] (atan((exp(x)*(243*a^12*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*
a^3*b^3 - 15*a^4*b^2)^(3/2) + 3840*b^12*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 -
15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 110560*a*b^11*(- 6*a*b^5 - 6*
a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 4050*a^11
*b*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)
^(3/2) + 976143*a^2*b^10*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20
*a^3*b^3 - 15*a^4*b^2)^(3/2) + 2740050*a^3*b^9*(- 6*a*b^5 - 6*a^5*b - a^6 -
 b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 4252775*a^4*b^8*(- 6*a
*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) +
4316760*a^5*b^7*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3
- 15*a^4*b^2)^(3/2) + 3087390*a^6*b^6*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15
*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 1608364*a^7*b^5*(- 6*a*b^5 - 6*
a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 615750*a^
8*b^4*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b
^2)^(3/2) + 171000*a^9*b^3*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 -
20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 33075*a^10*b^2*(- 6*a*b^5 - 6*a^5*b - a^6
- b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2)))/(81*a^19*(300*a*b^3 +
 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 256*b^19*(300*a*b^3 + 60
*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 9504*a*b^18*(300*a*b^3 + 60
*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 1809*a^18*b*(300*a*b^3 + 60
*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 134241*a^2*b^17*(300*a*b^3
+ 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 963809*a^3*b^16*(300*a*
b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 4252296*a^4*b^15*(3
00*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 12815304*a^5*b
^14*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 28102636
*a^6*b^13*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 46
681644*a^7*b^12*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2
) + 60321816*a^8*b^11*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2
)^(1/2) + 61717144*a^9*b^10*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a
^2*b^2)^(1/2) + 50559894*a^10*b^9*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 +
 190*a^2*b^2)^(1/2) + 33362646*a^11*b^8*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225
*b^4 + 190*a^2*b^2)^(1/2) + 17752184*a^12*b^7*(300*a*b^3 + 60*a^3*b + 9*a^4
 + 225*b^4 + 190*a^2*b^2)^(1/2) + 7586616*a^13*b^6*(300*a*b^3 + 60*a^3*b +
9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 2577804*a^14*b^5*(300*a*b^3 + 60*a^3
*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 683596*a^15*b^4*(300*a*b^3 + 60
*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 137064*a^16*b^3*(300*a*b^3
+ 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 19656*a^17*b^2*(300*a*b
^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2)))*(300*a*b^3 + 60*a^3*
b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2))/(4*(- 6*a*b^5 - 6*a^5*b - a^6 - b
^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(1/2)) - (4*exp(x))/((a + b)*(6*
exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) - ((b^5)^(1/2)*(2*atan(
(b^3*exp(x)*(a*(a + b)^6)^(1/2))/(2*a*(a + b)^3*(b^5)^(1/2))) - 2*atan((exp
(x)*((2*(16*b^14*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b
^3 + 15*a^5*b^2)^(1/2) + 321*a*b^13*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15
```

```
*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 1890*a^2*b^12*(a*b^6 + 6*a^6*b
+ a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 5685*a^3*
b^11*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*
b^2)^(1/2) + 10440*a^4*b^10*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4
 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 12690*a^5*b^9*(a*b^6 + 6*a^6*b + a^7 +
6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 10620*a^6*b^8*(a*
b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/
2) + 6210*a^7*b^7*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*
b^3 + 15*a^5*b^2)^(1/2) + 2520*a^8*b^6*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 +
 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 685*a^9*b^5*(a*b^6 + 6*a^6*b
 + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 114*a^10
*b^4*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*
b^2)^(1/2) + 9*a^11*b^3*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 2
0*a^4*b^3 + 15*a^5*b^2)^(1/2)))/(a^2*b*(a + b)^10*(a*b + a^2)*(b^5)^(1/2)*(
3*a*b^2 + 3*a^2*b + a^3 + b^3)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*
(225*a*b^4 + 60*a^4*b + 9*a^5 + 16*b^5 + 300*a^2*b^3 + 190*a^3*b^2)*(6*a*b^
5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)*(a*b^6 + 6*
a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2)) + (4
*(4032*a^5*(b^5)^(5/2) + 74990*a^10*(b^5)^(3/2) + 18*a^15*(b^5)^(1/2) + 288
*a^2*b^8*(b^5)^(3/2) + 1152*a^3*b^7*(b^5)^(3/2) + 2688*a^4*b^6*(b^5)^(3/2)
+ 4032*a^6*b^4*(b^5)^(3/2) + 2688*a^7*b^3*(b^5)^(3/2) + 1152*a^8*b^2*(b^5)^
(3/2) + 450*a^2*b^13*(b^5)^(1/2) + 4650*a^3*b^12*(b^5)^(1/2) + 21980*a^4*b^
11*(b^5)^(1/2) + 62940*a^5*b^10*(b^5)^(1/2) + 121878*a^6*b^9*(b^5)^(1/2) +
168702*a^7*b^8*(b^5)^(1/2) + 172008*a^8*b^7*(b^5)^(1/2) + 131112*a^9*b^6*(b
^5)^(1/2) + 31878*a^11*b^4*(b^5)^(1/2) + 9852*a^12*b^3*(b^5)^(1/2) + 2108*a
^13*b^2*(b^5)^(1/2) + 32*a*b^9*(b^5)^(3/2) + 288*a^9*b*(b^5)^(3/2) + 282*a^
14*b*(b^5)^(1/2)))/(a*b^4*(a + b)^7*(a*b + a^2)*(a*(a + b)^6)^(1/2)*(3*a*b^
2 + 3*a^2*b + a^3 + b^3)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(225*a
*b^4 + 60*a^4*b + 9*a^5 + 16*b^5 + 300*a^2*b^3 + 190*a^3*b^2)*(6*a*b^5 + 6*
a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)*(a*b^6 + 6*a^6*b
+ a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))) + (2*exp(
3*x)*(16*b^14*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3
+ 15*a^5*b^2)^(1/2) + 321*a*b^13*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^
3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 1890*a^2*b^12*(a*b^6 + 6*a^6*b + a
^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 5685*a^3*b^1
1*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2
)^(1/2) + 10440*a^4*b^10*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 +
20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 12690*a^5*b^9*(a*b^6 + 6*a^6*b + a^7 + 6*a
^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 10620*a^6*b^8*(a*b^6
 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2)
+ 6210*a^7*b^7*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3
 + 15*a^5*b^2)^(1/2) + 2520*a^8*b^6*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15
*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 685*a^9*b^5*(a*b^6 + 6*a^6*b +
a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2) + 114*a^10*b^
4*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2
)^(1/2) + 9*a^11*b^3*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a
^4*b^3 + 15*a^5*b^2)^(1/2)))/(a^2*b*(a + b)^10*(a*b + a^2)*(b^5)^(1/2)*(3*a
*b^2 + 3*a^2*b + a^3 + b^3)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(22
5*a*b^4 + 60*a^4*b + 9*a^5 + 16*b^5 + 300*a^2*b^3 + 190*a^3*b^2)*(6*a*b^5 +
 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)*(a*b^6 + 6*a^6
*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2)))*((a^17
*b*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^
2)^(1/2))/4 + (a^2*b^16*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 2
0*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (15*a^3*b^15*(a*b^6 + 6*a^6*b + a^7 + 6*
a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (105*a^4*b^14*(a
*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1
/2))/4 + (455*a^5*b^13*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20
*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (1365*a^6*b^12*(a*b^6 + 6*a^6*b + a^7 + 6
*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (3003*a^7*b^11*
```

```
(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^
(1/2))/4 + (5005*a^8*b^10*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 +
 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (6435*a^9*b^9*(a*b^6 + 6*a^6*b + a^7 +
 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (6435*a^10*b^
8*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2
)^(1/2))/4 + (5005*a^11*b^7*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4
 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (3003*a^12*b^6*(a*b^6 + 6*a^6*b + a^
7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (1365*a^13
*b^5*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*
b^2)^(1/2))/4 + (455*a^14*b^4*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b
^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (105*a^15*b^3*(a*b^6 + 6*a^6*b + a
^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)^(1/2))/4 + (15*a^16*
b^2*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b
^2)^(1/2))/4))))/(2*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^
4*b^3 + 15*a^5*b^2)^(1/2)) - (6*exp(x))/((a + b)*(3*exp(2*x) - 3*exp(4*x) +
 exp(6*x) - 1)) + (exp(x)*(10*a*b + 3*a^2 + 7*b^2))/(4*(a + b)^3*(exp(2*x)
- 1)) - (exp(x)*(a - 3*b))/(2*(a + b)^2*(exp(4*x) - 2*exp(2*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+b*cosh(x)**2),x)
[Out] Timed out
```

3.13 $\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=88

$$\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}-\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b^3}-\frac{(4a+7b)\sinh(x)\cosh(x)}{8b^2}+\frac{\sinh^3(x)\cosh(x)}{4b}$$

```
[Out] 1/8*(8*a^2+20*a*b+15*b^2)*x/b^3-1/8*(4*a+7*b)*cosh(x)*sinh(x)/b^2+1/4*cosh(
x)*sinh(x)^3/b-(a+b)^(5/2)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/b^3/a^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}-\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b^3}-\frac{(4a+7b)\sinh(x)\cosh(x)}{8b^2}+\frac{\sinh^3(x)\cosh(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^6/(a + b*Cosh[x]^2),x]
```

```
[Out] ((8*a^2 + 20*a*b + 15*b^2)*x)/(8*b^3) - ((a + b)^(5/2)*ArcTanh[(Sqrt[a]*Tan
h[x])/Sqrt[a + b]])/(Sqrt[a]*b^3) - ((4*a + 7*b)*Cosh[x]*Sinh[x])/(8*b^2) +
 (Cosh[x]*Sinh[x]^3)/(4*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx = -\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a-(a+b)x^2\right)}\,dx, x, \coth(x)\right)$$

$$= \frac{\cosh(x)\sinh^3(x)}{4b} + \frac{\text{Subst}\left(\int \frac{-a-4b-3(a+b)x^2}{\left(1-x^2\right)^2\left(a+(-a-b)x^2\right)}\,dx, x, \coth(x)\right)}{4b}$$

$$= -\frac{(4a+7b)\cosh(x)\sinh(x)}{8b^2} + \frac{\cosh(x)\sinh^3(x)}{4b} - \frac{\text{Subst}\left(\int \frac{4a^2+9ab+8b^2+(a+b)(4a+7b)x^2}{\left(1-x^2\right)\left(a+(-a-b)x^2\right)}\,dx, x\right.}{8b^2}$$

$$= -\frac{(4a+7b)\cosh(x)\sinh(x)}{8b^2} + \frac{\cosh(x)\sinh^3(x)}{4b} - \frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+(-a-b)x^2}\,dx, x, \text{coth}\right.}{b^3}$$

$$= \frac{\left(8a^2+20ab+15b^2\right)x}{8b^3} - \frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b^3} - \frac{(4a+7b)\cosh(x)\sinh(x)}{8b^2} + \text{cos}$$

Mathematica [A] time = 0.17, size = 76, normalized size = 0.86

$$\frac{4x\left(8a^2+20ab+15b^2\right)-8b(a+2b)\sinh(2x)-\frac{32(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}+b^2\sinh(4x)}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^6/(a + b*Cosh[x]^2),x]
```

```
[Out] (4*(8*a^2 + 20*a*b + 15*b^2)*x - (32*(a + b)^(5/2)*ArcTanh[(Sqrt[a]*Tanh[x]
)/Sqrt[a + b]])/Sqrt[a] - 8*b*(a + 2*b)*Sinh[2*x] + b^2*Sinh[4*x])/(32*b^3)
```

fricas [B] time = 0.51, size = 1308, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 - 8*(a*b + 2
*b^2)*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - 2*a*b - 4*b^2)*sinh(x)^6 + 8*(8*a^2
+ 20*a*b + 15*b^2)*x*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 - 6*(a*b + 2*b^2)*cosh(
x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 - 60*(a*b + 2*b^2)*cosh(x)^2 + 4*(8*a^2
 + 20*a*b + 15*b^2)*x)*sinh(x)^4 + 8*(7*b^2*cosh(x)^5 - 20*(a*b + 2*b^2)*co
sh(x)^3 + 4*(8*a^2 + 20*a*b + 15*b^2)*x*cosh(x))*sinh(x)^3 + 8*(a*b + 2*b^2
)*cosh(x)^2 + 4*(7*b^2*cosh(x)^6 - 30*(a*b + 2*b^2)*cosh(x)^4 + 12*(8*a^2 +
```

```
20*a*b + 15*b^2)*x*cosh(x)^2 + 2*a*b + 4*b^2)*sinh(x)^2 + 32*((a^2 + 2*a*b
 + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)^3*sinh(x) + 6*(a^2 + 2*a*
b + b^2)*cosh(x)^2*sinh(x)^2 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a
^2 + 2*a*b + b^2)*sinh(x)^4)*sqrt((a + b)/a)*log((b^2*cosh(x)^4 + 4*b^2*cos
h(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(
x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2
*a*b + b^2)*cosh(x))*sinh(x) + 4*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a
*b*sinh(x)^2 + 2*a^2 + a*b)*sqrt((a + b)/a))/(b*cosh(x)^4 + 4*b*cosh(x)*sin
h(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*
sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) - b^2 + 8*(b^
2*cosh(x)^7 - 6*(a*b + 2*b^2)*cosh(x)^5 + 4*(8*a^2 + 20*a*b + 15*b^2)*x*cos
h(x)^3 + 2*(a*b + 2*b^2)*cosh(x))*sinh(x))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)^3
*sinh(x) + 6*b^3*cosh(x)^2*sinh(x)^2 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x
)^4), 1/64*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 - 8*(a*
b + 2*b^2)*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - 2*a*b - 4*b^2)*sinh(x)^6 + 8*(8
*a^2 + 20*a*b + 15*b^2)*x*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 - 6*(a*b + 2*b^2)*
cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 - 60*(a*b + 2*b^2)*cosh(x)^2 + 4*(
8*a^2 + 20*a*b + 15*b^2)*x)*sinh(x)^4 + 8*(7*b^2*cosh(x)^5 - 20*(a*b + 2*b^
2)*cosh(x)^3 + 4*(8*a^2 + 20*a*b + 15*b^2)*x*cosh(x))*sinh(x)^3 + 8*(a*b +
2*b^2)*cosh(x)^2 + 4*(7*b^2*cosh(x)^6 - 30*(a*b + 2*b^2)*cosh(x)^4 + 12*(8*
a^2 + 20*a*b + 15*b^2)*x*cosh(x)^2 + 2*a*b + 4*b^2)*sinh(x)^2 - 64*((a^2 +
2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)^3*sinh(x) + 6*(a^2 +
 2*a*b + b^2)*cosh(x)^2*sinh(x)^2 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3
 + (a^2 + 2*a*b + b^2)*sinh(x)^4)*sqrt(-(a + b)/a)*arctan(1/2*(b*cosh(x)^2
+ 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-(a + b)/a)/(a + b)) -
b^2 + 8*(b^2*cosh(x)^7 - 6*(a*b + 2*b^2)*cosh(x)^5 + 4*(8*a^2 + 20*a*b + 15
*b^2)*x*cosh(x)^3 + 2*(a*b + 2*b^2)*cosh(x))*sinh(x))/(b^3*cosh(x)^4 + 4*b^
3*cosh(x)^3*sinh(x) + 6*b^3*cosh(x)^2*sinh(x)^2 + 4*b^3*cosh(x)*sinh(x)^3 +
 b^3*sinh(x)^4)]
```

giac [B] time = 0.13, size = 166, normalized size = 1.89

$$\frac{be^{(4x)} - 8\,ae^{(2x)} - 16\,be^{(2x)}}{64\,b^2} + \frac{\left(8\,a^2 + 20\,ab + 15\,b^2\right)x}{8\,b^3} - \frac{\left(48\,a^2e^{(4x)} + 120\,abe^{(4x)} + 90\,b^2e^{(4x)} - 8\,abe^{(2x)} - 16\,b^2e^{(2x)}\right.}{64\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/64*(b*e^(4*x) - 8*a*e^(2*x) - 16*b*e^(2*x))/b^2 + 1/8*(8*a^2 + 20*a*b + 1
5*b^2)*x/b^3 - 1/64*(48*a^2*e^(4*x) + 120*a*b*e^(4*x) + 90*b^2*e^(4*x) - 8*
a*b*e^(2*x) - 16*b^2*e^(2*x) + b^2)*e^(-4*x)/b^3 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b
)*b^3)
```

maple [B] time = 0.13, size = 575, normalized size = 6.53

$$\frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)-1\right)^4} - \frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} - \frac{a^{\frac{5}{2}}\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right) + 2\sqrt{a}\,\tanh\left(\frac{x}{2}\right) + \sqrt{a+b}\right)}{2b^3\sqrt{a+b}} + \frac{a^{\frac{5}{2}}\ln\left(-\sqrt{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a+b*cosh(x)^2),x)
```

```
[Out] 1/4/b/(tanh(1/2*x)-1)^4-1/4/b/(tanh(1/2*x)+1)^4+3/2/b^2*a^(3/2)/(a+b)^(1/2)
*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))-3/2/b^2*a
^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)
^(1/2))-3/2/b*a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*ta
nh(1/2*x)+(a+b)^(1/2))+3/2/b*a^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x
```

```
)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))-1/2/b^3*a^(5/2)/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))+1/2/b^3*a^(5/2)/(a+
b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))+1
/2/b/(tanh(1/2*x)-1)^3-5/8/b/(tanh(1/2*x)-1)^2-7/8/b/(tanh(1/2*x)-1)+1/2/b/
(tanh(1/2*x)+1)^3+5/8/b/(tanh(1/2*x)+1)^2-7/8/b/(tanh(1/2*x)+1)-15/8/b*ln(t
anh(1/2*x)-1)+15/8/b*ln(tanh(1/2*x)+1)-1/b^3*ln(tanh(1/2*x)-1)*a^2+1/b^3*ln
(tanh(1/2*x)+1)*a^2+1/2/a^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2
*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))-1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*ta
nh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))-1/2/b^2/(tanh(1/2*x)+1)*a+5/
2*a/b^2*ln(tanh(1/2*x)+1)-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b^2/(tanh(1/2*x)-
1)*a-5/2*a/b^2*ln(tanh(1/2*x)-1)+1/2/b^2/(tanh(1/2*x)+1)^2*a
```

maxima [B] time = 0.45, size = 651, normalized size = 7.40

$$-\frac{15\,(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}\,b}+\frac{5\,\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{32\,\sqrt{(a+b)a}}+\frac{3\,(2\,a+b)x}{2\,b^2}+\frac{15\,x}{16\,b}-\frac{\left(4\,(2\,a+b)e^{(-2\,x)}-b\right)}{64\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -15/64*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) +
 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) + 5/32*log((b*e^(-2*x) +
 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/s
qrt((a + b)*a) + 3/2*(2*a + b)*x/b^2 + 15/16*x/b - 1/64*(4*(2*a + b)*e^(-2*
x) - b)*e^(4*x)/b^2 - 3/16*e^(2*x)/b + 3/16*e^(-2*x)/b + 1/64*(4*(2*a + b)*
e^(2*x) - b)*e^(-4*x)/b^2 - 3/16*(2*a + b)*log(b*e^(4*x) + 2*(2*a + b)*e^(2
*x) + b)/b^2 + 3/16*(2*a + b)*log(2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^
2 - 3/64*(8*a^2 + 8*a*b + b^2)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a)
)/(b*e^(2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) + 3/64*(
8*a^2 + 8*a*b + b^2)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-
2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) + 1/8*(16*a^2 +
16*a*b + 3*b^2)*x/b^3 - 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(b*e^(4*x) + 2*(2
*a + b)*e^(2*x) + b)/b^3 + 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(2*(2*a + b)*e
^(-2*x) + b*e^(-4*x) + b)/b^3 - 1/128*(32*a^3 + 48*a^2*b + 18*a*b^2 + b^3)*
log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2*a + b + 2*sqrt
((a + b)*a)))/(sqrt((a + b)*a)*b^3) + 1/128*(32*a^3 + 48*a^2*b + 18*a*b^2 +
 b^3)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b
+ 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^3)
```

mupad [B] time = 1.72, size = 248, normalized size = 2.82

$$\frac{e^{4\,x}}{64\,b}-\frac{e^{-4\,x}}{64\,b}+\frac{x\,\left(8\,a^2+20\,a\,b+15\,b^2\right)}{8\,b^3}+\frac{e^{-2\,x}\,\left(a+2\,b\right)}{8\,b^2}-\frac{e^{2\,x}\,\left(a+2\,b\right)}{8\,b^2}+\frac{\ln\left(\frac{4\,(a+b)^5\,\left(2\,a\,b+8\,a^2\,e^{2\,x}+b^2\,e^{2\,x}+b^2+8\,a\,b\,e^{2\,x}\right)}{a\,b^8}-8\right.}{2\,\sqrt{a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a + b*cosh(x)^2),x)
```

```
[Out] exp(4*x)/(64*b) - exp(-4*x)/(64*b) + (x*(20*a*b + 8*a^2 + 15*b^2))/(8*b^3)
+ (exp(-2*x)*(a + 2*b))/(8*b^2) - (exp(2*x)*(a + 2*b))/(8*b^2) + (log((4*(a
 + b)^5*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a*
b^8) - (8*(a + b)^(11/2)*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(1/2)*b^8))*
(a + b)^(5/2))/(2*a^(1/2)*b^3) - (log((8*(a + b)^(11/2)*(b + 4*a*exp(2*x) +
 2*b*exp(2*x)))/(a^(1/2)*b^8) + (4*(a + b)^5*(2*a*b + 8*a^2*exp(2*x) + b^2*
exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a*b^8))*(a + b)^(5/2))/(2*a^(1/2)*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**6/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.14 $\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{x(2a+3b)}{2b^2}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

```
[Out] -1/2*(2*a+3*b)*x/b^2+1/2*cosh(x)*sinh(x)/b+(a+b)^(3/2)*arctanh(a^(1/2)*tanh
(x)/(a+b)^(1/2))/b^2/a^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3191, 414, 522, 206, 208}

$$-\frac{x(2a+3b)}{2b^2}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] -((2*a + 3*b)*x)/(2*b^2) + ((a + b)^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a
+ b]])/(Sqrt[a]*b^2) + (Cosh[x]*Sinh[x])/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-(a+b)x^2\right)}\,dx, x, \coth(x)\right)$$

$$= \frac{\cosh(x)\sinh(x)}{2b} - \frac{\text{Subst}\left(\int \frac{-a-2b+(-a-b)x^2}{\left(1-x^2\right)\left(a+(-a-b)x^2\right)}\,dx, x, \coth(x)\right)}{2b}$$

$$= \frac{\cosh(x)\sinh(x)}{2b} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+(-a-b)x^2}\,dx, x, \coth(x)\right)}{b^2} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{}{1-}\right.}{2b^2}$$

$$= -\frac{(2a+3b)x}{2b^2} + \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b^2} + \frac{\cosh(x)\sinh(x)}{2b}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.88

$$\frac{\frac{4(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}} - 4ax - 6bx + b\sinh(2x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] (-4*a*x - 6*b*x + (4*(a + b)^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/
Sqrt[a] + b*Sinh[2*x])/(4*b^2)
```

fricas [B] time = 0.48, size = 568, normalized size = 9.63

$$\left[\frac{b\cosh(x)^4 + 4\,b\cosh(x)\sinh(x)^3 + b\sinh(x)^4 - 4\,(2\,a+3\,b)x\cosh(x)^2 + 2\left(3\,b\cosh(x)^2 - 2\,(2\,a+3\,b)x\right)\sin}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a + 3*b)*x*c
osh(x)^2 + 2*(3*b*cosh(x)^2 - 2*(2*a + 3*b)*x)*sinh(x)^2 + 4*((a + b)*cosh(
x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2)*sqrt((a + b)/a)*log((
b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*c
osh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^
2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(a*b*cosh(x)^2 +
2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + 2*a^2 + a*b)*sqrt((a + b)/a))/(b*co
sh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(
3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*si
nh(x) + b)) + 4*(b*cosh(x)^3 - 2*(2*a + 3*b)*x*cosh(x))*sinh(x) - b)/(b^2*c
osh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2), 1/8*(b*cosh(x)^4 + 4*b*c
osh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a + 3*b)*x*cosh(x)^2 + 2*(3*b*cosh(x)
^2 - 2*(2*a + 3*b)*x)*sinh(x)^2 + 8*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*
sinh(x) + (a + b)*sinh(x)^2)*sqrt(-(a + b)/a)*arctan(1/2*(b*cosh(x)^2 + 2*b
*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-(a + b)/a)/(a + b)) + 4*(b*
```

```
cosh(x)^3 - 2*(2*a + 3*b)*x*cosh(x))*sinh(x) - b)/(b^2*cosh(x)^2 + 2*b^2*co
sh(x)*sinh(x) + b^2*sinh(x)^2)]
```

giac [B] time = 0.14, size = 103, normalized size = 1.75

$$-\frac{(2a+3b)x}{2b^2}+\frac{e^{(2x)}}{8b}+\frac{\left(4ae^{(2x)}+6be^{(2x)}-b\right)e^{(-2x)}}{8b^2}+\frac{\left(a^2+2ab+b^2\right)\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + 1/8*e^(2*x)/b + 1/8*(4*a*e^(2*x) + 6*b*e^(2*x) - b
)*e^(-2*x)/b^2 + (a^2 + 2*a*b + b^2)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(
-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^2)
```

maple [B] time = 0.11, size = 351, normalized size = 5.95

$$\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{2b\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*cosh(x)^2),x)
```

```
[Out] 1/2/b/(tanh(1/2*x)-1)^2+1/2/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)+3/2/b
*ln(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^2+1/2/b/(tanh(1/2*x)+1)-a/b^2*ln(t
anh(1/2*x)+1)-3/2/b*ln(tanh(1/2*x)+1)-1/2/b^2*a^(3/2)/(a+b)^(1/2)*ln(-(a+b)
^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))-1/b*a^(1/2)/(a+b)^(
1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))-1/2/a
^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b
)^(1/2))+1/2/b^2*a^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)
*tanh(1/2*x)+(a+b)^(1/2))+1/b*a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x
)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))+1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/
2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B] time = 0.43, size = 348, normalized size = 5.90

$$\frac{(2a+b)\log\left(\frac{be^{(2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{4\sqrt{(a+b)a}\,b}-\frac{3\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{16\sqrt{(a+b)a}}-\frac{(2a+b)x}{b^2}-\frac{x}{b}+\frac{e^{(2x)}}{8b}-\frac{e^{(-2x)}}{8b}+\frac{(2a+b)\log\left(be^{(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2*
a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 3/16*log((b*e^(-2*x) + 2*
a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/sqrt
((a + b)*a) - (2*a + b)*x/b^2 - x/b + 1/8*e^(2*x)/b - 1/8*e^(-2*x)/b + 1/8*
(2*a + b)*log(b*e^(4*x) + 2*(2*a + b)*e^(2*x) + b)/b^2 - 1/8*(2*a + b)*log(
2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^2 + 1/32*(8*a^2 + 8*a*b + b^2)*log
((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2*a + b + 2*sqrt((a
 + b)*a)))/(sqrt((a + b)*a)*b^2) - 1/32*(8*a^2 + 8*a*b + b^2)*log((b*e^(-2*
x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)
))/(sqrt((a + b)*a)*b^2)
```

mupad [B] time = 1.27, size = 146, normalized size = 2.47

$$\frac{e^{2x}}{8b}-\frac{e^{-2x}}{8b}-\frac{x\,(2a+3b)}{2b^2}+\frac{\ln\left(-\frac{4\,e^{2x}\,(a+b)^2}{b^3}-\frac{2\,(a+b)^{3/2}\left(b+2\,a\,e^{2x}+b\,e^{2x}\right)}{\sqrt{a}\,b^3}\right)(a+b)^{3/2}}{2\sqrt{a}\,b^2}-\frac{\ln\left(\frac{2\,(a+b)^{3/2}\left(b+2\,a\,e^{2x}+b\,e^{2x}\right)}{\sqrt{a}\,b^3}-\frac{4\,e^{2x}\,(\right.}{b^3}}{2\sqrt{a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*cosh(x)^2),x)
```

```
[Out] exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (x*(2*a + 3*b))/(2*b^2) + (log(- (4*exp(
2*x)*(a + b)^2)/b^3 - (2*(a + b)^(3/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(a^
(1/2)*b^3))*(a + b)^(3/2))/(2*a^(1/2)*b^2) - (log((2*(a + b)^(3/2)*(b + 2*a
*exp(2*x) + b*exp(2*x)))/(a^(1/2)*b^3) - (4*exp(2*x)*(a + b)^2)/b^3)*(a + b
)^(3/2))/(2*a^(1/2)*b^2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.15 $\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{x}{b}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b}$$

```
[Out] x/b-arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 391, 206, 208}

$$\frac{x}{b}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[x]^2),x]
```

```
[Out] x/b - (Sqrt[a + b]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(Sqrt[a]*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a-(a+b)x^2\right)}\,dx,x,\coth(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\coth(x)\right)}{b}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+(-a-b)x^2}\,dx,x,\coth(x)\right)}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.92

$$\frac{x-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[x]^2),x]
```

```
[Out] (x - (Sqrt[a + b]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/Sqrt[a])/b
```

fricas [A] time = 0.55, size = 300, normalized size = 7.69

$$\left[\frac{\sqrt{\frac{a+b}{a}}\log\left(\frac{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\left(2ab+b^2\right)\cosh(x)^2+2\left(3b^2\cosh(x)^2+2ab+b^2\right)\sinh(x)^2+8a^2+8ab+b^2+4\left(b^2\cosh\right.}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2(2a+b)\cosh(x)^2+2\left(3b\cosh(x)^2+2a\right.}\right.}{2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt((a + b)/a)*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*si
nh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sin
h(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*si
nh(x) + 4*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + 2*a^2 +
a*b)*sqrt((a + b)/a))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 +
2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x
)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 2*x)/b, -(sqrt(-(a + b)/a)*arctan(
1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-(a +
b)/a)/(a + b)) - x)/b]
```

giac [A] time = 0.13, size = 52, normalized size = 1.33

$$-\frac{(a+b)\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -(a + b)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*
b)*b) + x/b
```

maple [B] time = 0.10, size = 183, normalized size = 4.69

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{\sqrt{a}\ \ln\left(\sqrt{a+b}\ \left(\tanh^2\left(\frac{x}{2}\right)\right)-2\sqrt{a}\ \tanh\left(\frac{x}{2}\right)+\sqrt{a+b}\right)}{2b\sqrt{a+b}}-\frac{\sqrt{a}\ \ln\left(\sqrt{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(x)^2),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)+1/2/b*a^(1/2)/(a+b)^(1/2)*ln((
a+b)^(1/2)*tanh(1/2*x)^2-2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))-1/2/b*a^(1/2)/(
a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))+
1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2-2*a^(1/2)*tanh(1/2*x)+
(a+b)^(1/2))-1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)
*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B] time = 0.42, size = 120, normalized size = 3.08

$$-\frac{(2a+b)\log\left(\frac{be^{(2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{4\sqrt{(a+b)a}\,b}+\frac{\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{4\sqrt{(a+b)a}}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2
*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) + 1/4*log((b*e^(-2*x) + 2*
a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/sqrt
((a + b)*a) + x/b
```

mupad [B] time = 0.22, size = 79, normalized size = 2.03

$$\frac{x}{b}+\frac{\mathrm{atan}\left(\frac{\sqrt{-a\,b^2}}{2\,a\,\sqrt{a+b}}+\frac{\sqrt{-a\,b^2}}{b\,\sqrt{a+b}}+\frac{\mathrm{e}^{2\,x}\,\sqrt{-a\,b^2}}{2\,a\,\sqrt{a+b}}\right)\sqrt{a+b}}{\sqrt{-a\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*cosh(x)^2),x)
```

```
[Out] x/b + (atan((-a*b^2)^(1/2)/(2*a*(a + b)^(1/2)) + (-a*b^2)^(1/2)/(b*(a + b)^
(1/2)) + (exp(2*x)*(-a*b^2)^(1/2))/(2*a*(a + b)^(1/2)))*(a + b)^(1/2))/(-a*
b^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.16 $\int \frac{1}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a-(a+b)x^2}\,dx, x, \coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

fricas [B] time = 0.46, size = 293, normalized size = 10.10

$$\left[\frac{\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\left(2\,ab+b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+2\,ab+b^2\right)\sinh(x)^2+8\,a^2+8\,ab+b^2+4\left(b^2\cosh(x)^3+\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+2\,a+b\right)\sinh(x}\right.}{2\sqrt{a^2+ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*
b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 +
8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(b*cosh
(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*co
sh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(
3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*si
nh(x) + b))/sqrt(a^2 + a*b), sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^2 + 2*b
*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a*b))/(a^
2 + a*b)]
```

giac [A]   time = 0.13, size = 39, normalized size = 1.34

$$\frac{\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/sqrt(-a^2 - a*b)
```

maple [B]   time = 0.09, size = 78, normalized size = 2.69

$$-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\sqrt{a}\,\tanh\left(\frac{x}{2}\right)+\sqrt{a+b}\right)}{2\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right)+2\sqrt{a}\,\tanh\left(\frac{x}{2}\right)+\sqrt{a+b}\right)}{2\sqrt{a}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^2),x)
```

```
[Out] -1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2-2*a^(1/2)*tanh(1/2*x)
+(a+b)^(1/2))+1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2
)*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B]   time = 0.42, size = 53, normalized size = 1.83

$$-\frac{\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b +
 2*sqrt((a + b)*a)))/sqrt((a + b)*a)
```

mupad [B]   time = 0.35, size = 267, normalized size = 9.21

$$-\frac{\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^{2\,x}\,{\left(-a^2-b\,a\right)}^{3/2}\,\left(\frac{4\,\left(4\,a+2\,b\right)\,\left(8\,a^3+12\,a^2\,b+4\,a\,b^2\right)}{b^5\,{\left(-a^2-b\,a\right)}^{3/2}\,\sqrt{-a\,\left(a+b\right)}}+\frac{2\,\left(8\,a^2+8\,a\,b+b^2\right)\,\left(8\,a^2\,\sqrt{-a^2-b\,a}+b^2\,\sqrt{-a^2-b\,a}+8\,a\,b\,\sqrt{-a^2-b\,a}\right)}{a\,b^5\,\left(a+b\right)\,{\left(-a^2-b\,a\right)}^{3/2}}\right)}{4}+\frac{\left(2\,a^2\,b+2\,a\,b^2\right)\,\left(4\,a+2\,b\right)}{b^3\,\sqrt{-a\,\left(a+b\right)}}+ \cdots\right)}{\sqrt{-a^2-b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^2),x)
```

```
[Out] -atan((b^2*exp(2*x)*(- a*b - a^2)^(3/2)*((4*(4*a + 2*b)*(4*a*b^2 + 12*a^2*b
 + 8*a^3))/(b^5*(- a*b - a^2)^(3/2)*(-a*(a + b))^(1/2)) + (2*(8*a*b + 8*a^2
 + b^2)*(8*a^2*(- a*b - a^2)^(1/2) + b^2*(- a*b - a^2)^(1/2) + 8*a*b*(- a*b
 - a^2)^(1/2)))/(a*b^5*(a + b)*(- a*b - a^2)^(3/2))))/4 + ((2*a*b^2 + 2*a^2
*b)*(4*a + 2*b))/(b^3*(-a*(a + b))^(1/2)) + ((b^2*(- a*b - a^2)^(1/2) + 2*a
*b*(- a*b - a^2)^(1/2))*(8*a*b + 8*a^2 + b^2))/(2*a*b^3*(a + b)))/(- a*b -
a^2)^(1/2)
```

sympy [A] time = 46.72, size = 12026, normalized size = 414.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (2*tanh(
x/2)/(b*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-tanh(x/2)/(2*b) - 1/(2*b*tanh(x/2
)), Eq(a, -b)), (-5*I*a**(5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
- b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 5*I
*a**(5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2)
)/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(
a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**
(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*
b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 3*I*a**(5/2)*sqrt(b)*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(
b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(
a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2
*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*s
qrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b))) + 3*I*a**(5/2)*sqrt(b)*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*s
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qrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*
b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))) + 10*I*a**(3/2)*b**(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a
**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
+ a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
 a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 10*I*a**(3/2
)*b**(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*
a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b
**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a
/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*s
qrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
+ b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b))) + 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/
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(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)))
- I*sqrt(a)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(
x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I
*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b
))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b
) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) +
2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + I*sqrt(a)*b**(5/2)*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt
(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
- b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**
2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) + a/(a + b) - b/(a + b))) + I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*
b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))) - I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + t
anh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) -
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
 + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - a**3*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
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(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b))) + a**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b
/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + t
anh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) -
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
 + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - a**3*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b))) + a**3*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + t
anh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) -
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
 + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 10*a**2*b*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b*
*2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b))) - 10*a**2*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
```

```
 + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
 b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
+ 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 2*a**2*
b*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)
*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 1
0*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) + a/(a + b) - b/(a + b))) - 2*a**2*b*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b
) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)
/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
 b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
+ a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(
a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt
(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqr
t(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b
) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) -
 5*a*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*
a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b
**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a
/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 5*a*b**2*sqrt(-2*I*sqrt(a)*sqrt(
b)/(a + b) + a/(a + b) - b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
+ a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**
3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
```

```
a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
+ b))) + 3*a*b**2*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2
))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a*
*(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
- b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a
*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*s
qrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 3*a*b**2*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
 a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
 b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
- 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b))), True))
```

3.17 $\int \frac{\operatorname{csch}^4(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{\coth^3(x)}{3(a+b)}+\frac{(a+2b)\coth(x)}{(a+b)^2}$$

```
[Out] (a+2*b)*coth(x)/(a+b)^2-1/3*coth(x)^3/(a+b)+b^2*arctanh(a^(1/2)*tanh(x)/(a+
b)^(1/2))/(a+b)^(5/2)/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 390, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{\coth^3(x)}{3(a+b)}+\frac{(a+2b)\coth(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + ((a
+ 2*b)*Coth[x])/(a + b)^2 - Coth[x]^3/(3*(a + b))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a-(a+b)x^2}\,dx, x, \coth(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2} - \frac{x^2}{a+b} + \frac{b^2}{(a+b)^2\left(a-(a+b)x^2\right)}\right)dx, x, \coth(x)\right)$$
$$= \frac{(a+2b)\coth(x)}{(a+b)^2} - \frac{\coth^3(x)}{3(a+b)} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(a+b)x^2}\,dx, x, \coth(x)\right)}{(a+b)^2}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}} + \frac{(a+2b)\coth(x)}{(a+b)^2} - \frac{\coth^3(x)}{3(a+b)}$$

Mathematica [A] time = 0.22, size = 59, normalized size = 1.00

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}} - \frac{\coth(x)\left((a+b)\operatorname{csch}^2(x)-2a-5b\right)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) - (Cot
h[x]*(-2*a - 5*b + (a + b)*Csch[x]^2))/(3*(a + b)^2)
```

fricas [B] time = 0.54, size = 1875, normalized size = 31.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(x)^4 + 48*(a^2*b + a*b^2)*cosh(x)*sinh(x)^3 +
 12*(a^2*b + a*b^2)*sinh(x)^4 + 8*a^3 + 28*a^2*b + 20*a*b^2 - 24*(a^3 + 3*a
^2*b + 2*a*b^2)*cosh(x)^2 - 24*(a^3 + 3*a^2*b + 2*a*b^2 - 3*(a^2*b + a*b^2)
*cosh(x)^2)*sinh(x)^2 + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*si
nh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 3*b^2*cos
h(x)^2 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4
 - 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 - b^2 + 6*(b^2*cosh(x)^5 - 2*b^2*cosh(x
)^3 + b^2*cosh(x))*sinh(x))*sqrt(a^2 + a*b)*log((b^2*cosh(x)^4 + 4*b^2*cosh
(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x
)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*
a*b + b^2)*cosh(x))*sinh(x) - 4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh
(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*
sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 +
 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 48*((a^2*b + a*b^2)*co
sh(x)^3 - (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))/((a^4 + 3*a^3*b + 3*a
^2*b^2 + a*b^3)*cosh(x)^6 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*s
inh(x)^5 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^6 - 3*(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)*cosh(x)^4 - 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - 5*
(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^4 - a^4 - 3*a^3*b -
3*a^2*b^2 - a*b^3 + 4*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 - 3*
(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x))*sinh(x)^3 + 3*(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*cosh(x)^2 + 3*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*co
sh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - 6*(a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh
```

```
(x)^5 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*cosh(x))*sinh(x)), 1/3*(6*(a^2*b + a*b^2)*cosh(x)^4 + 24
*(a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + 6*(a^2*b + a*b^2)*sinh(x)^4 + 4*a^3 +
14*a^2*b + 10*a*b^2 - 12*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 - 12*(a^3 + 3*
a^2*b + 2*a*b^2 - 3*(a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 3*(b^2*cosh(x)^6
 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cos
h(x)^2 - b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh
(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 - 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 - b^
2 + 6*(b^2*cosh(x)^5 - 2*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a^2 -
a*b)*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)
*sqrt(-a^2 - a*b)/(a^2 + a*b)) + 24*((a^2*b + a*b^2)*cosh(x)^3 - (a^3 + 3*a
^2*b + 2*a*b^2)*cosh(x))*sinh(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh
(x)^6 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^5 + (a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^6 - 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
)*cosh(x)^4 - 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - 5*(a^4 + 3*a^3*b + 3*a
^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^4 - a^4 - 3*a^3*b - 3*a^2*b^2 - a*b^3 +
4*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 - 3*(a^4 + 3*a^3*b + 3*a
^2*b^2 + a*b^3)*cosh(x))*sinh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
cosh(x)^2 + 3*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3 - 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*
sinh(x)^2 + 6*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^5 - 2*(a^4 + 3*a
^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*c
osh(x))*sinh(x))]
```

giac [B] time = 0.39, size = 107, normalized size = 1.81

$$\frac{b^2 \arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\left(a^2+2ab+b^2\right)\sqrt{-a^2-ab}} + \frac{2\left(3be^{(4x)} - 6ae^{(2x)} - 12be^{(2x)} + 2a+5b\right)}{3\left(a^2+2ab+b^2\right)\left(e^{(2x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/((a^2 + 2*a*b + b^2)
*sqrt(-a^2 - a*b)) + 2/3*(3*b*e^(4*x) - 6*a*e^(2*x) - 12*b*e^(2*x) + 2*a +
5*b)/((a^2 + 2*a*b + b^2)*(e^(2*x) - 1)^3)
```

maple [B] time = 0.14, size = 177, normalized size = 3.00

$$-\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24\left(a+b\right)^2} - \frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{24\left(a+b\right)^2} + \frac{3a\tanh\left(\frac{x}{2}\right)}{8\left(a+b\right)^2} + \frac{7\tanh\left(\frac{x}{2}\right)b}{8\left(a+b\right)^2} - \frac{b^2\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right) - 2\sqrt{a}\,\tanh\left(\frac{x}{2}\right) + \sqrt{a}}{2\left(a+b\right)^{\frac{5}{2}}\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*cosh(x)^2),x)
```

```
[Out] -1/24/(a+b)^2*a*tanh(1/2*x)^3-1/24/(a+b)^2*tanh(1/2*x)^3*b+3/8/(a+b)^2*a*ta
nh(1/2*x)+7/8/(a+b)^2*tanh(1/2*x)*b-1/2*b^2/(a+b)^(5/2)/a^(1/2)*ln((a+b)^(1
/2)*tanh(1/2*x)^2-2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))+1/2*b^2/(a+b)^(5/2)/a^
(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))-1/24/
(a+b)/tanh(1/2*x)^3+3/8/(a+b)^2/tanh(1/2*x)*a+7/8/(a+b)^2/tanh(1/2*x)*b
```

maxima [B] time = 0.43, size = 161, normalized size = 2.73

$$-\frac{b^2\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\left(a^2+2ab+b^2\right)} - \frac{2\left(6(a+2b)e^{(-2x)} - 3be^{(-4x)} - 2a-5b\right)}{3\left(a^2+2ab+b^2 - 3\left(a^2+2ab+b^2\right)e^{(-2x)} + 3\left(a^2+2ab+b^2\right)e^{(-4x)} - \left(a^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a +
 b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2)) - 2/3*(6*(a
+ 2*b)*e^(-2*x) - 3*b*e^(-4*x) - 2*a - 5*b)/(a^2 + 2*a*b + b^2 - 3*(a^2 + 2
*a*b + b^2)*e^(-2*x) + 3*(a^2 + 2*a*b + b^2)*e^(-4*x) - (a^2 + 2*a*b + b^2)
*e^(-6*x))
```

mupad [B] time = 1.45, size = 245, normalized size = 4.15

$$\frac{2\,b}{(a+b)^2\,\left(e^{2\,x}-1\right)}-\frac{4}{(a+b)\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}-\frac{8}{3\,(a+b)\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)}-\frac{b^2\,\ln\left(\frac{4\,b^2\,\left(2\,a\,b+8\,a^2\,e^{2\,x}+b^2\,e^{2\,x}+b^2 \cdots}{a\,(a+b)^5} \cdots\right.}{2\,\sqrt{a}\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*cosh(x)^2)),x)
```

```
[Out] (2*b)/((a + b)^2*(exp(2*x) - 1)) - 4/((a + b)*(exp(4*x) - 2*exp(2*x) + 1))
- 8/(3*(a + b)*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (b^2*log((4*b^2*
(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a*(a + b)^
5) - (8*b^2*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(1/2)*(a + b)^(9/2))))/(2
*a^(1/2)*(a + b)^(5/2)) + (b^2*log((8*b^2*(b + 4*a*exp(2*x) + 2*b*exp(2*x))
)/(a^(1/2)*(a + b)^(9/2)) + (4*b^2*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) +
 b^2 + 8*a*b*exp(2*x)))/(a*(a + b)^5)))/(2*a^(1/2)*(a + b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.18 $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(a^2+3ab+3b^2\right)\coth(x)}{(a+b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{\coth^5(x)}{5(a+b)}+\frac{(2a+3b)\coth^3(x)}{3(a+b)^2}$$

```
[Out] -(a^2+3*a*b+3*b^2)*coth(x)/(a+b)^3+1/3*(2*a+3*b)*coth(x)^3/(a+b)^2-1/5*coth
(x)^5/(a+b)-b^3*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/(a+b)^(7/2)/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 390, 208}

$$-\frac{\left(a^2+3ab+3b^2\right)\coth(x)}{(a+b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{\coth^5(x)}{5(a+b)}+\frac{(2a+3b)\coth^3(x)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^6/(a + b*Cosh[x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(7/2))) - (
(a^2 + 3*a*b + 3*b^2)*Coth[x])/(a + b)^3 + ((2*a + 3*b)*Coth[x]^3)/(3*(a +
b)^2) - Coth[x]^5/(5*(a + b))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a-(a+b)x^2}\,dx, x, \coth(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{a^2+3ab+3b^2}{(a+b)^3} - \frac{(2a+3b)x^2}{(a+b)^2} + \frac{x^4}{a+b} + \frac{b^3}{(a+b)^3\left(a-(a+b)x^2\right)}\right)dx, x, \cot$$
$$= -\frac{\left(a^2+3ab+3b^2\right)\coth(x)}{(a+b)^3} + \frac{(2a+3b)\coth^3(x)}{3(a+b)^2} - \frac{\coth^5(x)}{5(a+b)} - \frac{b^3\operatorname{Subst}\left(\int \frac{1}{a-(a+b)x^2}\,dx, x\right.}{(a+b)^3}$$
$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{7/2}} - \frac{\left(a^2+3ab+3b^2\right)\coth(x)}{(a+b)^3} + \frac{(2a+3b)\coth^3(x)}{3(a+b)^2} - \frac{\coth^5(x)}{5(a+b)}$$

Mathematica [A] time = 0.37, size = 92, normalized size = 1.03

$$-\frac{\coth(x)\left(-\left(4a^2+13ab+9b^2\right)\operatorname{csch}^2(x)+8a^2+3(a+b)^2\operatorname{csch}^4(x)+26ab+33b^2\right)}{15(a+b)^3} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^6/(a + b*Cosh[x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(7/2))) - (
Coth[x]*(8*a^2 + 26*a*b + 33*b^2 - (4*a^2 + 13*a*b + 9*b^2)*Csch[x]^2 + 3*(
a + b)^2*Csch[x]^4))/(15*(a + b)^3)
```

fricas [B] time = 0.53, size = 4977, normalized size = 55.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(60*(a^2*b^2 + a*b^3)*cosh(x)^8 + 480*(a^2*b^2 + a*b^3)*cosh(x)*sinh
(x)^7 + 60*(a^2*b^2 + a*b^3)*sinh(x)^8 - 120*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*
cosh(x)^6 - 120*(a^3*b + 4*a^2*b^2 + 3*a*b^3 - 14*(a^2*b^2 + a*b^3)*cosh(x)
^2)*sinh(x)^6 + 240*(14*(a^2*b^2 + a*b^3)*cosh(x)^3 - 3*(a^3*b + 4*a^2*b^2
+ 3*a*b^3)*cosh(x))*sinh(x)^5 + 40*(8*a^4 + 31*a^3*b + 47*a^2*b^2 + 24*a*b^
3)*cosh(x)^4 + 40*(105*(a^2*b^2 + a*b^3)*cosh(x)^4 + 8*a^4 + 31*a^3*b + 47*
a^2*b^2 + 24*a*b^3 - 45*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sinh(x)^4
+ 32*a^4 + 136*a^3*b + 236*a^2*b^2 + 132*a*b^3 + 160*(21*(a^2*b^2 + a*b^3)*
cosh(x)^5 - 15*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^3 + (8*a^4 + 31*a^3*b
+ 47*a^2*b^2 + 24*a*b^3)*cosh(x))*sinh(x)^3 - 40*(4*a^4 + 17*a^3*b + 28*a^2
*b^2 + 15*a*b^3)*cosh(x)^2 + 40*(42*(a^2*b^2 + a*b^3)*cosh(x)^6 - 45*(a^3*b
 + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^4 - 4*a^4 - 17*a^3*b - 28*a^2*b^2 - 15*a*b^
3 + 6*(8*a^4 + 31*a^3*b + 47*a^2*b^2 + 24*a*b^3)*cosh(x)^2)*sinh(x)^2 - 15*
(b^3*cosh(x)^10 + 10*b^3*cosh(x)*sinh(x)^9 + b^3*sinh(x)^10 - 5*b^3*cosh(x)
^8 + 10*b^3*cosh(x)^6 + 5*(9*b^3*cosh(x)^2 - b^3)*sinh(x)^8 + 40*(3*b^3*cos
h(x)^3 - b^3*cosh(x))*sinh(x)^7 - 10*b^3*cosh(x)^4 + 10*(21*b^3*cosh(x)^4 -
 14*b^3*cosh(x)^2 + b^3)*sinh(x)^6 + 4*(63*b^3*cosh(x)^5 - 70*b^3*cosh(x)^3
 + 15*b^3*cosh(x))*sinh(x)^5 + 5*b^3*cosh(x)^2 + 10*(21*b^3*cosh(x)^6 - 35*
b^3*cosh(x)^4 + 15*b^3*cosh(x)^2 - b^3)*sinh(x)^4 + 40*(3*b^3*cosh(x)^7 - 7
*b^3*cosh(x)^5 + 5*b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x)^3 - b^3 + 5*(9*b^3*
cosh(x)^8 - 28*b^3*cosh(x)^6 + 30*b^3*cosh(x)^4 - 12*b^3*cosh(x)^2 + b^3)*s
inh(x)^2 + 10*(b^3*cosh(x)^9 - 4*b^3*cosh(x)^7 + 6*b^3*cosh(x)^5 - 4*b^3*co
sh(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(a^2 + a*b)*log((b^2*cosh(x)^4 + 4*b^2*
```

```
cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*co
sh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 +
 (2*a*b + b^2)*cosh(x))*sinh(x) + 4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*
sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
+ b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)
^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 80*(6*(a^2*b^2 + a
*b^3)*cosh(x)^7 - 9*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^5 + 2*(8*a^4 + 31
*a^3*b + 47*a^2*b^2 + 24*a*b^3)*cosh(x)^3 - (4*a^4 + 17*a^3*b + 28*a^2*b^2
+ 15*a*b^3)*cosh(x))*sinh(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b
^4)*cosh(x)^10 + 10*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)
*sinh(x)^9 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sinh(x)^10 - 5
*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^8 - 5*(a^5 + 4*a^4
*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 9*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b
^3 + a*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*cosh(x)^3 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*c
osh(x))*sinh(x)^7 + 10*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh
(x)^6 + 10*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 21*(a^5 + 4*a^4
*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^4 - 14*(a^5 + 4*a^4*b + 6*a^3*b
^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^5 + 4*a^4*b + 6*a^3
*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^5 - 70*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*cosh(x)^3 + 15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4
)*cosh(x))*sinh(x)^5 - a^5 - 4*a^4*b - 6*a^3*b^2 - 4*a^2*b^3 - a*b^4 - 10*(
a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^4 + 10*(21*(a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^6 - a^5 - 4*a^4*b - 6*a^3*b^
2 - 4*a^2*b^3 - a*b^4 - 35*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*
cosh(x)^4 + 15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^2)*s
inh(x)^4 + 40*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^7
- 7*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^5 + 5*(a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^3 - (a^5 + 4*a^4*b + 6*a^3*b
^2 + 4*a^2*b^3 + a*b^4)*cosh(x))*sinh(x)^3 + 5*(a^5 + 4*a^4*b + 6*a^3*b^2 +
 4*a^2*b^3 + a*b^4)*cosh(x)^2 + 5*(9*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + a*b^4)*cosh(x)^8 - 28*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*co
sh(x)^6 + a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 30*(a^5 + 4*a^4*b
 + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^4 - 12*(a^5 + 4*a^4*b + 6*a^3*b^2
 + 4*a^2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^5 + 4*a^4*b + 6*a^3*b^2
 + 4*a^2*b^3 + a*b^4)*cosh(x)^9 - 4*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
+ a*b^4)*cosh(x)^7 + 6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh
(x)^5 - 4*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^3 + (a^5
+ 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x))*sinh(x)), -1/15*(30*(a^
2*b^2 + a*b^3)*cosh(x)^8 + 240*(a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^7 + 30*(a^
2*b^2 + a*b^3)*sinh(x)^8 - 60*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^6 - 60*
(a^3*b + 4*a^2*b^2 + 3*a*b^3 - 14*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^6 +
120*(14*(a^2*b^2 + a*b^3)*cosh(x)^3 - 3*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(
x))*sinh(x)^5 + 20*(8*a^4 + 31*a^3*b + 47*a^2*b^2 + 24*a*b^3)*cosh(x)^4 + 2
0*(105*(a^2*b^2 + a*b^3)*cosh(x)^4 + 8*a^4 + 31*a^3*b + 47*a^2*b^2 + 24*a*b
^3 - 45*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sinh(x)^4 + 16*a^4 + 68*a^
3*b + 118*a^2*b^2 + 66*a*b^3 + 80*(21*(a^2*b^2 + a*b^3)*cosh(x)^5 - 15*(a^3
*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^3 + (8*a^4 + 31*a^3*b + 47*a^2*b^2 + 24*a
*b^3)*cosh(x))*sinh(x)^3 - 20*(4*a^4 + 17*a^3*b + 28*a^2*b^2 + 15*a*b^3)*co
sh(x)^2 + 20*(42*(a^2*b^2 + a*b^3)*cosh(x)^6 - 45*(a^3*b + 4*a^2*b^2 + 3*a*
b^3)*cosh(x)^4 - 4*a^4 - 17*a^3*b - 28*a^2*b^2 - 15*a*b^3 + 6*(8*a^4 + 31*a
^3*b + 47*a^2*b^2 + 24*a*b^3)*cosh(x)^2)*sinh(x)^2 + 15*(b^3*cosh(x)^10 + 1
0*b^3*cosh(x)*sinh(x)^9 + b^3*sinh(x)^10 - 5*b^3*cosh(x)^8 + 10*b^3*cosh(x)
^6 + 5*(9*b^3*cosh(x)^2 - b^3)*sinh(x)^8 + 40*(3*b^3*cosh(x)^3 - b^3*cosh(x
))*sinh(x)^7 - 10*b^3*cosh(x)^4 + 10*(21*b^3*cosh(x)^4 - 14*b^3*cosh(x)^2 +
 b^3)*sinh(x)^6 + 4*(63*b^3*cosh(x)^5 - 70*b^3*cosh(x)^3 + 15*b^3*cosh(x))*
sinh(x)^5 + 5*b^3*cosh(x)^2 + 10*(21*b^3*cosh(x)^6 - 35*b^3*cosh(x)^4 + 15*
b^3*cosh(x)^2 - b^3)*sinh(x)^4 + 40*(3*b^3*cosh(x)^7 - 7*b^3*cosh(x)^5 + 5*
b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x)^3 - b^3 + 5*(9*b^3*cosh(x)^8 - 28*b^3*
```

```
cosh(x)^6 + 30*b^3*cosh(x)^4 - 12*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 10*(b^3*
cosh(x)^9 - 4*b^3*cosh(x)^7 + 6*b^3*cosh(x)^5 - 4*b^3*cosh(x)^3 + b^3*cosh(
x))*sinh(x))*sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x)
 + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a*b)) + 40*(6*(a^2*b^2 +
a*b^3)*cosh(x)^7 - 9*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^5 + 2*(8*a^4 + 3
1*a^3*b + 47*a^2*b^2 + 24*a*b^3)*cosh(x)^3 - (4*a^4 + 17*a^3*b + 28*a^2*b^2
 + 15*a*b^3)*cosh(x))*sinh(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*
b^4)*cosh(x)^10 + 10*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x
)*sinh(x)^9 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sinh(x)^10 -
5*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^8 - 5*(a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 9*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*
b^3 + a*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + a*b^4)*cosh(x)^3 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*
cosh(x))*sinh(x)^7 + 10*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos
h(x)^6 + 10*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 21*(a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^4 - 14*(a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^5 - 70*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + a*b^4)*cosh(x)^3 + 15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^
4)*cosh(x))*sinh(x)^5 - a^5 - 4*a^4*b - 6*a^3*b^2 - 4*a^2*b^3 - a*b^4 - 10*
(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^4 + 10*(21*(a^5 + 4
*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^6 - a^5 - 4*a^4*b - 6*a^3*b
^2 - 4*a^2*b^3 - a*b^4 - 35*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)
*cosh(x)^4 + 15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^2)*
sinh(x)^4 + 40*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^7
 - 7*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^5 + 5*(a^5 + 4
*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^3 - (a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4)*cosh(x))*sinh(x)^3 + 5*(a^5 + 4*a^4*b + 6*a^3*b^2
+ 4*a^2*b^3 + a*b^4)*cosh(x)^2 + 5*(9*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^
3 + a*b^4)*cosh(x)^8 - 28*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*c
osh(x)^6 + a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 30*(a^5 + 4*a^4*
b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^4 - 12*(a^5 + 4*a^4*b + 6*a^3*b^
2 + 4*a^2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^5 + 4*a^4*b + 6*a^3*b^
2 + 4*a^2*b^3 + a*b^4)*cosh(x)^9 - 4*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + a*b^4)*cosh(x)^7 + 6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos
h(x)^5 - 4*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x)^3 + (a^5
 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cosh(x))*sinh(x))]
```

giac [B] time = 0.59, size = 189, normalized size = 2.12

$$-\frac{b^3 \arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{-a^2-ab}} - \frac{2\left(15b^2e^{(8x)} - 30abe^{(6x)} - 90b^2e^{(6x)} + 80a^2e^{(4x)} + 230abe^{(4x)} + 240b^2e^{(\cdots}\right.}{15\left(a^3+3a^2b+3ab^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*sqrt(-a^2 - a*b)) - 2/15*(15*b^2*e^(8*x) - 30*a*b*e^(6*x) - 9
0*b^2*e^(6*x) + 80*a^2*e^(4*x) + 230*a*b*e^(4*x) + 240*b^2*e^(4*x) - 40*a^2
*e^(2*x) - 130*a*b*e^(2*x) - 150*b^2*e^(2*x) + 8*a^2 + 26*a*b + 33*b^2)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*x) - 1)^5)
```

maple [B] time = 0.15, size = 307, normalized size = 3.45

$$-\frac{a^2\left(\tanh^5\left(\frac{x}{2}\right)\right)}{160\left(a+b\right)^3} - \frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)ab}{80\left(a+b\right)^3} - \frac{b^2\left(\tanh^5\left(\frac{x}{2}\right)\right)}{160\left(a+b\right)^3} + \frac{5a^2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{96\left(a+b\right)^3} + \frac{7a\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{48\left(a+b\right)^3} + \frac{3b^2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{32\left(a+b\right)^3} - \frac{5a^2\tanh\cdots}{16\left(a+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^6/(a+b*cosh(x)^2),x)
```

```
[Out] -1/160/(a+b)^3*a^2*tanh(1/2*x)^5-1/80/(a+b)^3*tanh(1/2*x)^5*a*b-1/160/(a+b)
^3*b^2*tanh(1/2*x)^5+5/96/(a+b)^3*a^2*tanh(1/2*x)^3+7/48/(a+b)^3*a*tanh(1/2
*x)^3*b+3/32/(a+b)^3*b^2*tanh(1/2*x)^3-5/16/(a+b)^3*a^2*tanh(1/2*x)-1/(a+b)
^3*a*b*tanh(1/2*x)-19/16/(a+b)^3*b^2*tanh(1/2*x)+1/2*b^3/(a+b)^(7/2)/a^(1/2
)*ln((a+b)^(1/2)*tanh(1/2*x)^2-2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))-1/2*b^3/(
a+b)^(7/2)/a^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)
^(1/2))-1/160/(a+b)/tanh(1/2*x)^5+5/96/(a+b)^2/tanh(1/2*x)^3*a+3/32/(a+b)^2
/tanh(1/2*x)^3*b-5/16/(a+b)^3/tanh(1/2*x)*a^2-1/(a+b)^3/tanh(1/2*x)*a*b-19/
16/(a+b)^3/tanh(1/2*x)*b^2
```

maxima [B] time = 0.48, size = 307, normalized size = 3.45

$$\frac{b^3 \log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{(a+b)a}} - \frac{2\left(15b^2e^{(-8x)}+8a^2+26ab+3\right.}{15\left(a^3+3a^2b+3ab^2+b^3-5\left(a^3+3a^2b+3ab^2+b^3\right)e^{(-2x)}+10\left(a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a +
b + 2*sqrt((a + b)*a)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*a)) -
 2/15*(15*b^2*e^(-8*x) + 8*a^2 + 26*a*b + 33*b^2 - 10*(4*a^2 + 13*a*b + 15*
b^2)*e^(-2*x) + 10*(8*a^2 + 23*a*b + 24*b^2)*e^(-4*x) - 30*(a*b + 3*b^2)*e^
(-6*x))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
e^(-2*x) + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*x) - 10*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*e^(-6*x) + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-8*x) - (a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-10*x))
```

mupad [B] time = 1.55, size = 333, normalized size = 3.74

$$\frac{4\left(b^2+a\,b\right)}{{(a+b)}^3\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)} - \frac{16}{(a+b)\,\left(6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1\right)} - \frac{2\,b^2}{{(a+b)}^3\,\left(e^{2\,x}-1\right)} - \frac{}{5\,(a+b)\,\left(5\,e^{2\,x}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^6*(a + b*cosh(x)^2)),x)
```

```
[Out] (4*(a*b + b^2))/((a + b)^3*(exp(4*x) - 2*exp(2*x) + 1)) - 16/((a + b)*(6*ex
p(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) - (2*b^2)/((a + b)^3*(exp
(2*x) - 1)) - 32/(5*(a + b)*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp
(8*x) + exp(10*x) - 1)) - (8*(4*a + 3*b))/(3*(a + b)^2*(3*exp(2*x) - 3*exp(
4*x) + exp(6*x) - 1)) + (b^3*log((4*b^4*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2
*x) + b^2 + 8*a*b*exp(2*x)))/(a*(a + b)^7) - (8*b^4*(b + 4*a*exp(2*x) + 2*b
*exp(2*x)))/(a^(1/2)*(a + b)^(13/2))))/(2*a^(1/2)*(a + b)^(7/2)) - (b^3*log
((8*b^4*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(1/2)*(a + b)^(13/2)) + (4*b^
4*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a*(a + b
)^7)))/(2*a^(1/2)*(a + b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**6/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.19 $\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx$

Optimal. Leaf size=98

$$\frac{\log\left(3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}\right)}{12\sqrt[3]{6}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{6}\cosh(x)+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}}$$

```
[Out] 1/12*arctan(1/3*(1+6^(1/3)*cosh(x))*3^(1/2))*2^(2/3)*3^(1/6)-1/36*ln(2^(2/3
)-3^(1/3)*cosh(x))*6^(2/3)+1/72*ln(2*2^(1/3)+2^(2/3)*3^(1/3)*cosh(x)+3^(2/3
)*cosh(x)^2)*6^(2/3)
```

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3223, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}\right)}{12\sqrt[3]{6}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{6}\cosh(x)+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(4 - 3*Cosh[x]^3),x]
```

```
[Out] ArcTan[(1 + 6^(1/3)*Cosh[x])/Sqrt[3]]/(2*2^(1/3)*3^(5/6)) - Log[2^(2/3) - 3
^(1/3)*Cosh[x]]/(6*6^(1/3)) + Log[2*2^(1/3) + 2^(2/3)*3^(1/3)*Cosh[x] + 3^(
2/3)*Cosh[x]^2]/(12*6^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{4-3x^3}\,dx, x, \cosh(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{3}x}\,dx, x, \cosh(x)\right)}{6\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{2\,2^{2/3}+\sqrt[3]{3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \cosh(x)\right)}{6\sqrt[3]{2}}$$

$$= -\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}} + \frac{\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \cosh(x)\right)}{2\,2^{2/3}} + \frac{\text{Subst}\left(\int \frac{}{2}\right.}{}$$

$$= -\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}} + \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\cosh(x)+3^{2/3}\cosh^2(x)\right)}{12\sqrt[3]{6}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{6}\cosh(x)}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}} - \frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}} + \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\cosh(x)+3^{2/3}\right.}{12\sqrt[3]{6}}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 0.79

$$\frac{1}{72}\left(6^{2/3}\left(\log\left(6^{2/3}\cosh^2(x)+2\sqrt[3]{6}\cosh(x)+4\right)-2\log\left(2-\sqrt[3]{6}\cosh(x)\right)\right)+6\,2^{2/3}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{6}\cosh(x)+}{\sqrt{3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(4 - 3*Cosh[x]^3),x]
```

```
[Out] (6*2^(2/3)*3^(1/6)*ArcTan[(1 + 6^(1/3)*Cosh[x])/Sqrt[3]] + 6^(2/3)*(-2*Log[
2 - 6^(1/3)*Cosh[x]] + Log[4 + 2*6^(1/3)*Cosh[x] + 6^(2/3)*Cosh[x]^2]))/72
```

fricas [B] time = 0.52, size = 305, normalized size = 3.11

$$\frac{1}{12}\cdot 6^{\frac{1}{6}}\sqrt{2}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{12}\cdot 6^{\frac{1}{6}}\left(6^{\frac{2}{3}}\sqrt{2}\,(-1)^{\frac{2}{3}}\cosh(x)^3+6^{\frac{2}{3}}\sqrt{2}\,(-1)^{\frac{2}{3}}\sinh(x)^3+\left(3\cdot 6^{\frac{2}{3}}\sqrt{2}\,(-1)^{\frac{2}{3}}\cosh(x)+4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(4-3*cosh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/12*6^(1/6)*sqrt(2)*(-1)^(1/3)*arctan(1/12*6^(1/6)*(6^(2/3)*sqrt(2)*(-1)^(
2/3)*cosh(x)^3 + 6^(2/3)*sqrt(2)*(-1)^(2/3)*sinh(x)^3 + (3*6^(2/3)*sqrt(2)*
```

```
(-1)^(2/3)*cosh(x) + 4*6^(1/3)*sqrt(2))*sinh(x)^2 + 4*6^(1/3)*sqrt(2)*cosh(
x)^2 + (6^(2/3)*sqrt(2)*(-1)^(2/3) - 16*sqrt(2)*(-1)^(1/3))*cosh(x) + (3*6^
(2/3)*sqrt(2)*(-1)^(2/3)*cosh(x)^2 + 6^(2/3)*sqrt(2)*(-1)^(2/3) + 8*6^(1/3)
*sqrt(2)*cosh(x) - 16*sqrt(2)*(-1)^(1/3))*sinh(x) + 2*6^(1/3)*sqrt(2))) - 1
/12*6^(1/6)*sqrt(2)*(-1)^(1/3)*arctan(1/12*6^(1/6)*(6^(2/3)*sqrt(2)*(-1)^(2
/3)*cosh(x) + 6^(2/3)*sqrt(2)*(-1)^(2/3)*sinh(x) + 2*6^(1/3)*sqrt(2))) - 1/
72*6^(2/3)*(-1)^(1/3)*log(-2*(2*6^(2/3)*(-1)^(1/3)*cosh(x) - 3*cosh(x)^2 -
3*sinh(x)^2 - 4*6^(1/3)*(-1)^(2/3) - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 1/36*6^(2/3)*(-1)^(1/3)*log(2*(6^(2/3)*(-1)^(1/3) + 3*cosh(x))/
(cosh(x) - sinh(x)))
```

giac [A] time = 0.15, size = 80, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{4}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{2}{3}}\left(\left(\frac{4}{3}\right)^{\frac{1}{3}}+e^{(-x)}+e^{x}\right)\right)+\frac{1}{72}\cdot 36^{\frac{1}{3}}\log\left(\left(e^{(-x)}+e^{x}\right)^{2}+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(e^{(-x)}+e^{x}\right)+4\left(\frac{4}{3}\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(4-3*cosh(x)^3),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(4/3)^(1/3)*arctan(1/4*sqrt(3)*(4/3)^(2/3)*((4/3)^(1/3) + e^(-
x) + e^x)) + 1/72*36^(1/3)*log((e^(-x) + e^x)^2 + 2*(4/3)^(1/3)*(e^(-x) + e
^x) + 4*(4/3)^(2/3)) - 1/12*(4/3)^(1/3)*log(abs(-2*(4/3)^(1/3) + e^(-x) + e
^x))
```

maple [A] time = 0.05, size = 80, normalized size = 0.82

$$-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cosh(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cosh^{2}(x)+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cosh(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}+\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cosh(x)}{2}+1\right)}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(4-3*cosh(x)^3),x)
```

```
[Out] -1/36*4^(1/3)*3^(2/3)*ln(cosh(x)-1/3*4^(1/3)*3^(2/3))+1/72*4^(1/3)*3^(2/3)*
ln(cosh(x)^2+1/3*4^(1/3)*3^(2/3)*cosh(x)+1/3*4^(2/3)*3^(1/3))+1/12*4^(1/3)*
3^(1/6)*arctan(1/3*3^(1/2)*(1/2*4^(2/3)*3^(1/3)*cosh(x)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sinh(x)}{3\cosh(x)^{3}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(4-3*cosh(x)^3),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(x)/(3*cosh(x)^3 - 4), x)
```

mupad [B] time = 3.51, size = 205, normalized size = 2.09

$$-\frac{6^{2/3}\ln\left(\frac{256\,e^{2x}}{81}-\frac{128\,e^{x}}{27}+\frac{6^{2/3}\left(\frac{256\,e^{2x}}{9}-\frac{2048\,e^{x}}{27}+\frac{6^{2/3}\left(256\,e^{2x}-\frac{2048\,e^{x}}{3}+256\right)}{36}+\frac{256}{9}\right)}{36}+\frac{256}{81}\right)}{36}-\frac{6^{2/3}\ln\left(\frac{256\,e^{2x}}{81}-\frac{128\,e^{x}}{27}+\frac{6^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(x)/(3*cosh(x)^3 - 4),x)
```

```
[Out] (6^(2/3)*log((256*exp(2*x))/81 - (128*exp(x))/27 - (6^(2/3)*((3^(1/2)*1i)/2
 + 1/2)*((256*exp(2*x))/9 - (2048*exp(x))/27 - (6^(2/3)*((3^(1/2)*1i)/2 + 1
/2)*(256*exp(2*x) - (2048*exp(x))/3 + 256))/36 + 256/9))/36 + 256/81)*((3^(
1/2)*1i)/2 + 1/2))/36 - (6^(2/3)*log((256*exp(2*x))/81 - (128*exp(x))/27 +
(6^(2/3)*((3^(1/2)*1i)/2 - 1/2)*((256*exp(2*x))/9 - (2048*exp(x))/27 + (6^(
2/3)*((3^(1/2)*1i)/2 - 1/2)*(256*exp(2*x) - (2048*exp(x))/3 + 256))/36 + 25
6/9))/36 + 256/81)*((3^(1/2)*1i)/2 - 1/2))/36 - (6^(2/3)*log((256*exp(2*x))
/81 - (128*exp(x))/27 + (6^(2/3)*((256*exp(2*x))/9 - (2048*exp(x))/27 + (6^
(2/3)*(256*exp(2*x) - (2048*exp(x))/3 + 256))/36 + 256/9))/36 + 256/81))/36
```

sympy [A] time = 1.54, size = 85, normalized size = 0.87

$$-\frac{6^{\frac{2}{3}}\log\left(\cosh\left(x\right)-\frac{6^{\frac{2}{3}}}{3}\right)}{36}+\frac{6^{\frac{2}{3}}\log\left(36\cosh^{2}\left(x\right)+12\cdot6^{\frac{2}{3}}\cosh\left(x\right)+24\sqrt[3]{6}\right)}{72}+\frac{2^{\frac{2}{3}}\sqrt[6]{3}\operatorname{atan}\left(\frac{\sqrt[3]{2}\cdot3^{\frac{5}{6}}\cosh\left(x\right)}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(4-3*cosh(x)**3),x)
```

```
[Out] -6**(2/3)*log(cosh(x) - 6**(2/3)/3)/36 + 6**(2/3)*log(36*cosh(x)**2 + 12*6*
*(2/3)*cosh(x) + 24*6**(1/3))/72 + 2**(2/3)*3**(1/6)*atan(2**(1/3)*3**(5/6)
*cosh(x)/3 + sqrt(3)/3)/12
```

3.20 $\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{\left(a^2-ab+b^2\right)\sinh(x)}{b^3}-\frac{(a-2b)\sinh^3(x)}{3b^2}+\frac{\sinh^5(x)}{5b}$$

```
[Out] (a^2-a*b+b^2)*sinh(x)/b^3-1/3*(a-2*b)*sinh(x)^3/b^2+1/5*sinh(x)^5/b-a^3*arc
tan(sinh(x)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 390, 205}

$$\frac{\left(a^2-ab+b^2\right)\sinh(x)}{b^3}-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}-\frac{(a-2b)\sinh^3(x)}{3b^2}+\frac{\sinh^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^7/(a + b*Cosh[x]^2),x]
```

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b])) + ((a^
2 - a*b + b^2)*Sinh[x])/b^3 - ((a - 2*b)*Sinh[x]^3)/(3*b^2) + Sinh[x]^5/(5*
b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{a+b+bx^2}\,dx,x,\sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{b^3}-\frac{(a-2b)x^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+b+bx^2\right)}\right)dx,x,\sinh(x)\right)$$
$$= \frac{\left(a^2-ab+b^2\right)\sinh(x)}{b^3}-\frac{(a-2b)\sinh^3(x)}{3b^2}+\frac{\sinh^5(x)}{5b}-\frac{a^3\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,x,\sin\right.}{b^3}$$
$$= -\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{\left(a^2-ab+b^2\right)\sinh(x)}{b^3}-\frac{(a-2b)\sinh^3(x)}{3b^2}+\frac{\sinh^5(x)}{5b}$$

Mathematica [A] time = 0.29, size = 86, normalized size = 1.10

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{a+b}\,\text{csch}(x)}{\sqrt{b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{\left(8a^2-6ab+5b^2\right)\sinh(x)}{8b^3}-\frac{(4a-5b)\sinh(3x)}{48b^2}+\frac{\sinh(5x)}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^7/(a + b*Cosh[x]^2),x]

[Out] (a^3*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/(b^(7/2)*Sqrt[a + b]) + ((8*a^2
 - 6*a*b + 5*b^2)*Sinh[x])/(8*b^3) - ((4*a - 5*b)*Sinh[3*x])/(48*b^2) + Sin
h[5*x]/(80*b)
```

fricas [B] time = 0.52, size = 2508, normalized size = 32.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*cosh(x)^2),x, algorithm="fricas")

[Out] [1/480*(3*(a*b^3 + b^4)*cosh(x)^10 + 30*(a*b^3 + b^4)*cosh(x)*sinh(x)^9 + 3
*(a*b^3 + b^4)*sinh(x)^10 - 5*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^8 - 5*(4*
a^2*b^2 - a*b^3 - 5*b^4 - 27*(a*b^3 + b^4)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a*
b^3 + b^4)*cosh(x)^3 - (4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x))*sinh(x)^7 + 30*
(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^6 + 10*(63*(a*b^3 + b^4)*cosh
(x)^4 + 24*a^3*b + 6*a^2*b^2 - 3*a*b^3 + 15*b^4 - 14*(4*a^2*b^2 - a*b^3 - 5
*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a*b^3 + b^4)*cosh(x)^5 - 70*(4*a^2*b^2
 - a*b^3 - 5*b^4)*cosh(x)^3 + 45*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh
(x))*sinh(x)^5 - 30*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^4 + 10*(6
3*(a*b^3 + b^4)*cosh(x)^6 - 35*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^4 - 24*a
^3*b - 6*a^2*b^2 + 3*a*b^3 - 15*b^4 + 45*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b
^4)*cosh(x)^2)*sinh(x)^4 - 3*a*b^3 - 3*b^4 + 40*(9*(a*b^3 + b^4)*cosh(x)^7
- 7*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^5 + 15*(8*a^3*b + 2*a^2*b^2 - a*b^3
 + 5*b^4)*cosh(x)^3 - 3*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x))*sinh
(x)^3 + 5*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^2 + 5*(27*(a*b^3 + b^4)*cosh(
x)^8 - 28*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^6 + 90*(8*a^3*b + 2*a^2*b^2 -
 a*b^3 + 5*b^4)*cosh(x)^4 + 4*a^2*b^2 - a*b^3 - 5*b^4 - 36*(8*a^3*b + 2*a^2
*b^2 - a*b^3 + 5*b^4)*cosh(x)^2)*sinh(x)^2 - 240*(a^3*cosh(x)^5 + 5*a^3*cos
h(x)^4*sinh(x) + 10*a^3*cosh(x)^3*sinh(x)^2 + 10*a^3*cosh(x)^2*sinh(x)^3 +
5*a^3*cosh(x)*sinh(x)^4 + a^3*sinh(x)^5)*sqrt(-a*b - b^2)*log((b*cosh(x)^4
+ 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*co
sh(x)^2 - 2*a - 3*b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh
(x) + 4*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*si
```

```
nh(x) - cosh(x))*sqrt(-a*b - b^2) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x
)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 10*(3*(a*b^3 + b^
4)*cosh(x)^9 - 4*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^7 + 18*(8*a^3*b + 2*a^
2*b^2 - a*b^3 + 5*b^4)*cosh(x)^5 - 12*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)
*cosh(x)^3 + (4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x))*sinh(x))/((a*b^4 + b^5)*c
osh(x)^5 + 5*(a*b^4 + b^5)*cosh(x)^4*sinh(x) + 10*(a*b^4 + b^5)*cosh(x)^3*s
inh(x)^2 + 10*(a*b^4 + b^5)*cosh(x)^2*sinh(x)^3 + 5*(a*b^4 + b^5)*cosh(x)*s
inh(x)^4 + (a*b^4 + b^5)*sinh(x)^5), 1/480*(3*(a*b^3 + b^4)*cosh(x)^10 + 30
*(a*b^3 + b^4)*cosh(x)*sinh(x)^9 + 3*(a*b^3 + b^4)*sinh(x)^10 - 5*(4*a^2*b^
2 - a*b^3 - 5*b^4)*cosh(x)^8 - 5*(4*a^2*b^2 - a*b^3 - 5*b^4 - 27*(a*b^3 + b
^4)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a*b^3 + b^4)*cosh(x)^3 - (4*a^2*b^2 - a*b
^3 - 5*b^4)*cosh(x))*sinh(x)^7 + 30*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*c
osh(x)^6 + 10*(63*(a*b^3 + b^4)*cosh(x)^4 + 24*a^3*b + 6*a^2*b^2 - 3*a*b^3
+ 15*b^4 - 14*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a*
b^3 + b^4)*cosh(x)^5 - 70*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^3 + 45*(8*a^3
*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x))*sinh(x)^5 - 30*(8*a^3*b + 2*a^2*b^
2 - a*b^3 + 5*b^4)*cosh(x)^4 + 10*(63*(a*b^3 + b^4)*cosh(x)^6 - 35*(4*a^2*b
^2 - a*b^3 - 5*b^4)*cosh(x)^4 - 24*a^3*b - 6*a^2*b^2 + 3*a*b^3 - 15*b^4 + 4
5*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^2)*sinh(x)^4 - 3*a*b^3 - 3*
b^4 + 40*(9*(a*b^3 + b^4)*cosh(x)^7 - 7*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)
^5 + 15*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^3 - 3*(8*a^3*b + 2*a^
2*b^2 - a*b^3 + 5*b^4)*cosh(x))*sinh(x)^3 + 5*(4*a^2*b^2 - a*b^3 - 5*b^4)*c
osh(x)^2 + 5*(27*(a*b^3 + b^4)*cosh(x)^8 - 28*(4*a^2*b^2 - a*b^3 - 5*b^4)*c
osh(x)^6 + 90*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^4 + 4*a^2*b^2 -
 a*b^3 - 5*b^4 - 36*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^2)*sinh(x
)^2 - 480*(a^3*cosh(x)^5 + 5*a^3*cosh(x)^4*sinh(x) + 10*a^3*cosh(x)^3*sinh(
x)^2 + 10*a^3*cosh(x)^2*sinh(x)^3 + 5*a^3*cosh(x)*sinh(x)^4 + a^3*sinh(x)^5
)*sqrt(a*b + b^2)*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(
x)^3 + (4*a + 3*b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + 3*b)*sinh(x))/sqrt(a*b
+ b^2)) - 480*(a^3*cosh(x)^5 + 5*a^3*cosh(x)^4*sinh(x) + 10*a^3*cosh(x)^3*s
inh(x)^2 + 10*a^3*cosh(x)^2*sinh(x)^3 + 5*a^3*cosh(x)*sinh(x)^4 + a^3*sinh(
x)^5)*sqrt(a*b + b^2)*arctan(1/2*sqrt(a*b + b^2)*(cosh(x) + sinh(x))/(a + b
)) + 10*(3*(a*b^3 + b^4)*cosh(x)^9 - 4*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^
7 + 18*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^5 - 12*(8*a^3*b + 2*a^
2*b^2 - a*b^3 + 5*b^4)*cosh(x)^3 + (4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x))*sin
h(x))/((a*b^4 + b^5)*cosh(x)^5 + 5*(a*b^4 + b^5)*cosh(x)^4*sinh(x) + 10*(a*
b^4 + b^5)*cosh(x)^3*sinh(x)^2 + 10*(a*b^4 + b^5)*cosh(x)^2*sinh(x)^3 + 5*(
a*b^4 + b^5)*cosh(x)*sinh(x)^4 + (a*b^4 + b^5)*sinh(x)^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-54,60]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-64,24]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.12, size = 317, normalized size = 4.06

$$-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}+\frac{a}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{7}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{11}{12b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{}{3b^{2}\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(a+b*cosh(x)^2),x)
```

```
[Out] -1/2/b/(tanh(1/2*x)-1)^4-1/5/b/(tanh(1/2*x)-1)^5+1/2/b^2/(tanh(1/2*x)-1)^2*
a-7/8/b/(tanh(1/2*x)-1)^2-11/12/b/(tanh(1/2*x)-1)^3+1/3/b^2/(tanh(1/2*x)-1)
^3*a-1/b^3/(tanh(1/2*x)-1)*a^2+1/b^2/(tanh(1/2*x)-1)*a-1/b/(tanh(1/2*x)-1)-
1/5/b/(tanh(1/2*x)+1)^5+1/2/b/(tanh(1/2*x)+1)^4-1/2/b^2/(tanh(1/2*x)+1)^2*a
+7/8/b/(tanh(1/2*x)+1)^2-11/12/b/(tanh(1/2*x)+1)^3+1/3/b^2/(tanh(1/2*x)+1)^
3*a-1/b^3/(tanh(1/2*x)+1)*a^2+1/b^2/(tanh(1/2*x)+1)*a-1/b/(tanh(1/2*x)+1)+a
^3/b^(7/2)/(a+b)^(1/2)*arctan(1/2*(-2*(a+b)^(1/2)*tanh(1/2*x)+2*a^(1/2))/b^
(1/2))-a^3/b^(7/2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)+2*a^(1
/2))/b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,b^2e^{(10\,x)}-3\,b^2-5\left(4\,ab-5\,b^2\right)e^{(8\,x)}+30\left(8\,a^2-6\,ab+5\,b^2\right)e^{(6\,x)}-30\left(8\,a^2-6\,ab+5\,b^2\right)e^{(4\,x)}+5\left(4\,ab-5\right.}{480\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*x) - 3*b^2 - 5*(4*a*b - 5*b^2)*e^(8*x) + 30*(8*a^2 - 6*a
*b + 5*b^2)*e^(6*x) - 30*(8*a^2 - 6*a*b + 5*b^2)*e^(4*x) + 5*(4*a*b - 5*b^2
)*e^(2*x))*e^(-5*x)/b^3 - 1/128*integrate(256*(a^3*e^(3*x) + a^3*e^x)/(b^4*
e^(4*x) + b^4 + 2*(2*a*b^3 + b^4)*e^(2*x)), x)
```

mupad [B] time = 1.39, size = 293, normalized size = 3.76

$$\frac{e^{5\,x}}{160\,b}-\frac{e^{-5\,x}}{160\,b}-\frac{e^{-x}\,\left(8\,a^2-6\,a\,b+5\,b^2\right)}{16\,b^3}+\frac{\left(2\,\mathrm{atan}\left(\frac{\left(b^9\,\sqrt{b^8+a\,b^7}+a\,b^8\,\sqrt{b^8+a\,b^7}\right)\left(e^x\left(\frac{2\,a^7}{b^{11}\,{(a+b)}^2\,\sqrt{a^6}}-\frac{4\left(2\,a^4\,b^4\,\sqrt{a^6}+2\,a^5\,b^3\,\sqrt{a^6}\right)}{a^3\,b^8\,(a+b)\,\sqrt{b^7\,(a+b)}\,\sqrt{b^8+a\,b^7}}\right.\right.}{4\,a^4}\right.\right.}{2\,\sqrt{b^8+a\,b^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(a + b*cosh(x)^2),x)
```

```
[Out] exp(5*x)/(160*b) - exp(-5*x)/(160*b) - (exp(-x)*(8*a^2 - 6*a*b + 5*b^2))/(1
6*b^3) + ((2*atan(((b^9*(a*b^7 + b^8)^(1/2) + a*b^8*(a*b^7 + b^8)^(1/2))*(e
xp(x)*((2*a^7)/(b^11*(a + b)^2*(a^6)^(1/2)) - (4*(2*a^4*b^4*(a^6)^(1/2) + 2
*a^5*b^3*(a^6)^(1/2)))/(a^3*b^8*(a + b)*(b^7*(a + b))^(1/2)*(a*b^7 + b^8)^(
1/2))) - (2*a^7*exp(3*x))/(b^11*(a + b)^2*(a^6)^(1/2))))/(4*a^4)) - 2*atan(
(a^3*exp(x)*(b^7*(a + b))^(1/2))/(2*b^3*(a + b)*(a^6)^(1/2))))*(a^6)^(1/2))
/(2*(a*b^7 + b^8)^(1/2)) + (exp(-3*x)*(4*a - 5*b))/(96*b^2) - (exp(3*x)*(4*
a - 5*b))/(96*b^2) + (exp(x)*(8*a^2 - 6*a*b + 5*b^2))/(16*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**7/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.21 $\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=88

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^3\sqrt{a+b}}+\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}-\frac{(4a-3b)\sinh(x)\cosh(x)}{8b^2}+\frac{\sinh(x)\cosh^3(x)}{4b}$$

```
[Out] 1/8*(8*a^2-4*a*b+3*b^2)*x/b^3-1/8*(4*a-3*b)*cosh(x)*sinh(x)/b^2+1/4*cosh(x)
^3*sinh(x)/b-a^(5/2)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/b^3/(a+b)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3187, 470, 578, 522, 206, 208}

$$\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^3\sqrt{a+b}}-\frac{(4a-3b)\sinh(x)\cosh(x)}{8b^2}+\frac{\sinh(x)\cosh^3(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(a + b*Cosh[x]^2),x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sq
rt[a + b]])/(b^3*Sqrt[a + b]) - ((4*a - 3*b)*Cosh[x]*Sinh[x])/(8*b^2) + (Co
sh[x]^3*Sinh[x])/(4*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
```

```
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3\left(a-(a+b)x^2\right)}\,dx, x, \coth(x)\right)\\
&= \frac{\cosh^3(x)\sinh(x)}{4b} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(a-3b)x^2\right)}{\left(1-x^2\right)^2\left(a+(-a-b)x^2\right)}\,dx, x, \coth(x)\right)}{4b}\\
&= -\frac{(4a-3b)\cosh(x)\sinh(x)}{8b^2} + \frac{\cosh^3(x)\sinh(x)}{4b} + \frac{\text{Subst}\left(\int \frac{-a(4a-3b)+\left(-4a^2+ab-3b^2\right)x^2}{\left(1-x^2\right)\left(a+(-a-b)x^2\right)}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(4a-3b)\cosh(x)\sinh(x)}{8b^2} + \frac{\cosh^3(x)\sinh(x)}{4b} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+(-a-b)x^2}\,dx, x, \coth(x)\right)}{b^3}\\
&= \frac{\left(8a^2-4ab+3b^2\right)x}{8b^3} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^3\sqrt{a+b}} - \frac{(4a-3b)\cosh(x)\sinh(x)}{8b^2} + \frac{\cosh^3(x
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 76, normalized size = 0.86

$$
\frac{-\frac{32a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 4x\left(8a^2-4ab+3b^2\right) - 8b(a-b)\sinh(2x) + b^2\sinh(4x)}{32b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(a + b*Cosh[x]^2),x]
```

```
[Out] (4*(8*a^2 - 4*a*b + 3*b^2)*x - (32*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a
 + b]])/Sqrt[a + b] - 8*(a - b)*b*Sinh[2*x] + b^2*Sinh[4*x])/(32*b^3)
```

fricas [B] time = 0.48, size = 1245, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 - 8*(a*b - b
^2)*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - 2*a*b + 2*b^2)*sinh(x)^6 + 8*(8*a^2 -
4*a*b + 3*b^2)*x*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 - 6*(a*b - b^2)*cosh(x))*si
nh(x)^5 + 2*(35*b^2*cosh(x)^4 - 60*(a*b - b^2)*cosh(x)^2 + 4*(8*a^2 - 4*a*b
```

```
 + 3*b^2)*x)*sinh(x)^4 + 8*(7*b^2*cosh(x)^5 - 20*(a*b - b^2)*cosh(x)^3 + 4*
(8*a^2 - 4*a*b + 3*b^2)*x*cosh(x))*sinh(x)^3 + 8*(a*b - b^2)*cosh(x)^2 + 4*
(7*b^2*cosh(x)^6 - 30*(a*b - b^2)*cosh(x)^4 + 12*(8*a^2 - 4*a*b + 3*b^2)*x*
cosh(x)^2 + 2*a*b - 2*b^2)*sinh(x)^2 + 32*(a^2*cosh(x)^4 + 4*a^2*cosh(x)^3*
sinh(x) + 6*a^2*cosh(x)^2*sinh(x)^2 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)
^4)*sqrt(a/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh
(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(
x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh
(x) + 4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2
)*sinh(x)^2 + 2*a^2 + 3*a*b + b^2)*sqrt(a/(a + b)))/(b*cosh(x)^4 + 4*b*cosh
(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*
a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) - b^2
+ 8*(b^2*cosh(x)^7 - 6*(a*b - b^2)*cosh(x)^5 + 4*(8*a^2 - 4*a*b + 3*b^2)*x*
cosh(x)^3 + 2*(a*b - b^2)*cosh(x))*sinh(x))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)^
3*sinh(x) + 6*b^3*cosh(x)^2*sinh(x)^2 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(
x)^4), 1/64*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 - 8*(a
*b - b^2)*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - 2*a*b + 2*b^2)*sinh(x)^6 + 8*(8*
a^2 - 4*a*b + 3*b^2)*x*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 - 6*(a*b - b^2)*cosh(
x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 - 60*(a*b - b^2)*cosh(x)^2 + 4*(8*a^2 -
 4*a*b + 3*b^2)*x)*sinh(x)^4 + 8*(7*b^2*cosh(x)^5 - 20*(a*b - b^2)*cosh(x)^
3 + 4*(8*a^2 - 4*a*b + 3*b^2)*x*cosh(x))*sinh(x)^3 + 8*(a*b - b^2)*cosh(x)^
2 + 4*(7*b^2*cosh(x)^6 - 30*(a*b - b^2)*cosh(x)^4 + 12*(8*a^2 - 4*a*b + 3*b
^2)*x*cosh(x)^2 + 2*a*b - 2*b^2)*sinh(x)^2 - 64*(a^2*cosh(x)^4 + 4*a^2*cosh
(x)^3*sinh(x) + 6*a^2*cosh(x)^2*sinh(x)^2 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*s
inh(x)^4)*sqrt(-a/(a + b))*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
b*sinh(x)^2 + 2*a + b)*sqrt(-a/(a + b))/a) - b^2 + 8*(b^2*cosh(x)^7 - 6*(a*
b - b^2)*cosh(x)^5 + 4*(8*a^2 - 4*a*b + 3*b^2)*x*cosh(x)^3 + 2*(a*b - b^2)*
cosh(x))*sinh(x))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)^3*sinh(x) + 6*b^3*cosh(x)^
2*sinh(x)^2 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4)]
```

giac [B] time = 0.13, size = 150, normalized size = 1.70

$$-\frac{a^{3} \arctan\left(\frac{b e^{(2x)}+2a+b}{2\sqrt{-a^{2}-ab}}\right)}{\sqrt{-a^{2}-ab}\,b^{3}}+\frac{b e^{(4x)}-8\,a e^{(2x)}+8\,b e^{(2x)}}{64\,b^{2}}+\frac{\left(8\,a^{2}-4\,ab+3\,b^{2}\right)x}{8\,b^{3}}-\frac{\left(48\,a^{2} e^{(4x)}-24\,ab e^{(4x)}+18\,b^{2} e^{(4x)}-\cdots\right.}{64\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -a^3*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b
^3) + 1/64*(b*e^(4*x) - 8*a*e^(2*x) + 8*b*e^(2*x))/b^2 + 1/8*(8*a^2 - 4*a*b
 + 3*b^2)*x/b^3 - 1/64*(48*a^2*e^(4*x) - 24*a*b*e^(4*x) + 18*b^2*e^(4*x) -
8*a*b*e^(2*x) + 8*b^2*e^(2*x) + b^2)*e^(-4*x)/b^3
```

maple [B] time = 0.11, size = 326, normalized size = 3.70

$$\frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{a}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{7}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{a}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{5}{8b\left(\tanh \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(a+b*cosh(x)^2),x)
```

```
[Out] 1/4/b/(tanh(1/2*x)-1)^4+1/2/b/(tanh(1/2*x)-1)^3-1/2/b^2/(tanh(1/2*x)-1)^2*a
+7/8/b/(tanh(1/2*x)-1)^2-1/2/b^2/(tanh(1/2*x)-1)*a+5/8/b/(tanh(1/2*x)-1)-1/
b^3*ln(tanh(1/2*x)-1)*a^2+1/2*a/b^2*ln(tanh(1/2*x)-1)-3/8/b*ln(tanh(1/2*x)-
1)-1/4/b/(tanh(1/2*x)+1)^4+1/2/b/(tanh(1/2*x)+1)^3+1/2/b^2/(tanh(1/2*x)+1)^
2*a-7/8/b/(tanh(1/2*x)+1)^2-1/2/b^2/(tanh(1/2*x)+1)*a+5/8/b/(tanh(1/2*x)+1)
+1/b^3*ln(tanh(1/2*x)+1)*a^2-1/2*a/b^2*ln(tanh(1/2*x)+1)+3/8/b*ln(tanh(1/2*
```

```
x)+1)+1/2/b^3*a^(5/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*t
anh(1/2*x)-(a+b)^(1/2))-1/2/b^3*a^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2
*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B] time = 0.50, size = 651, normalized size = 7.40

$$-\frac{15\,(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}\,b}-\frac{5\,\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{32\,\sqrt{(a+b)a}}-\frac{3\,(2\,a+b)x}{2\,b^2}+\frac{15\,x}{16\,b}-\frac{\left(4\,(2\,a+b)e^{(-2\,x)}-\right.}{64\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -15/64*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) +
 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 5/32*log((b*e^(-2*x) +
 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/s
qrt((a + b)*a) - 3/2*(2*a + b)*x/b^2 + 15/16*x/b - 1/64*(4*(2*a + b)*e^(-2*
x) - b)*e^(4*x)/b^2 + 3/16*e^(2*x)/b - 3/16*e^(-2*x)/b + 1/64*(4*(2*a + b)*
e^(2*x) - b)*e^(-4*x)/b^2 + 3/16*(2*a + b)*log(b*e^(4*x) + 2*(2*a + b)*e^(2
*x) + b)/b^2 - 3/16*(2*a + b)*log(2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^
2 + 3/64*(8*a^2 + 8*a*b + b^2)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a)
)/(b*e^(2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) - 3/64*(
8*a^2 + 8*a*b + b^2)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-
2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) + 1/8*(16*a^2 +
16*a*b + 3*b^2)*x/b^3 - 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(b*e^(4*x) + 2*(2
*a + b)*e^(2*x) + b)/b^3 + 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(2*(2*a + b)*e
^(-2*x) + b*e^(-4*x) + b)/b^3 - 1/128*(32*a^3 + 48*a^2*b + 18*a*b^2 + b^3)*
log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2*a + b + 2*sqrt
((a + b)*a)))/(sqrt((a + b)*a)*b^3) + 1/128*(32*a^3 + 48*a^2*b + 18*a*b^2 +
 b^3)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b
+ 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^3)
```

mupad [B] time = 1.33, size = 178, normalized size = 2.02

$$\frac{e^{4\,x}}{64\,b}-\frac{e^{-4\,x}}{64\,b}+\frac{x\,\left(8\,a^2-4\,a\,b+3\,b^2\right)}{8\,b^3}+\frac{e^{-2\,x}\,\left(a-b\right)}{8\,b^2}-\frac{e^{2\,x}\,\left(a-b\right)}{8\,b^2}+\frac{a^{5/2}\,\ln\left(\frac{4\,a^3\,e^{2\,x}}{b^4}-\frac{2\,a^{5/2}\,\left(b+2\,a\,e^{2\,x}+b\,e^{2\,x}\right)}{b^4\,\sqrt{a+b}}\right)}{2\,b^3\,\sqrt{a+b}}-\frac{a^{5/2}\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(a + b*cosh(x)^2),x)
```

```
[Out] exp(4*x)/(64*b) - exp(-4*x)/(64*b) + (x*(8*a^2 - 4*a*b + 3*b^2))/(8*b^3) +
(exp(-2*x)*(a - b))/(8*b^2) - (exp(2*x)*(a - b))/(8*b^2) + (a^(5/2)*log((4*
a^3*exp(2*x))/b^4 - (2*a^(5/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(b^4*(a + b
)^(1/2))))/(2*b^3*(a + b)^(1/2)) - (a^(5/2)*log((4*a^3*exp(2*x))/b^4 + (2*a
^(5/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(b^4*(a + b)^(1/2))))/(2*b^3*(a + b
)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.22 $\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sinh(x)}{b^2}+\frac{\sinh^3(x)}{3b}$$

```
[Out] -(a-b)*sinh(x)/b^2+1/3*sinh(x)^3/b+a^2*arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))/
b^(5/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 390, 205}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sinh(x)}{b^2}+\frac{\sinh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]) - ((a - b
)*Sinh[x])/b^2 + Sinh[x]^3/(3*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^2}{a+b+bx^2}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{a-b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+b+bx^2)}\right)dx, x, \sinh(x)\right)$$
$$= -\frac{(a-b)\sinh(x)}{b^2}+\frac{\sinh^3(x)}{3b}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \sinh(x)\right)}{b^2}$$
$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sinh(x)}{b^2}+\frac{\sinh^3(x)}{3b}$$

Mathematica [A] time = 0.17, size = 61, normalized size = 1.09

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\text{csch}(x)}{\sqrt{b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(4a-3b)\sinh(x)}{4b^2}+\frac{\sinh(3x)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] -((a^2*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/(b^(5/2)*Sqrt[a + b])) - ((4*
a - 3*b)*Sinh[x])/(4*b^2) + Sinh[3*x]/(12*b)
```

fricas [B] time = 0.53, size = 1184, normalized size = 21.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2
 + b^3)*sinh(x)^6 - 3*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^4 - 3*(4*a^2*b + a*
b^2 - 3*b^3 - 5*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cos
h(x)^3 - 3*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x))*sinh(x)^3 - a*b^2 - b^3 + 3*(
4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^2 + 3*(5*(a*b^2 + b^3)*cosh(x)^4 + 4*a^2*b
 + a*b^2 - 3*b^3 - 6*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^2)*sinh(x)^2 - 12*(a
^2*cosh(x)^3 + 3*a^2*cosh(x)^2*sinh(x) + 3*a^2*cosh(x)*sinh(x)^2 + a^2*sinh
(x)^3)*sqrt(-a*b - b^2)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x
)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 + 4
*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*(cosh(x)^3 + 3*cosh(x)*sin
h(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))*sqrt(-a*b - b^2)
+ b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(
x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*c
osh(x))*sinh(x) + b)) + 6*((a*b^2 + b^3)*cosh(x)^5 - 2*(4*a^2*b + a*b^2 - 3
*b^3)*cosh(x)^3 + (4*a^2*b + a*b^2 - 3*b^3)*cosh(x))*sinh(x))/((a*b^3 + b^4
)*cosh(x)^3 + 3*(a*b^3 + b^4)*cosh(x)^2*sinh(x) + 3*(a*b^3 + b^4)*cosh(x)*s
inh(x)^2 + (a*b^3 + b^4)*sinh(x)^3), 1/24*((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b
^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 - 3*(4*a^2*b + a*b^2
- 3*b^3)*cosh(x)^4 - 3*(4*a^2*b + a*b^2 - 3*b^3 - 5*(a*b^2 + b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 - 3*(4*a^2*b + a*b^2 - 3*b^3)*co
sh(x))*sinh(x)^3 - a*b^2 - b^3 + 3*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^2 + 3*
(5*(a*b^2 + b^3)*cosh(x)^4 + 4*a^2*b + a*b^2 - 3*b^3 - 6*(4*a^2*b + a*b^2 -
 3*b^3)*cosh(x)^2)*sinh(x)^2 + 24*(a^2*cosh(x)^3 + 3*a^2*cosh(x)^2*sinh(x)
+ 3*a^2*cosh(x)*sinh(x)^2 + a^2*sinh(x)^3)*sqrt(a*b + b^2)*arctan(1/2*(b*co
```

```
sh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + 3*b)*cosh(x) + (3*b*
cosh(x)^2 + 4*a + 3*b)*sinh(x))/sqrt(a*b + b^2)) + 24*(a^2*cosh(x)^3 + 3*a^
2*cosh(x)^2*sinh(x) + 3*a^2*cosh(x)*sinh(x)^2 + a^2*sinh(x)^3)*sqrt(a*b + b
^2)*arctan(1/2*sqrt(a*b + b^2)*(cosh(x) + sinh(x))/(a + b)) + 6*((a*b^2 + b
^3)*cosh(x)^5 - 2*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^3 + (4*a^2*b + a*b^2 -
3*b^3)*cosh(x))*sinh(x))/((a*b^3 + b^4)*cosh(x)^3 + 3*(a*b^3 + b^4)*cosh(x)
^2*sinh(x) + 3*(a*b^3 + b^4)*cosh(x)*sinh(x)^2 + (a*b^3 + b^4)*sinh(x)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-97,37]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-81,22]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.10, size = 176, normalized size = 3.14

$$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{a}{b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+b*cosh(x)^2),x)
```

```
[Out] -1/3/b/(tanh(1/2*x)-1)^3-1/2/b/(tanh(1/2*x)-1)^2+1/b^2/(tanh(1/2*x)-1)*a-1/
b/(tanh(1/2*x)-1)-1/3/b/(tanh(1/2*x)+1)^3+1/2/b/(tanh(1/2*x)+1)^2+1/b^2/(ta
nh(1/2*x)+1)*a-1/b/(tanh(1/2*x)+1)+a^2/b^(5/2)/(a+b)^(1/2)*arctan(1/2*(2*(a
+b)^(1/2)*tanh(1/2*x)-2*a^(1/2))/b^(1/2))+a^2/b^(5/2)/(a+b)^(1/2)*arctan(1/
2*(2*(a+b)^(1/2)*tanh(1/2*x)+2*a^(1/2))/b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(be^{(6\,x)}-3\,(4\,a-3\,b)e^{(4\,x)}+3\,(4\,a-3\,b)e^{(2\,x)}-b\right)e^{(-3\,x)}}{24\,b^{2}}+\frac{1}{32}\int\frac{64\left(a^{2}e^{(3\,x)}+a^{2}e^{x}\right)}{b^{3}e^{(4\,x)}+b^{3}+2\left(2\,ab^{2}+b^{3}\right)e^{(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(b*e^(6*x) - 3*(4*a - 3*b)*e^(4*x) + 3*(4*a - 3*b)*e^(2*x) - b)*e^(-3*
x)/b^2 + 1/32*integrate(64*(a^2*e^(3*x) + a^2*e^x)/(b^3*e^(4*x) + b^3 + 2*(
2*a*b^2 + b^3)*e^(2*x)), x)
```

mupad [B] time = 1.25, size = 243, normalized size = 4.34

$$\frac{e^{3\,x}}{24\,b}-\frac{e^{-3\,x}}{24\,b}+\frac{e^{-x}\,\left(4\,a-3\,b\right)}{8\,b^{2}}+\frac{\sqrt{a^{4}}\,\left(2\,\mathrm{atan}\left(\frac{a^{2}\,e^{x}\,\sqrt{b^{5}\,(a+b)}}{2\,b^{2}\,(a+b)\,\sqrt{a^{4}}}\right)-2\,\mathrm{atan}\left(\left(\left(\frac{b^{7}\,\sqrt{b^{6}+a\,b^{5}}}{4}+\frac{a\,b^{6}\,\sqrt{b^{6}+a\,b^{5}}}{4}\right)\left(e^{x}\left(\frac{2\,a^{2}}{b^{8}\,(a+b)^{2}\,\sqrt{a^{4}}}-\right.\right.\right.\right.\right.}{2\,\sqrt{b^{6}+a\,b^{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a + b*cosh(x)^2),x)
```

```
[Out] exp(3*x)/(24*b) - exp(-3*x)/(24*b) + (exp(-x)*(4*a - 3*b))/(8*b^2) + ((a^4)
^(1/2)*(2*atan((a^2*exp(x)*(b^5*(a + b))^(1/2))/(2*b^2*(a + b)*(a^4)^(1/2))
) - 2*atan(((b^7*(a*b^5 + b^6)^(1/2))/4 + (a*b^6*(a*b^5 + b^6)^(1/2))/4)*(e
xp(x)*((2*a^2)/(b^8*(a + b)^2*(a^4)^(1/2)) - (4*(2*a^3*b^3*(a^4)^(1/2) + 2*
a^4*b^2*(a^4)^(1/2)))/(a^5*b^6*(a + b)*(b^5*(a + b))^(1/2)*(a*b^5 + b^6)^(1
/2))) - (2*a^2*exp(3*x))/(b^8*(a + b)^2*(a^4)^(1/2))))))/(2*(a*b^5 + b^6)^(
1/2)) - (exp(x)*(4*a - 3*b))/(8*b^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.23 $\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

```
[Out] -1/2*(2*a-b)*x/b^2+1/2*cosh(x)*sinh(x)/b+a^(3/2)*arctanh(a^(1/2)*tanh(x)/(a
+b)^(1/2))/b^2/(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3187, 470, 522, 206, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] -((2*a - b)*x)/(2*b^2) + (a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(
b^2*Sqrt[a + b]) + (Cosh[x]*Sinh[x])/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2\left(a-(a+b)x^2\right)}\,dx, x, \coth(x)\right)$$

$$= \frac{\cosh(x)\sinh(x)}{2b} + \frac{\text{Subst}\left(\int \frac{a+(a-b)x^2}{\left(1-x^2\right)\left(a+(-a-b)x^2\right)}\,dx, x, \coth(x)\right)}{2b}$$

$$= \frac{\cosh(x)\sinh(x)}{2b} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(-a-b)x^2}\,dx, x, \coth(x)\right)}{b^2} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.}{2b^2}$$

$$= -\frac{(2a-b)x}{2b^2} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}} + \frac{\cosh(x)\sinh(x)}{2b}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.88

$$\frac{\frac{4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 2x(b-2a) + b\sinh(2x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] (2*(-2*a + b)*x + (4*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/Sqrt[a
 + b] + b*Sinh[2*x])/(4*b^2)
```

fricas [B] time = 0.58, size = 573, normalized size = 9.71

$$\left[\frac{b\cosh(x)^4 + 4\,b\cosh(x)\sinh(x)^3 + b\sinh(x)^4 - 4\,(2\,a-b)x\cosh(x)^2 + 2\left(3\,b\cosh(x)^2 - 2\,(2\,a-b)x\right)\sinh(x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a - b)*x*cos
h(x)^2 + 2*(3*b*cosh(x)^2 - 2*(2*a - b)*x)*sinh(x)^2 + 4*(a*cosh(x)^2 + 2*a
*cosh(x)*sinh(x) + a*sinh(x)^2)*sqrt(a/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*
cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*co
sh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 +
 (2*a*b + b^2)*cosh(x))*sinh(x) - 4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*
cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + 2*a^2 + 3*a*b + b^2)*sqrt(a/(a +
b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(
x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*c
osh(x))*sinh(x) + b)) + 4*(b*cosh(x)^3 - 2*(2*a - b)*x*cosh(x))*sinh(x) - b
)/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2), 1/8*(b*cosh(x)^4
 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a - b)*x*cosh(x)^2 + 2*(3*b*c
osh(x)^2 - 2*(2*a - b)*x)*sinh(x)^2 + 8*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x)
+ a*sinh(x)^2)*sqrt(-a/(a + b))*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(
```

```
x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a/(a + b))/a) + 4*(b*cosh(x)^3 - 2*(2*a -
 b)*x*cosh(x))*sinh(x) - b)/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*si
nh(x)^2)]
```

giac [B] time = 0.13, size = 95, normalized size = 1.61

$$\frac{a^2 \arctan\left(\frac{be^{(2x)}+2\,a+b}{2\,\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,b^2}-\frac{(2\,a-b)x}{2\,b^2}+\frac{e^{(2x)}}{8\,b}+\frac{\left(4\,ae^{(2x)}-2\,be^{(2x)}-b\right)e^{(-2x)}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] a^2*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^
2) - 1/2*(2*a - b)*x/b^2 + 1/8*e^(2*x)/b + 1/8*(4*a*e^(2*x) - 2*b*e^(2*x) -
 b)*e^(-2*x)/b^2
```

maple [B] time = 0.10, size = 188, normalized size = 3.19

$$\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*cosh(x)^2),x)
```

```
[Out] 1/2/b/(tanh(1/2*x)-1)^2+1/2/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)-1/2/b
*ln(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^2+1/2/b/(tanh(1/2*x)+1)-a/b^2*ln(t
anh(1/2*x)+1)+1/2/b*ln(tanh(1/2*x)+1)-1/2/b^2*a^(3/2)/(a+b)^(1/2)*ln(-(a+b)
^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))+1/2/b^2*a^(3/2)/(a+
b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B] time = 0.56, size = 347, normalized size = 5.88

$$-\frac{(2\,a+b)\log\left(\frac{be^{(2x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}\,b}-\frac{3\,\log\left(\frac{be^{(-2x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{16\,\sqrt{(a+b)a}}-\frac{(2\,a+b)x}{b^2}+\frac{x}{b}+\frac{e^{(2x)}}{8\,b}-\frac{e^{(-2x)}}{8\,b}+\frac{(2\,a+b)\log\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2
*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 3/16*log((b*e^(-2*x) + 2
*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/sqr
t((a + b)*a) - (2*a + b)*x/b^2 + x/b + 1/8*e^(2*x)/b - 1/8*e^(-2*x)/b + 1/8
*(2*a + b)*log(b*e^(4*x) + 2*(2*a + b)*e^(2*x) + b)/b^2 - 1/8*(2*a + b)*log
(2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^2 + 1/32*(8*a^2 + 8*a*b + b^2)*lo
g((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2*a + b + 2*sqrt((
a + b)*a)))/(sqrt((a + b)*a)*b^2) - 1/32*(8*a^2 + 8*a*b + b^2)*log((b*e^(-2
*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a
)))/(sqrt((a + b)*a)*b^2)
```

mupad [B] time = 1.17, size = 142, normalized size = 2.41

$$\frac{e^{2x}}{8\,b}-\frac{e^{-2x}}{8\,b}-\frac{x\,(2\,a-b)}{2\,b^2}+\frac{a^{3/2}\,\ln\left(-\frac{4\,a^2\,e^{2x}}{b^3}-\frac{2\,a^{3/2}\,\left(b+2\,a\,e^{2x}+b\,e^{2x}\right)}{b^3\,\sqrt{a+b}}\right)}{2\,b^2\,\sqrt{a+b}}-\frac{a^{3/2}\,\ln\left(\frac{2\,a^{3/2}\,\left(b+2\,a\,e^{2x}+b\,e^{2x}\right)}{b^3\,\sqrt{a+b}}-\frac{4\,a^2\,e^{2x}}{b^3}\right)}{2\,b^2\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*cosh(x)^2),x)
```

```
[Out] exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (x*(2*a - b))/(2*b^2) + (a^(3/2)*log(- (
4*a^2*exp(2*x))/b^3 - (2*a^(3/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(b^3*(a +
 b)^(1/2))))/(2*b^2*(a + b)^(1/2)) - (a^(3/2)*log((2*a^(3/2)*(b + 2*a*exp(2
*x) + b*exp(2*x)))/(b^3*(a + b)^(1/2)) - (4*a^2*exp(2*x))/b^3))/(2*b^2*(a +
 b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.24 $\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=38

$$\frac{\sinh(x)}{b}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

```
[Out] sinh(x)/b-a*arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 388, 205}

$$\frac{\sinh(x)}{b}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sinh[x
]/b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{a+b+bx^2}\,dx,x,\sinh(x)\right)\\ &= \frac{\sinh(x)}{b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,x,\sinh(x)\right)}{b}\\ &= -\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}+\frac{\sinh(x)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.00

$$\frac{\sinh(x)}{b}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sinh[x
]/b
```

fricas [B] time = 0.45, size = 498, normalized size = 13.11

$$\left[\frac{\left(ab+b^2\right)\cosh(x)^2+2\left(ab+b^2\right)\cosh(x)\sinh(x)+\left(ab+b^2\right)\sinh(x)^2-\sqrt{-ab-b^2}\,(a\cosh(x)+a\sinh(x))\,\mathrm{lc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*s
inh(x)^2 - sqrt(-a*b - b^2)*(a*cosh(x) + a*sinh(x))*log((b*cosh(x)^4 + 4*b*
cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^
2 - 2*a - 3*b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) +
4*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x)
- cosh(x))*sqrt(-a*b - b^2) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*s
inh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 +
4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) - a*b - b^2)/((a*b^2 + b^
3)*cosh(x) + (a*b^2 + b^3)*sinh(x)), 1/2*((a*b + b^2)*cosh(x)^2 + 2*(a*b +
b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 - 2*sqrt(a*b + b^2)*(a*cosh(x)
 + a*sinh(x))*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3
 + (4*a + 3*b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + 3*b)*sinh(x))/sqrt(a*b + b^
2)) - 2*sqrt(a*b + b^2)*(a*cosh(x) + a*sinh(x))*arctan(1/2*sqrt(a*b + b^2)*
(cosh(x) + sinh(x))/(a + b)) - a*b - b^2)/((a*b^2 + b^3)*cosh(x) + (a*b^2 +
 b^3)*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[81,-22]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[55,-12]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.09, size = 96, normalized size = 2.53

$$-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)-2\sqrt{a}}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}\sqrt{a+b}}-\frac{a\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)+2\sqrt{a}}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}\sqrt{a+b}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*cosh(x)^2),x)
```

```
[Out] -1/b/(tanh(1/2*x)+1)-a/b^(3/2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1
/2*x)-2*a^(1/2))/b^(1/2))-a/b^(3/2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*t
anh(1/2*x)+2*a^(1/2))/b^(1/2))-1/b/(tanh(1/2*x)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2x)}-1\right)e^{(-x)}}{2b}-\frac{1}{8}\int\frac{16\left(ae^{(3x)}+ae^{x}\right)}{b^2e^{(4x)}+b^2+2\left(2ab+b^2\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x)/b - 1/8*integrate(16*(a*e^(3*x) + a*e^x)/(b^2*e^(4
*x) + b^2 + 2*(2*a*b + b^2)*e^(2*x)), x)
```

mupad [B] time = 1.16, size = 204, normalized size = 5.37

$$\frac{e^x}{2b}-\frac{e^{-x}}{2b}-\frac{\left(2\,\mathrm{atan}\left(\frac{a^3\,e^x\,\sqrt{b^3\,(a+b)}}{2\,b\,(a+b)\,\left(a^2\right)^{3/2}}\right)-2\,\mathrm{atan}\left(\left(\frac{b^5\,\sqrt{b^4+a\,b^3}}{4}+\frac{a\,b^4\,\sqrt{b^4+a\,b^3}}{4}\right)\left(e^x\left(\frac{2\,a^3}{b^5\,(a+b)^2\,\left(a^2\right)^{3/2}}-\frac{4\left(2\,b^2\left(a^2\right)^{3/2}+2\,a\,b\left(a^2\right)^{3/2}\right)}{a^3\,b^4\,(a+b)\,\sqrt{b^3\,(a+b)}\,\sqrt{b^4+a}}\right.\right.\right.}{2\,\sqrt{b^4+a\,b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*cosh(x)^2),x)
```

```
[Out] exp(x)/(2*b) - exp(-x)/(2*b) - ((2*atan((a^3*exp(x)*(b^3*(a + b))^(1/2))/(2
*b*(a + b)*(a^2)^(3/2))) - 2*atan(((b^5*(a*b^3 + b^4)^(1/2))/4 + (a*b^4*(a*
b^3 + b^4)^(1/2))/4)*(exp(x)*((2*a^3)/(b^5*(a + b)^2*(a^2)^(3/2)) - (4*(2*b
^2*(a^2)^(3/2) + 2*a*b*(a^2)^(3/2)))/(a^3*b^4*(a + b)*(b^3*(a + b))^(1/2)*(
a*b^3 + b^4)^(1/2))) - (2*a^3*exp(3*x))/(b^5*(a + b)^2*(a^2)^(3/2)))))*(a^2
)^(1/2))/(2*(a*b^3 + b^4)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.25 $\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}$$

```
[Out] x/b-arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))*a^(1/2)/b/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = $\{3171, 3181, 208\}$

$$\frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[x]^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(b*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh^2(x)}\,dx}{b}\\ &= \frac{x}{b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-(a+b)x^2}\,dx,x,\coth(x)\right)}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 0.92

$$\frac{x-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[x]^2),x]
```

```
[Out] (x - (Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/Sqrt[a + b])/b
```

fricas [A] time = 0.49, size = 317, normalized size = 8.13

$$\left[\frac{\sqrt{\frac{a}{a+b}}\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\left(2\,ab+b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+2\,ab+b^2\right)\sinh(x)^2+8\,a^2+8\,ab+b^2+4\left(b^2\cosh(x\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\left(3\,b\cosh(\right.}\right.}{2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*si
nh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sin
h(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*si
nh(x) + 4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b
^2)*sinh(x)^2 + 2*a^2 + 3*a*b + b^2)*sqrt(a/(a + b)))/(b*cosh(x)^4 + 4*b*co
sh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 +
2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 2*
x)/b, -(sqrt(-a/(a + b))*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*
sinh(x)^2 + 2*a + b)*sqrt(-a/(a + b))/a) - x)/b]
```

giac [A] time = 0.13, size = 50, normalized size = 1.28

$$-\frac{a\arctan\left(\frac{be^{(2\,x)}+2\,a+b}{2\,\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b)
+ x/b
```

maple [B] time = 0.10, size = 110, normalized size = 2.82

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{\sqrt{a}\;\ln\left(-\sqrt{a+b}\;\left(\tanh^2\left(\frac{x}{2}\right)\right)+2\sqrt{a}\;\tanh\left(\frac{x}{2}\right)-\sqrt{a+b}\right)}{2b\sqrt{a+b}}-\frac{\sqrt{a}\;\ln\left(\sqrt{a+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(x)^2),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)+1/2/b*a^(1/2)/(a+b)^(1/2)*ln(-
(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))-1/2/b*a^(1/2)/
(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B] time = 0.51, size = 120, normalized size = 3.08

$$-\frac{(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}\,b}-\frac{\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2
*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 1/4*log((b*e^(-2*x) + 2*
a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/sqrt
((a + b)*a) + x/b
```

mupad [B] time = 1.38, size = 376, normalized size = 9.64

$$\frac{x}{b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(b^5\,\sqrt{-b^3-a\,b^2}+a\,b^4\,\sqrt{-b^3-a\,b^2}\right)\left(\mathrm{e}^{2\,x}\left(\frac{2\left(8\,a^{5/2}\,\sqrt{-b^3-a\,b^2}+\sqrt{a}\,b^2\,\sqrt{-b^3-a\,b^2}+8\,a^{3/2}\,b\,\sqrt{-b^3-a\,b^2}\right)\left(8\,a^2+8\,a\,b+b^2\right)}{b^8\,{\left(a+b\right)}^2\,\sqrt{-b^3-a\,b^2}}+\frac{4\,\sqrt{a}\,\left(4\,a+2\,b\right)\left(8\,a^3\,b+\cdots\right.}{b^7\,\left(a+b\right)\,\sqrt{-b^2\,\left(a+b\cdots\right.}}\cdots\right)\right.}{4\,a}\cdots\right)}{\sqrt{-b^3-a\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*cosh(x)^2),x)
```

```
[Out] x/b + (a^(1/2)*atan(((b^5*(- a*b^2 - b^3)^(1/2) + a*b^4*(- a*b^2 - b^3)^(1/
2))*(exp(2*x)*((2*(8*a^(5/2)*(- a*b^2 - b^3)^(1/2) + a^(1/2)*b^2*(- a*b^2 -
 b^3)^(1/2) + 8*a^(3/2)*b*(- a*b^2 - b^3)^(1/2))*(8*a*b + 8*a^2 + b^2))/(b^
8*(a + b)^2*(- a*b^2 - b^3)^(1/2)) + (4*a^(1/2)*(4*a + 2*b)*(4*a*b^3 + 8*a^
3*b + 12*a^2*b^2))/(b^7*(a + b)*(-b^2*(a + b))^(1/2)*(- a*b^2 - b^3)^(1/2))
) + (2*(a^(1/2)*b^2*(- a*b^2 - b^3)^(1/2) + 2*a^(3/2)*b*(- a*b^2 - b^3)^(1/
2))*(8*a*b + 8*a^2 + b^2))/(b^8*(a + b)^2*(- a*b^2 - b^3)^(1/2)) + (4*a^(1/
2)*(2*a*b^3 + 2*a^2*b^2)*(4*a + 2*b))/(b^7*(a + b)*(-b^2*(a + b))^(1/2)*(-
a*b^2 - b^3)^(1/2))))/(4*a)))/(- a*b^2 - b^3)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.26 $\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

```
[Out] arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))/b^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3186, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \sinh(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

fricas [B] time = 0.46, size = 337, normalized size = 11.62

$$\left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2\,(2\,a+3\,b)\cosh(x)^2+2\,\left(3\,b\cosh(x)^2-2\,a-3\,b\right)\sinh(x)^2+4\,\left(b\cosh(x)^3-(2\,a+\cdots\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\,\left(3\,b\cosh\cdots\right.}\right.}{2\,\left(ab+b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b - b^2)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)
^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 + 4*
(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*(cosh(x)^3 + 3*cosh(x)*sinh
(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))*sqrt(-a*b - b^2) +
 b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x
)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*co
sh(x))*sinh(x) + b))/(a*b + b^2), (sqrt(a*b + b^2)*arctan(1/2*(b*cosh(x)^3
+ 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + 3*b)*cosh(x) + (3*b*cosh(x)^
2 + 4*a + 3*b)*sinh(x))/sqrt(a*b + b^2)) + sqrt(a*b + b^2)*arctan(1/2*sqrt(
a*b + b^2)*(cosh(x) + sinh(x))/(a + b)))/(a*b + b^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-58,31]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-85,-18]Undef/Unsigned Inf encountered in limitLimit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.07, size = 66, normalized size = 2.28

$$\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)-2\sqrt{a}}{2\sqrt{b}}\right)}{\sqrt{a+b}\,\sqrt{b}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)+2\sqrt{a}}{2\sqrt{b}}\right)}{\sqrt{a+b}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x)^2),x)
```

```
[Out] 1/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)-2*a^(1/2))/b^(1
/2))+1/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)+2*a^(1/2))
/b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{b\cosh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/(b*cosh(x)^2 + a), x)
```

mupad [B] time = 1.19, size = 87, normalized size = 3.00

$$-\frac{\ln\left(-\frac{4\left(a-a\,\mathrm{e}^{2\,x}\right)}{b^2\,(a+b)}-\frac{8\,a\,\mathrm{e}^x}{(-b)^{5/2}\,\sqrt{a+b}}\right)-\ln\left(\frac{8\,a\,\mathrm{e}^x}{(-b)^{5/2}\,\sqrt{a+b}}-\frac{4\left(a-a\,\mathrm{e}^{2\,x}\right)}{b^2\,(a+b)}\right)}{2\,\sqrt{-b}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*cosh(x)^2),x)
```

```
[Out] -(log(- (4*(a - a*exp(2*x)))/(b^2*(a + b)) - (8*a*exp(x))/((-b)^(5/2)*(a +
b)^(1/2))) - log((8*a*exp(x))/((-b)^(5/2)*(a + b)^(1/2)) - (4*(a - a*exp(2*
x)))/(b^2*(a + b))))/(2*(-b)^(1/2)*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.27 $\int \frac{1}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^2)^(-1),x]
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a-(a+b)x^2}\,dx, x, \coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^2)^(-1),x]
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

fricas [B] time = 0.56, size = 293, normalized size = 10.10

$$\left[\frac{\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\left(2\,ab+b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+2\,ab+b^2\right)\sinh(x)^2+8\,a^2+8\,ab+b^2+4\left(b^2\cosh(x)^3+\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+2\,a+b\right)\sinh(x}\right.}{2\sqrt{a^2+ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*
b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 +
8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(b*cosh
(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*co
sh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(
3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*si
nh(x) + b))/sqrt(a^2 + a*b), sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^2 + 2*b
*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a*b))/(a^
2 + a*b)]
```

giac [A] time = 0.13, size = 39, normalized size = 1.34

$$\frac{\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/sqrt(-a^2 - a*b)
```

maple [B] time = 0.08, size = 78, normalized size = 2.69

$$-\frac{\ln\left(\sqrt{a+b}\ \left(\tanh^2\left(\frac{x}{2}\right)\right)-2\sqrt{a}\ \tanh\left(\frac{x}{2}\right)+\sqrt{a+b}\right)}{2\sqrt{a}\ \sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\ \left(\tanh^2\left(\frac{x}{2}\right)\right)+2\sqrt{a}\ \tanh\left(\frac{x}{2}\right)+\sqrt{a+b}\right)}{2\sqrt{a}\ \sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^2),x)
```

```
[Out] -1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2-2*a^(1/2)*tanh(1/2*x)
+(a+b)^(1/2))+1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2
)*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B] time = 0.41, size = 53, normalized size = 1.83

$$-\frac{\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b +
 2*sqrt((a + b)*a)))/sqrt((a + b)*a)
```

mupad [B] time = 0.00, size = 267, normalized size = 9.21

$$-\frac{\operatorname{atan}\left(\frac{b^2\,e^{2x}\left(-a^2-b\,a\right)^{3/2}\left(\frac{4\,(4\,a+2\,b)\left(8\,a^3+12\,a^2\,b+4\,a\,b^2\right)}{b^5\left(-a^2-b\,a\right)^{3/2}\sqrt{-a\,(a+b)}}+\frac{2\left(8\,a^2+8\,a\,b+b^2\right)\left(8\,a^2\,\sqrt{-a^2-b\,a}+b^2\,\sqrt{-a^2-b\,a}+8\,a\,b\,\sqrt{-a^2-b\,a}\right)}{a\,b^5\,(a+b)\left(-a^2-b\,a\right)^{3/2}}\right)}{4}+\frac{\left(2\,a^2\,b+2\,a\,b^2\right)(4\,a+2\,b)}{b^3\,\sqrt{-a\,(a+b)}}+\cdots\right)}{\sqrt{-a^2-b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^2),x)
```

```
[Out] -atan((b^2*exp(2*x)*(- a*b - a^2)^(3/2)*((4*(4*a + 2*b)*(4*a*b^2 + 12*a^2*b
 + 8*a^3))/(b^5*(- a*b - a^2)^(3/2)*(-a*(a + b))^(1/2)) + (2*(8*a*b + 8*a^2
 + b^2)*(8*a^2*(- a*b - a^2)^(1/2) + b^2*(- a*b - a^2)^(1/2) + 8*a*b*(- a*b
 - a^2)^(1/2)))/(a*b^5*(a + b)*(- a*b - a^2)^(3/2))))/4 + ((2*a*b^2 + 2*a^2
*b)*(4*a + 2*b))/(b^3*(-a*(a + b))^(1/2)) + ((b^2*(- a*b - a^2)^(1/2) + 2*a
*b*(- a*b - a^2)^(1/2))*(8*a*b + 8*a^2 + b^2))/(2*a*b^3*(a + b)))/(- a*b -
a^2)^(1/2)
```

sympy [A] time = 44.77, size = 12026, normalized size = 414.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (2*tanh(
x/2)/(b*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-tanh(x/2)/(2*b) - 1/(2*b*tanh(x/2
)), Eq(a, -b)), (-5*I*a**(5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
- b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 5*I
*a**(5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2)
)/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(
a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**
(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*
b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 3*I*a**(5/2)*sqrt(b)*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(
b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(
a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2
*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*s
qrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b))) + 3*I*a**(5/2)*sqrt(b)*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*s
```

```
qrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*
b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))) + 10*I*a**(3/2)*b**(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a
**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
+ a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
 a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 10*I*a**(3/2
)*b**(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*
a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b
**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a
/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*s
qrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
+ b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b))) + 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/
```

```
(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)))
- I*sqrt(a)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(
x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I
*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b
))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b
) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) +
2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + I*sqrt(a)*b**(5/2)*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt
(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
- b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**
2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) + a/(a + b) - b/(a + b))) + I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*
b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))) - I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + t
anh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) -
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
 + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - a**3*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
```

```
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b))) + a**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b
/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + t
anh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) -
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
 + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - a**3*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/
(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b))) + a**3*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + t
anh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) -
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
 + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 10*a**2*b*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b*
*2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b))) - 10*a**2*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
```

```
 + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
 b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
+ 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 2*a**2*
b*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)
*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a
+ b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 1
0*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) + a/(a + b) - b/(a + b))) - 2*a**2*b*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b
) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)
/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
 b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
+ a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(
a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt
(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqr
t(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(
a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b
) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) -
 5*a*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(-s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*
a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b
**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a
/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
 b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) + a/(a + b) - b/(a + b))) + 5*a*b**2*sqrt(-2*I*sqrt(a)*sqrt(
b)/(a + b) + a/(a + b) - b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
+ a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**
3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) +
```

```
a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a
+ b))) + 3*a*b**2*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + tanh(x/2
))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 8*I*a*
*(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) + a/(a + b) - b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
- b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) -
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a
*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*s
qrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))) - 3*a*b**2*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b)) + tanh(x/2))/(8*I*a**(7/2)*sqrt(b)*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) + a/(a + b) - b/(a + b)) - 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*
sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) +
 a/(a + b) - b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a +
 b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))
- 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b)) - 10*a**2*b**2*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) + a/(a + b) - b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) + a/(a + b) - b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) + a/(a + b)
 - b/(a + b))), True))
```

3.28 $\int \frac{\text{sech}(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

```
[Out] arctan(sinh(x))/a-arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a/(a+b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3186, 391, 203, 205}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] ArcTan[Sinh[x]]/a - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(a*Sqrt
[a + b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \sinh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \sinh(x)\right)}{a}$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

Mathematica [A] time = 0.10, size = 45, normalized size = 1.10

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(x)}{\sqrt{b}}\right)}{a\sqrt{a+b}} + \frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Cosh[x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/(a*Sqrt[a + b]) + (2*ArcTan
[Tanh[x/2]])/a
```

fricas [B] time = 0.47, size = 360, normalized size = 8.78

$$\left[\sqrt{-\frac{b}{a+b}}\log\left(\frac{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2(2a+3b)\cosh(x)^2+2\left(3b\cosh(x)^2-2a-3b\right)\sinh(x)^2+4\left(b\cosh(x)^3-(2a+3b)\cosh(x)\right.}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2(2a+b)\cosh(x)^2+2\left(3b\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/(a + b))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)
^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 + 4*
(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*((a + b)*cosh(x)^3 + 3*(a +
 b)*cosh(x)*sinh(x)^2 + (a + b)*sinh(x)^3 - (a + b)*cosh(x) + (3*(a + b)*co
sh(x)^2 - a - b)*sinh(x))*sqrt(-b/(a + b)) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*
sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a +
b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 4*arctan
(cosh(x) + sinh(x)))/a, -(sqrt(b/(a + b))*arctan(1/2*sqrt(b/(a + b))*(cosh(
x) + sinh(x))) + sqrt(b/(a + b))*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh
(x)^2 + b*sinh(x)^3 + (4*a + 3*b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + 3*b)*sin
h(x))*sqrt(b/(a + b))/b) - 2*arctan(cosh(x) + sinh(x)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-97,37]Warning, need to choose a branch for the root of a p
```

```
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-85,-18]Undef/Unsigned Inf encountered in limitLimit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.13, size = 84, normalized size = 2.05

$$-\frac{\sqrt{b}\,\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)-2\sqrt{a}}{2\sqrt{b}}\right)}{a\sqrt{a+b}}-\frac{\sqrt{b}\,\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)+2\sqrt{a}}{2\sqrt{b}}\right)}{a\sqrt{a+b}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*cosh(x)^2),x)
```

```
[Out] -1/a*b^(1/2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)-2*a^(1/2))/b
^(1/2))-1/a*b^(1/2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)+2*a^(
1/2))/b^(1/2))+2/a*arctan(tanh(1/2*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\arctan\left(e^{x}\right)}{a}-2\int\frac{be^{(3x)}+be^{x}}{abe^{(4x)}+ab+2\left(2a^{2}+ab\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^x)/a - 2*integrate((b*e^(3*x) + b*e^x)/(a*b*e^(4*x) + a*b + 2*(2
*a^2 + a*b)*e^(2*x)), x)
```

mupad [B] time = 1.30, size = 208, normalized size = 5.07

$$\frac{2\,\mathrm{atan}\left(\frac{e^{x}\left(16\left(a^{2}\right)^{3/2}+9b^{2}\sqrt{a^{2}}+24ab\sqrt{a^{2}}\right)}{16a^{3}+24a^{2}b+9ab^{2}}\right)}{\sqrt{a^{2}}}-\frac{\sqrt{b}\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,e^{x}\sqrt{a^{2}(a+b)}}{2a(a+b)}\right)+2\,\mathrm{atan}\left(\frac{4a^{4}e^{x}+8a^{3}be^{x}+4a^{2}b^{2}e^{x}-be^{x}\sqrt{a^{2}(a+b}}{\sqrt{b}\sqrt{a^{2}(a+b)}}\right.\right.}{2\sqrt{a^{3}+ba^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*cosh(x)^2)),x)
```

```
[Out] (2*atan((exp(x)*(16*(a^2)^(3/2) + 9*b^2*(a^2)^(1/2) + 24*a*b*(a^2)^(1/2)))/
(9*a*b^2 + 24*a^2*b + 16*a^3)))/(a^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp
(x)*(a^2*(a + b))^(1/2))/(2*a*(a + b))) + 2*atan((4*a^4*exp(x) + 8*a^3*b*ex
p(x) + 4*a^2*b^2*exp(x) - b*exp(x)*(a^2*(a + b))^(1/2)*(a^2*b + a^3)^(1/2)
+ b*exp(3*x)*(a^2*(a + b))^(1/2)*(a^2*b + a^3)^(1/2))/(b^(1/2)*(a^2*(a + b)
)^(1/2)*(2*a*b + 2*a^2)))))/(2*(a^2*b + a^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}(x)}{a+b\cosh^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(sech(x)/(a + b*cosh(x)**2), x)
```

3.29 $\int \frac{\text{sech}^2(x)}{a+b\cosh^2(x)} \, dx$

Optimal. Leaf size=38

$$\frac{\tanh(x)}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}$$

```
[Out] -b*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(3/2)/(a+b)^(1/2)+tanh(x)/a
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3187, 453, 208}

$$\frac{\tanh(x)}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Cosh[x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b])) + Tanh[
x]/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^2(x)}{a+b\cosh^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x^2\left(a-(a+b)x^2\right)}\,dx,x,\coth(x)\right)\\
&= \frac{\tanh(x)}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+(-a-b)x^2}\,dx,x,\coth(x)\right)}{a}\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tanh(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 38, normalized size = 1.00

$$\frac{\tanh(x)}{a} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Cosh[x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b])) + Tanh[
x]/a
```

fricas [B] time = 0.45, size = 457, normalized size = 12.03

$$\left[\frac{\left(b\cosh(x)^2 + 2\,b\cosh(x)\sinh(x) + b\sinh(x)^2 + b\right)\sqrt{a^2+ab}\,\log\left(\frac{b^2\cosh(x)^4 + 4\,b^2\cosh(x)\sinh(x)^3 + b^2\sinh(x)^4 + 2\,(2\,ab+}{b\cosh(x)^4+}\right.}{2\left(a^3+a^2b+\left(a^3+a^2b\right)\text{co}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a^2 + a*b)
*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b +
b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*
b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) + 4*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*cosh(x
)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*
cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x
) + b)) - 4*a^2 - 4*a*b)/(a^3 + a^2*b + (a^3 + a^2*b)*cosh(x)^2 + 2*(a^3 +
a^2*b)*cosh(x)*sinh(x) + (a^3 + a^2*b)*sinh(x)^2), -((b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^2 +
2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a*b))
+ 2*a^2 + 2*a*b)/(a^3 + a^2*b + (a^3 + a^2*b)*cosh(x)^2 + 2*(a^3 + a^2*b)*c
osh(x)*sinh(x) + (a^3 + a^2*b)*sinh(x)^2)]
```

giac [A] time = 0.12, size = 58, normalized size = 1.53

$$-\frac{b\arctan\left(\frac{be^{(2\,x)}+2\,a+b}{2\,\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a} - \frac{2}{a\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a)
- 2/(a*(e^(2*x) + 1))
```

maple [B] time = 0.14, size = 102, normalized size = 2.68

$$\frac{b\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right)+2\sqrt{a}\,\tanh\left(\frac{x}{2}\right)-\sqrt{a+b}\right)}{2a^{\frac{3}{2}}\sqrt{a+b}} - \frac{b\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right)+2\sqrt{a}\,\tanh\left(\frac{x}{2}\right)+\sqrt{a+b}\right.}{2a^{\frac{3}{2}}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*cosh(x)^2),x)
```

```
[Out] 1/2/a^(3/2)*b/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*
x)-(a+b)^(1/2))-1/2/a^(3/2)*b/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^
(1/2)*tanh(1/2*x)+(a+b)^(1/2))+2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [B] time = 0.56, size = 70, normalized size = 1.84

$$\frac{b \log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\,a} + \frac{2}{ae^{(-2x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b
+ 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*a) + 2/(a*e^(-2*x) + a)
```

mupad [B] time = 0.28, size = 108, normalized size = 2.84

$$\frac{b\,\ln\left(\frac{4\,e^{2\,x}}{a}-\frac{2\,\left(b+2\,a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^{3/2}\,\sqrt{a+b}}\right)}{2\,a^{3/2}\,\sqrt{a+b}}-\frac{2}{a\,\left(e^{2\,x}+1\right)}-\frac{b\,\ln\left(\frac{4\,e^{2\,x}}{a}+\frac{2\,\left(b+2\,a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^{3/2}\,\sqrt{a+b}}\right)}{2\,a^{3/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*cosh(x)^2)),x)
```

```
[Out] (b*log((4*exp(2*x))/a - (2*(b + 2*a*exp(2*x) + b*exp(2*x)))/(a^(3/2)*(a + b
)^(1/2))))/(2*a^(3/2)*(a + b)^(1/2)) - 2/(a*(exp(2*x) + 1)) - (b*log((4*exp
(2*x))/a + (2*(b + 2*a*exp(2*x) + b*exp(2*x)))/(a^(3/2)*(a + b)^(1/2))))/(2
*a^(3/2)*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(a + b*cosh(x)**2), x)
```

3.30 $\int \frac{\mathrm{sech}^3(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}}+\frac{(a-2b)\tan^{-1}(\sinh(x))}{2a^2}+\frac{\tanh(x)\mathrm{sech}(x)}{2a}$$

```
[Out] 1/2*(a-2*b)*arctan(sinh(x))/a^2+b^(3/2)*arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))
/a^2/(a+b)^(1/2)+1/2*sech(x)*tanh(x)/a
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3186, 414, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}}+\frac{(a-2b)\tan^{-1}(\sinh(x))}{2a^2}+\frac{\tanh(x)\mathrm{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTan[Sinh[x]])/(2*a^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Sinh[x])/Sqr
t[a + b]])/(a^2*Sqrt[a + b]) + (Sech[x]*Tanh[x])/(2*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
```

```
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx = \operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+bx^2\right)}\,dx, x, \sinh(x)\right)$$
$$= \frac{\operatorname{sech}(x)\tanh(x)}{2a} - \frac{\operatorname{Subst}\left(\int \frac{-a+b-bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)}\,dx, x, \sinh(x)\right)}{2a}$$
$$= \frac{\operatorname{sech}(x)\tanh(x)}{2a} + \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)}{2a^2} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \sin\right.}{a^2}$$
$$= \frac{(a-2b)\tan^{-1}(\sinh(x))}{2a^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}} + \frac{\operatorname{sech}(x)\tanh(x)}{2a}$$

Mathematica [A] time = 0.17, size = 58, normalized size = 0.98

$$\frac{-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(x)}{\sqrt{b}}\right)}{\sqrt{a+b}} + 2(a-2b)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + a\tanh(x)\operatorname{sech}(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Cosh[x]^2),x]
```

```
[Out] ((-2*b^(3/2)*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/Sqrt[a + b] + 2*(a - 2*
b)*ArcTan[Tanh[x/2]] + a*Sech[x]*Tanh[x])/(2*a^2)
```

fricas [B] time = 0.53, size = 963, normalized size = 16.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*cosh(x)^3 + 6*a*cosh(x)*sinh(x)^2 + 2*a*sinh(x)^3 + (b*cosh(x)^4
+ 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 +
b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-b/(a + b))*lo
g((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a + 3*b)*cosh(x
)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (2*a + 3*b
)*cosh(x))*sinh(x) + 4*((a + b)*cosh(x)^3 + 3*(a + b)*cosh(x)*sinh(x)^2 + (
a + b)*sinh(x)^3 - (a + b)*cosh(x) + (3*(a + b)*cosh(x)^2 - a - b)*sinh(x))
*sqrt(-b/(a + b)) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 +
 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(
x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 2*((a - 2*b)*cosh(x)^4 + 4*(a - 2
*b)*cosh(x)*sinh(x)^3 + (a - 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*
(a - 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 + 4*((a - 2*b)*cosh(x)^3 + (a - 2*
b)*cosh(x))*sinh(x) + a - 2*b)*arctan(cosh(x) + sinh(x)) - 2*a*cosh(x) + 2*
(3*a*cosh(x)^2 - a)*sinh(x))/(a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2
*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 +
4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x)), (a*cosh(x)^3 + 3*a*cosh(x)*sinh(x
)^2 + a*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*
b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))
*sinh(x) + b)*sqrt(b/(a + b))*arctan(1/2*sqrt(b/(a + b))*(cosh(x) + sinh(x)
```

```
)) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2
*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*s
qrt(b/(a + b))*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^
3 + (4*a + 3*b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + 3*b)*sinh(x))*sqrt(b/(a +
b))/b) + ((a - 2*b)*cosh(x)^4 + 4*(a - 2*b)*cosh(x)*sinh(x)^3 + (a - 2*b)*s
inh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a - 2*b)*cosh(x)^2 + a - 2*b)*sinh
(x)^2 + 4*((a - 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a - 2*b)*arct
an(cosh(x) + sinh(x)) - a*cosh(x) + (3*a*cosh(x)^2 - a)*sinh(x))/(a^2*cosh(
x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2
*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x)
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[37,-9]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[63,73]Undef/Unsigned Inf encountered in limitLimit: Max order reache
d or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.14, size = 131, normalized size = 2.22

$$\frac{b^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)-2\sqrt{a}}{2\sqrt{b}}\right)}{a^{2}\sqrt{a+b}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)+2\sqrt{a}}{2\sqrt{b}}\right)}{a^{2}\sqrt{a+b}}-\frac{\tanh^{3}\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}+\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*cosh(x)^2),x)
```

```
[Out] b^(3/2)/a^2/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)-2*a^(1/2))/b^
(1/2))+b^(3/2)/a^2/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)+2*a^(1
/2))/b^(1/2))-1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3+1/a/(tanh(1/2*x)^2+1)^2
*tanh(1/2*x)+1/a*arctan(tanh(1/2*x))-2/a^2*arctan(tanh(1/2*x))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3\,x)}-e^{x}}{ae^{(4\,x)}+2\,ae^{(2\,x)}+a}+\frac{(a-2\,b)\arctan\left(e^{x}\right)}{a^{2}}+8\int\frac{b^{2}e^{(3\,x)}+b^{2}e^{x}}{4\left(a^{2}be^{(4\,x)}+a^{2}b+2\left(2\,a^{3}+a^{2}b\right)e^{(2\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] (e^(3*x) - e^x)/(a*e^(4*x) + 2*a*e^(2*x) + a) + (a - 2*b)*arctan(e^x)/a^2 +
 8*integrate(1/4*(b^2*e^(3*x) + b^2*e^x)/(a^2*b*e^(4*x) + a^2*b + 2*(2*a^3
+ a^2*b)*e^(2*x)), x)
```

mupad [B] time = 1.56, size = 447, normalized size = 7.58

$$\frac{\mathrm{atan}\left(\frac{e^{x}\left(a^{7}\left(a^{4}\right)^{3/2}-12\,b^{3}\left(a^{4}\right)^{5/2}-18\,b^{7}\left(a^{4}\right)^{3/2}+36\,a^{2}\,b^{5}\left(a^{4}\right)^{3/2}-30\,a^{3}\,b^{4}\left(a^{4}\right)^{3/2}+21\,a^{5}\,b^{2}\left(a^{4}\right)^{3/2}+9\,a\,b^{6}\left(a^{4}\right)^{3/2}-8\,a^{6}\,b\left(a^{4}\right)^{3/2}\right)}{a^{12}\,\sqrt{a^{2}-4\,a\,b+4\,b^{2}}-6\,a^{11}\,b\,\sqrt{a^{2}-4\,a\,b+4\,b^{2}}+9\,a^{6}\,b^{6}\,\sqrt{a^{2}-4\,a\,b+4\,b^{2}}-18\,a^{8}\,b^{4}\,\sqrt{a^{2}-4\,a\,b+4\,b^{2}}+6\,a^{9}\,b^{3}\,\sqrt{a^{2}-4\,a\,b+4\,b^{2}}+9\,a^{10}\,b^{2}\,\sqrt{a^{2}-4\,a\,b}}\right.}{\sqrt{a^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*cosh(x)^2)),x)
```

```
[Out] (atan((exp(x)*(a^7*(a^4)^(3/2) - 12*b^3*(a^4)^(5/2) - 18*b^7*(a^4)^(3/2) +
36*a^2*b^5*(a^4)^(3/2) - 30*a^3*b^4*(a^4)^(3/2) + 21*a^5*b^2*(a^4)^(3/2) +
9*a*b^6*(a^4)^(3/2) - 8*a^6*b*(a^4)^(3/2)))/(a^12*(a^2 - 4*a*b + 4*b^2)^(1/
2) - 6*a^11*b*(a^2 - 4*a*b + 4*b^2)^(1/2) + 9*a^6*b^6*(a^2 - 4*a*b + 4*b^2)
^(1/2) - 18*a^8*b^4*(a^2 - 4*a*b + 4*b^2)^(1/2) + 6*a^9*b^3*(a^2 - 4*a*b +
4*b^2)^(1/2) + 9*a^10*b^2*(a^2 - 4*a*b + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^
2)^(1/2))/(a^4)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1)) + exp(x)
/(a*(exp(2*x) + 1)) - ((-b)^(3/2)*log((64*(exp(2*x) - 1)*(a^3 - 3*a^2*b + 3
*b^3))/(a^5*(a + b)^2) - (128*exp(x)*(a^3 - 3*a^2*b + 3*b^3))/(a^5*(-b)^(1/
2)*(a + b)^(3/2))))/(2*a^2*(a + b)^(1/2)) + ((-b)^(3/2)*log((64*(exp(2*x) -
 1)*(a^3 - 3*a^2*b + 3*b^3))/(a^5*(a + b)^2) + (128*exp(x)*(a^3 - 3*a^2*b +
 3*b^3))/(a^5*(-b)^(1/2)*(a + b)^(3/2))))/(2*a^2*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(sech(x)**3/(a + b*cosh(x)**2), x)
```

3.31 $\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=55

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tanh(x)}{a^2}-\frac{\tanh^3(x)}{3a}$$

```
[Out] b^2*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(5/2)/(a+b)^(1/2)+(a-b)*tanh(x)/
a^2-1/3*tanh(x)^3/a
```

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3187, 461, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tanh(x)}{a^2}-\frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + ((a -
b)*Tanh[x])/a^2 - Tanh[x]^3/(3*a)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4\left(a-(a+b)x^2\right)}\,dx, x, \coth(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{-a+b}{a^2x^2}+\frac{b^2}{a^2\left(a-(a+b)x^2\right)}\right)dx, x, \coth(x)\right)$$
$$= \frac{(a-b)\tanh(x)}{a^2}-\frac{\tanh^3(x)}{3a}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(a+b)x^2}\,dx, x, \coth(x)\right)}{a^2}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tanh(x)}{a^2}-\frac{\tanh^3(x)}{3a}$$

Mathematica [A] time = 0.15, size = 55, normalized size = 1.00

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{\tanh(x)\left(a\operatorname{sech}^2(x)+2a-3b\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Cosh[x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + ((2*a
- 3*b + a*Sech[x]^2)*Tanh[x])/(3*a^2)
```

fricas [B] time = 0.44, size = 1377, normalized size = 25.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(x)^4 + 48*(a^2*b + a*b^2)*cosh(x)*sinh(x)^3 +
 12*(a^2*b + a*b^2)*sinh(x)^4 - 8*a^3 + 4*a^2*b + 12*a*b^2 - 24*(a^3 - a*b^
2)*cosh(x)^2 - 24*(a^3 - a*b^2 - 3*(a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 3
*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4
 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)
^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)
*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x
))*sqrt(a^2 + a*b)*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(
x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x
)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(
x) - 4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2
 + a*b))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*c
osh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a +
b)*cosh(x))*sinh(x) + b)) + 48*((a^2*b + a*b^2)*cosh(x)^3 - (a^3 - a*b^2)*c
osh(x))*sinh(x))/((a^4 + a^3*b)*cosh(x)^6 + 6*(a^4 + a^3*b)*cosh(x)*sinh(x)
^5 + (a^4 + a^3*b)*sinh(x)^6 + 3*(a^4 + a^3*b)*cosh(x)^4 + 3*(a^4 + a^3*b +
 5*(a^4 + a^3*b)*cosh(x)^2)*sinh(x)^4 + a^4 + a^3*b + 4*(5*(a^4 + a^3*b)*co
sh(x)^3 + 3*(a^4 + a^3*b)*cosh(x))*sinh(x)^3 + 3*(a^4 + a^3*b)*cosh(x)^2 +
3*(5*(a^4 + a^3*b)*cosh(x)^4 + a^4 + a^3*b + 6*(a^4 + a^3*b)*cosh(x)^2)*sin
h(x)^2 + 6*((a^4 + a^3*b)*cosh(x)^5 + 2*(a^4 + a^3*b)*cosh(x)^3 + (a^4 + a^
3*b)*cosh(x))*sinh(x)), 1/3*(6*(a^2*b + a*b^2)*cosh(x)^4 + 24*(a^2*b + a*b^
2)*cosh(x)*sinh(x)^3 + 6*(a^2*b + a*b^2)*sinh(x)^4 - 4*a^3 + 2*a^2*b + 6*a*
b^2 - 12*(a^3 - a*b^2)*cosh(x)^2 - 12*(a^3 - a*b^2 - 3*(a^2*b + a*b^2)*cosh
(x)^2)*sinh(x)^2 + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)
```

```
^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^
2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*
b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 +
 b^2*cosh(x))*sinh(x))*sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(
x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a*b)) + 24*((a^
2*b + a*b^2)*cosh(x)^3 - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 + a^3*b)*cos
h(x)^6 + 6*(a^4 + a^3*b)*cosh(x)*sinh(x)^5 + (a^4 + a^3*b)*sinh(x)^6 + 3*(a
^4 + a^3*b)*cosh(x)^4 + 3*(a^4 + a^3*b + 5*(a^4 + a^3*b)*cosh(x)^2)*sinh(x)
^4 + a^4 + a^3*b + 4*(5*(a^4 + a^3*b)*cosh(x)^3 + 3*(a^4 + a^3*b)*cosh(x))*
sinh(x)^3 + 3*(a^4 + a^3*b)*cosh(x)^2 + 3*(5*(a^4 + a^3*b)*cosh(x)^4 + a^4
+ a^3*b + 6*(a^4 + a^3*b)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 + a^3*b)*cosh(x)^5
 + 2*(a^4 + a^3*b)*cosh(x)^3 + (a^4 + a^3*b)*cosh(x))*sinh(x))]
```

giac [A] time = 0.42, size = 87, normalized size = 1.58

$$\frac{b^2 \arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a^2} + \frac{2\left(3\,be^{(4x)} - 6\,ae^{(2x)} + 6\,be^{(2x)} - 2\,a + 3\,b\right)}{3\,a^2\left(e^{(2x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^
2) + 2/3*(3*b*e^(4*x) - 6*a*e^(2*x) + 6*b*e^(2*x) - 2*a + 3*b)/(a^2*(e^(2*x
) + 1)^3)
```

maple [B] time = 0.16, size = 139, normalized size = 2.53

$$-\frac{b^2 \ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right) + 2\sqrt{a}\,\tanh\left(\frac{x}{2}\right) - \sqrt{a+b}\right)}{2a^{\frac{5}{2}}\sqrt{a+b}} + \frac{b^2 \ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right) + 2\sqrt{a}\,\tanh\left(\frac{x}{2}\right) + \sqrt{a}\right.}{2a^{\frac{5}{2}}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*cosh(x)^2),x)
```

```
[Out] -1/2*b^2/a^(5/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1
/2*x)-(a+b)^(1/2))+1/2*b^2/a^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2
+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))-2/a^2*((-a+b)*tanh(1/2*x)^5+(-2/3*a+2*b
)*tanh(1/2*x)^3+(-a+b)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.42, size = 119, normalized size = 2.16

$$-\frac{b^2 \log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\,a^2} + \frac{2\left(6\,(a-b)e^{(-2x)} - 3\,be^{(-4x)} + 2\,a - 3\,b\right)}{3\left(3\,a^2e^{(-2x)} + 3\,a^2e^{(-4x)} + a^2e^{(-6x)} + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a +
 b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*a^2) + 2/3*(6*(a - b)*e^(-2*x) -
3*b*e^(-4*x) + 2*a - 3*b)/(3*a^2*e^(-2*x) + 3*a^2*e^(-4*x) + a^2*e^(-6*x) +
 a^2)
```

mupad [B] time = 1.32, size = 239, normalized size = 4.35

$$\frac{8}{3\,a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} - \frac{4}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)} + \frac{2\,b}{a^2\,\left(e^{2\,x}+1\right)} - \frac{b^2\,\ln\left(\frac{4\,b^2\,\left(2\,a\,b+8\,a^2\,e^{2\,x}+b^2\,e^{2\,x}+b^2+8\,a\,b\,e^{2\,x}\right)}{a^5\,(a+b)} - \frac{8\,b\right.}{2\,a^{5/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*cosh(x)^2)),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - 4/(a*(2*exp(2*x) + exp(4
*x) + 1)) + (2*b)/(a^2*(exp(2*x) + 1)) - (b^2*log((4*b^2*(2*a*b + 8*a^2*exp
(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a^5*(a + b)) - (8*b^2*(b + 4
*a*exp(2*x) + 2*b*exp(2*x)))/(a^(9/2)*(a + b)^(1/2))))/(2*a^(5/2)*(a + b)^(
1/2)) + (b^2*log((8*b^2*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(9/2)*(a + b)
^(1/2)) + (4*b^2*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2
*x)))/(a^5*(a + b))))/(2*a^(5/2)*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(sech(x)**4/(a + b*cosh(x)**2), x)
```

3.32 $\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=90

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}}+\frac{(3a-4b)\tanh(x)\operatorname{sech}(x)}{8a^2}+\frac{\left(3a^2-4ab+8b^2\right)\tan^{-1}(\sinh(x))}{8a^3}+\frac{\tanh(x)\operatorname{sech}^3(x)}{4a}$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*arctan(sinh(x))/a^3-b^(5/2)*arctan(sinh(x)*b^(1/2)/
(a+b)^(1/2))/a^3/(a+b)^(1/2)+1/8*(3*a-4*b)*sech(x)*tanh(x)/a^2+1/4*sech(x)^
3*tanh(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3186, 414, 527, 522, 203, 205}

$$\frac{\left(3a^2-4ab+8b^2\right)\tan^{-1}(\sinh(x))}{8a^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}}+\frac{(3a-4b)\tanh(x)\operatorname{sech}(x)}{8a^2}+\frac{\tanh(x)\operatorname{sech}^3(x)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*ArcTan[Sinh[x]])/(8*a^3) - (b^(5/2)*ArcTan[(Sqrt[b
]*Sinh[x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]) + ((3*a - 4*b)*Sech[x]*Tanh[x])/
(8*a^2) + (Sech[x]^3*Tanh[x])/(4*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+b+bx^2\right)}\,dx, x, \sinh(x)\right)$$
$$= \frac{\operatorname{sech}^3(x)\tanh(x)}{4a} - \frac{\text{Subst}\left(\int \frac{-3a+b-3bx^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)}\,dx, x, \sinh(x)\right)}{4a}$$
$$= \frac{(3a-4b)\operatorname{sech}(x)\tanh(x)}{8a^2} + \frac{\operatorname{sech}^3(x)\tanh(x)}{4a} + \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2+(3a-4b)bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)}\,dx, x, \text{sinh}\right.}{8a^2}$$
$$= \frac{(3a-4b)\operatorname{sech}(x)\tanh(x)}{8a^2} + \frac{\operatorname{sech}^3(x)\tanh(x)}{4a} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \sinh(x)\right)}{a^3} + (3$$
$$= \frac{\left(3a^2-4ab+8b^2\right)\tan^{-1}(\sinh(x))}{8a^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}} + \frac{(3a-4b)\operatorname{sech}(x)\tanh(x)}{8a^2} +$$

Mathematica [A] time = 0.31, size = 86, normalized size = 0.96

$$\frac{2\left(3a^2-4ab+8b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+2a^2\tanh(x)\operatorname{sech}^3(x)+\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(x)}{\sqrt{b}}\right)}{\sqrt{a+b}}+a(3a-4b)\tanh(x)\operatorname{sech}(x)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Cosh[x]^2),x]
```

```
[Out] ((8*b^(5/2)*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/Sqrt[a + b] + 2*(3*a^2 -
 4*a*b + 8*b^2)*ArcTan[Tanh[x/2]] + a*(3*a - 4*b)*Sech[x]*Tanh[x] + 2*a^2*S
ech[x]^3*Tanh[x])/(8*a^3)
```

fricas [B] time = 0.58, size = 3239, normalized size = 35.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^2 - 4*a*b)*cosh(x)^7 + 7*(3*a^2 - 4*a*b)*cosh(x)*sinh(x)^6 + (3*
a^2 - 4*a*b)*sinh(x)^7 + (11*a^2 - 4*a*b)*cosh(x)^5 + (21*(3*a^2 - 4*a*b)*c
osh(x)^2 + 11*a^2 - 4*a*b)*sinh(x)^5 + 5*(7*(3*a^2 - 4*a*b)*cosh(x)^3 + (11
*a^2 - 4*a*b)*cosh(x))*sinh(x)^4 - (11*a^2 - 4*a*b)*cosh(x)^3 + (35*(3*a^2
```

```
- 4*a*b)*cosh(x)^4 + 10*(11*a^2 - 4*a*b)*cosh(x)^2 - 11*a^2 + 4*a*b)*sinh(x
)^3 + (21*(3*a^2 - 4*a*b)*cosh(x)^5 + 10*(11*a^2 - 4*a*b)*cosh(x)^3 - 3*(11
*a^2 - 4*a*b)*cosh(x))*sinh(x)^2 + 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)
^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6
+ 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b
^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7
*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cos
h(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2
*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt
(-b/(a + b))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*
a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 + 4*(b*cosh(x)
^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*((a + b)*cosh(x)^3 + 3*(a + b)*cosh(x
)*sinh(x)^2 + (a + b)*sinh(x)^3 - (a + b)*cosh(x) + (3*(a + b)*cosh(x)^2 -
a - b)*sinh(x))*sqrt(-b/(a + b)) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
+ b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)
^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + ((3*a^2 - 4*a*b +
8*b^2)*cosh(x)^8 + 8*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 - 4
*a*b + 8*b^2)*sinh(x)^8 + 4*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^6 + 4*(7*(3*a^2
 - 4*a*b + 8*b^2)*cosh(x)^2 + 3*a^2 - 4*a*b + 8*b^2)*sinh(x)^6 + 8*(7*(3*a^
2 - 4*a*b + 8*b^2)*cosh(x)^3 + 3*(3*a^2 - 4*a*b + 8*b^2)*cosh(x))*sinh(x)^5
 + 6*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 4*a*b + 8*b^2)*cosh
(x)^4 + 30*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^2 + 9*a^2 - 12*a*b + 24*b^2)*sin
h(x)^4 + 8*(7*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^5 + 10*(3*a^2 - 4*a*b + 8*b^2
)*cosh(x)^3 + 3*(3*a^2 - 4*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 4*a
*b + 8*b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^6 + 15*(3*a^2
- 4*a*b + 8*b^2)*cosh(x)^4 + 9*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^2 + 3*a^2 -
4*a*b + 8*b^2)*sinh(x)^2 + 3*a^2 - 4*a*b + 8*b^2 + 8*((3*a^2 - 4*a*b + 8*b^
2)*cosh(x)^7 + 3*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^5 + 3*(3*a^2 - 4*a*b + 8*b
^2)*cosh(x)^3 + (3*a^2 - 4*a*b + 8*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) +
sinh(x)) - (3*a^2 - 4*a*b)*cosh(x) + (7*(3*a^2 - 4*a*b)*cosh(x)^6 + 5*(11*a
^2 - 4*a*b)*cosh(x)^4 - 3*(11*a^2 - 4*a*b)*cosh(x)^2 - 3*a^2 + 4*a*b)*sinh(
x))/(a^3*cosh(x)^8 + 8*a^3*cosh(x)*sinh(x)^7 + a^3*sinh(x)^8 + 4*a^3*cosh(x
)^6 + 6*a^3*cosh(x)^4 + 4*(7*a^3*cosh(x)^2 + a^3)*sinh(x)^6 + 8*(7*a^3*cosh
(x)^3 + 3*a^3*cosh(x))*sinh(x)^5 + 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(x)^4 +
30*a^3*cosh(x)^2 + 3*a^3)*sinh(x)^4 + 8*(7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3
 + 3*a^3*cosh(x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 +
 9*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 8*(a^3*cosh(x)^7 + 3*a^3*cosh(x)^5 + 3*
a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x)), 1/4*((3*a^2 - 4*a*b)*cosh(x)^7 + 7*(
3*a^2 - 4*a*b)*cosh(x)*sinh(x)^6 + (3*a^2 - 4*a*b)*sinh(x)^7 + (11*a^2 - 4*
a*b)*cosh(x)^5 + (21*(3*a^2 - 4*a*b)*cosh(x)^2 + 11*a^2 - 4*a*b)*sinh(x)^5
+ 5*(7*(3*a^2 - 4*a*b)*cosh(x)^3 + (11*a^2 - 4*a*b)*cosh(x))*sinh(x)^4 - (1
1*a^2 - 4*a*b)*cosh(x)^3 + (35*(3*a^2 - 4*a*b)*cosh(x)^4 + 10*(11*a^2 - 4*a
*b)*cosh(x)^2 - 11*a^2 + 4*a*b)*sinh(x)^3 + (21*(3*a^2 - 4*a*b)*cosh(x)^5 +
 10*(11*a^2 - 4*a*b)*cosh(x)^3 - 3*(11*a^2 - 4*a*b)*cosh(x))*sinh(x)^2 - 4*
(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6
+ 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^
3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b
^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3
*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cos
h(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*
cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(b/(a + b))*arctan(1/2*sqrt(b/(a + b)
)*(cosh(x) + sinh(x))) - 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*s
inh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*co
sh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)
^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(
x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 1
5*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7
 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(b/(a + b)
)*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + 3*
```

```
b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + 3*b)*sinh(x))*sqrt(b/(a + b))/b) + ((3*
a^2 - 4*a*b + 8*b^2)*cosh(x)^8 + 8*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)*sinh(x)^
7 + (3*a^2 - 4*a*b + 8*b^2)*sinh(x)^8 + 4*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^6
 + 4*(7*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^2 + 3*a^2 - 4*a*b + 8*b^2)*sinh(x)^
6 + 8*(7*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^3 + 3*(3*a^2 - 4*a*b + 8*b^2)*cosh
(x))*sinh(x)^5 + 6*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 4*a*b
 + 8*b^2)*cosh(x)^4 + 30*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^2 + 9*a^2 - 12*a*b
 + 24*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^5 + 10*(3*a^2 -
 4*a*b + 8*b^2)*cosh(x)^3 + 3*(3*a^2 - 4*a*b + 8*b^2)*cosh(x))*sinh(x)^3 +
4*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^
6 + 15*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^4 + 9*(3*a^2 - 4*a*b + 8*b^2)*cosh(x
)^2 + 3*a^2 - 4*a*b + 8*b^2)*sinh(x)^2 + 3*a^2 - 4*a*b + 8*b^2 + 8*((3*a^2
- 4*a*b + 8*b^2)*cosh(x)^7 + 3*(3*a^2 - 4*a*b + 8*b^2)*cosh(x)^5 + 3*(3*a^2
 - 4*a*b + 8*b^2)*cosh(x)^3 + (3*a^2 - 4*a*b + 8*b^2)*cosh(x))*sinh(x))*arc
tan(cosh(x) + sinh(x)) - (3*a^2 - 4*a*b)*cosh(x) + (7*(3*a^2 - 4*a*b)*cosh(
x)^6 + 5*(11*a^2 - 4*a*b)*cosh(x)^4 - 3*(11*a^2 - 4*a*b)*cosh(x)^2 - 3*a^2
+ 4*a*b)*sinh(x))/(a^3*cosh(x)^8 + 8*a^3*cosh(x)*sinh(x)^7 + a^3*sinh(x)^8
+ 4*a^3*cosh(x)^6 + 6*a^3*cosh(x)^4 + 4*(7*a^3*cosh(x)^2 + a^3)*sinh(x)^6 +
 8*(7*a^3*cosh(x)^3 + 3*a^3*cosh(x))*sinh(x)^5 + 4*a^3*cosh(x)^2 + 2*(35*a^
3*cosh(x)^4 + 30*a^3*cosh(x)^2 + 3*a^3)*sinh(x)^4 + 8*(7*a^3*cosh(x)^5 + 10
*a^3*cosh(x)^3 + 3*a^3*cosh(x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 + 15*a
^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 8*(a^3*cosh(x)^7 + 3*a^3*
cosh(x)^5 + 3*a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-54,60]Precision problem choosing root in common_EXT, curre
nt precision 14Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,8]Undef/Unsigned Inf encountered in limitLimit: Max order reached or un
able to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.15, size = 274, normalized size = 3.04

$$-\frac{b^{\frac{5}{2}} \arctan\left(\frac{2\sqrt{a+b}\ \tanh\left(\frac{x}{2}\right)-2\sqrt{a}}{2\sqrt{b}}\right)}{a^{3}\sqrt{a+b}}-\frac{b^{\frac{5}{2}} \arctan\left(\frac{2\sqrt{a+b}\ \tanh\left(\frac{x}{2}\right)+2\sqrt{a}}{2\sqrt{b}}\right)}{a^{3}\sqrt{a+b}}-\frac{5\left(\tanh^{7}\left(\frac{x}{2}\right)\right)}{4a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{\left(\tanh^{7}\left(\frac{x}{2}\right)\right)b}{a^{2}\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{3}{4a(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*cosh(x)^2),x)
```

```
[Out] -b^(5/2)/a^3/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)-2*a^(1/2))/b
^(1/2))-b^(5/2)/a^3/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*x)+2*a^(
1/2))/b^(1/2))-5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7+1/a^2/(tanh(1/2*x)^2
+1)^4*tanh(1/2*x)^7*b+3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5+1/a^2/(tanh(1
/2*x)^2+1)^4*tanh(1/2*x)^5*b-3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3-1/a^2/
(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*b+5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)-1
/a^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*b+3/4/a*arctan(tanh(1/2*x))-1/a^2*arct
an(tanh(1/2*x))*b+2/a^3*arctan(tanh(1/2*x))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(3a-4b)e^{(7x)}+(11a-4b)e^{(5x)}-(11a-4b)e^{(3x)}-(3a-4b)e^{x}}{4\left(a^2e^{(8x)}+4a^2e^{(6x)}+6a^2e^{(4x)}+4a^2e^{(2x)}+a^2\right)}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(e^x\right)}{4a^3}-32\int\frac{}{16(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((3*a - 4*b)*e^(7*x) + (11*a - 4*b)*e^(5*x) - (11*a - 4*b)*e^(3*x) - (3
*a - 4*b)*e^x)/(a^2*e^(8*x) + 4*a^2*e^(6*x) + 6*a^2*e^(4*x) + 4*a^2*e^(2*x)
 + a^2) + 1/4*(3*a^2 - 4*a*b + 8*b^2)*arctan(e^x)/a^3 - 32*integrate(1/16*(
b^3*e^(3*x) + b^3*e^x)/(a^3*b*e^(4*x) + a^3*b + 2*(2*a^4 + a^3*b)*e^(2*x)),
 x)
```

mupad [B] time = 36.10, size = 1305, normalized size = 14.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5*(a + b*cosh(x)^2)),x)
```

```
[Out] (atan((exp(x)*(243*a^12*(a^6)^(1/2) + 5024*b^6*(a^6)^(3/2) + 18432*b^12*(a^
6)^(1/2) + 6912*a^2*b^10*(a^6)^(1/2) + 30720*a^3*b^9*(a^6)^(1/2) - 26880*a^
4*b^8*(a^6)^(1/2) + 24192*a^5*b^7*(a^6)^(1/2) - 13408*a^7*b^5*(a^6)^(1/2) +
 17160*a^8*b^4*(a^6)^(1/2) - 9540*a^9*b^3*(a^6)^(1/2) + 4563*a^10*b^2*(a^6)
^(1/2) - 9216*a*b^11*(a^6)^(1/2) - 1134*a^11*b*(a^6)^(1/2)))/(81*a^13*(9*a^
4 - 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) - 270*a^12*b*(9*a^4 -
24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 2304*a^3*b^10*(9*a^4 - 2
4*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 3840*a^6*b^7*(9*a^4 - 24*
a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) - 1440*a^7*b^6*(9*a^4 - 24*a^
3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 864*a^8*b^5*(9*a^4 - 24*a^3*b
 - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1600*a^9*b^4*(9*a^4 - 24*a^3*b -
 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) - 1200*a^10*b^3*(9*a^4 - 24*a^3*b -
64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 945*a^11*b^2*(9*a^4 - 24*a^3*b - 64
*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2)))*(9*a^4 - 24*a^3*b - 64*a*b^3 + 64*b^4
 + 64*a^2*b^2)^(1/2))/(4*(a^6)^(1/2)) - (6*exp(x))/(a*(3*exp(2*x) + 3*exp(4
*x) + exp(6*x) + 1)) + ((b^5)^(1/2)*(2*atan((exp(x)*((2*(48*b^8*(a^6*b + a^
7)^(1/2) + 40*a^3*b^5*(a^6*b + a^7)^(1/2) - 15*a^4*b^4*(a^6*b + a^7)^(1/2)
+ 9*a^5*b^3*(a^6*b + a^7)^(1/2)))/(a^11*b*(a + b)*(a*b + a^2)*(a^6*b + a^7)
^(1/2)*(b^5)^(1/2)*(48*a*b^5 - 6*a^5*b + 9*a^6 + 48*b^6 + 40*a^3*b^3 + 25*a
^4*b^2)) - (4*(96*a^4*(b^5)^(3/2) + 18*a^9*(b^5)^(1/2) + 80*a^6*b^3*(b^5)^(
1/2) + 50*a^7*b^2*(b^5)^(1/2) + 96*a^3*b*(b^5)^(3/2) - 12*a^8*b*(b^5)^(1/2)
))/(a^8*b^4*(a + b)*(a*b + a^2)*(a^6*(a + b))^(1/2)*(a^6*b + a^7)^(1/2)*(9*
a^5 - 15*a^4*b + 48*b^5 + 40*a^3*b^2))) - (2*exp(3*x)*(48*b^8*(a^6*b + a^7)
^(1/2) + 40*a^3*b^5*(a^6*b + a^7)^(1/2) - 15*a^4*b^4*(a^6*b + a^7)^(1/2) +
9*a^5*b^3*(a^6*b + a^7)^(1/2)))/(a^11*b*(a + b)*(a*b + a^2)*(a^6*b + a^7)^(
1/2)*(b^5)^(1/2)*(48*a*b^5 - 6*a^5*b + 9*a^6 + 48*b^6 + 40*a^3*b^3 + 25*a^4
*b^2)))*((a^11*b*(a^6*b + a^7)^(1/2))/4 + (a^9*b^3*(a^6*b + a^7)^(1/2))/4 +
 (a^10*b^2*(a^6*b + a^7)^(1/2))/2)) - 2*atan((b^3*exp(x)*(a^6*(a + b))^(1/2
)*(9*a^5 - 15*a^4*b + 48*b^5 + 40*a^3*b^2))/(2*a^3*(b^5)^(1/2)*(48*a*b^5 -
6*a^5*b + 9*a^6 + 48*b^6 + 40*a^3*b^3 + 25*a^4*b^2)))))/(2*(a^6*b + a^7)^(1
/2)) + (4*exp(x))/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
 + (exp(x)*(a + 4*b))/(2*a^2*(2*exp(2*x) + exp(4*x) + 1)) - (exp(x)*(4*a*b
- 3*a^2))/(4*a^3*(exp(2*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

```
[Out] 1/2*(2*a+b)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)-1/2*b*
cosh(x)*sinh(x)/a/(a+b)/(a+b*cosh(x)^2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3184, 12, 3181, 208}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^2)^(-2),x]
```

```
[Out] ((2*a + b)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^(3/2)
) - (b*Cosh[x]*Sinh[x])/(2*a*(a + b)*(a + b*Cosh[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx = -\frac{b\cosh(x)\sinh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)} - \frac{\int \frac{-2a-b}{a+b\cosh^2(x)}\,dx}{2a(a+b)}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)} + \frac{(2a+b)\int \frac{1}{a+b\cosh^2(x)}\,dx}{2a(a+b)}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{a-(a+b)x^2}\,dx, x, \coth(x)\right)}{2a(a+b)}$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{b\cosh(x)\sinh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

Mathematica [A] time = 0.23, size = 68, normalized size = 1.05

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{b\sinh(2x)}{2a(a+b)(2a+b\cosh(2x)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^2)^(-2),x]
```

```
[Out] ((2*a + b)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^(3/2)
) - (b*Sinh[2*x])/(2*a*(a + b)*(2*a + b + b*Cosh[2*x]))
```

fricas [B] time = 0.47, size = 1239, normalized size = 19.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b + 4*a*b^2 + 4*(2*a^3 + 3*a^2*b + a*b^2)*cosh(x)^2 + 8*(2*a^3
+ 3*a^2*b + a*b^2)*cosh(x)*sinh(x) + 4*(2*a^3 + 3*a^2*b + a*b^2)*sinh(x)^2
+ ((2*a*b + b^2)*cosh(x)^4 + 4*(2*a*b + b^2)*cosh(x)*sinh(x)^3 + (2*a*b + b
^2)*sinh(x)^4 + 2*(4*a^2 + 4*a*b + b^2)*cosh(x)^2 + 2*(3*(2*a*b + b^2)*cosh
(x)^2 + 4*a^2 + 4*a*b + b^2)*sinh(x)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cos
h(x)^3 + (4*a^2 + 4*a*b + b^2)*cosh(x))*sinh(x))*sqrt(a^2 + a*b)*log((b^2*c
osh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x
)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4
*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*cosh(x)^4 + 4*b*c
osh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 +
 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)))/(a
^4*b + 2*a^3*b^2 + a^2*b^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 4*(a
^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^
3)*sinh(x)^4 + 2*(2*a^5 + 5*a^4*b + 4*a^3*b^2 + a^2*b^3)*cosh(x)^2 + 2*(2*a
^5 + 5*a^4*b + 4*a^3*b^2 + a^2*b^3 + 3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x
)^2)*sinh(x)^2 + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (2*a^5 + 5*a^
4*b + 4*a^3*b^2 + a^2*b^3)*cosh(x))*sinh(x)), 1/2*(2*a^2*b + 2*a*b^2 + 2*(2
*a^3 + 3*a^2*b + a*b^2)*cosh(x)^2 + 4*(2*a^3 + 3*a^2*b + a*b^2)*cosh(x)*sin
h(x) + 2*(2*a^3 + 3*a^2*b + a*b^2)*sinh(x)^2 + ((2*a*b + b^2)*cosh(x)^4 + 4
*(2*a*b + b^2)*cosh(x)*sinh(x)^3 + (2*a*b + b^2)*sinh(x)^4 + 2*(4*a^2 + 4*a
*b + b^2)*cosh(x)^2 + 2*(3*(2*a*b + b^2)*cosh(x)^2 + 4*a^2 + 4*a*b + b^2)*s
inh(x)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(x)^3 + (4*a^2 + 4*a*b + b^2)
```

```
*cosh(x))*sinh(x))*sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*s
inh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a*b)))/(a^4*b + 2*a
^3*b^2 + a^2*b^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 4*(a^4*b + 2*a
^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*sinh(x)
^4 + 2*(2*a^5 + 5*a^4*b + 4*a^3*b^2 + a^2*b^3)*cosh(x)^2 + 2*(2*a^5 + 5*a^4
*b + 4*a^3*b^2 + a^2*b^3 + 3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(
x)^2 + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (2*a^5 + 5*a^4*b + 4*a^
3*b^2 + a^2*b^3)*cosh(x))*sinh(x))]
```

giac [A] time = 0.14, size = 104, normalized size = 1.60

$$\frac{(2\,a+b)\arctan\left(\frac{be^{(2\,x)}+2\,a+b}{2\,\sqrt{-a^2-ab}}\right)}{2\left(a^2+ab\right)\sqrt{-a^2-ab}}+\frac{2\,ae^{(2\,x)}+be^{(2\,x)}+b}{\left(a^2+ab\right)\left(be^{(4\,x)}+4\,ae^{(2\,x)}+2\,be^{(2\,x)}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a + b)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/((a^2 + a*
b)*sqrt(-a^2 - a*b)) + (2*a*e^(2*x) + b*e^(2*x) + b)/((a^2 + a*b)*(b*e^(4*x
) + 4*a*e^(2*x) + 2*b*e^(2*x) + b))
```

maple [B] time = 0.11, size = 236, normalized size = 3.63

$$-\frac{2\left(\frac{b\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2a(a+b)}+\frac{b\tanh\left(\frac{x}{2}\right)}{2a(a+b)}\right)}{\left(\tanh^4\left(\frac{x}{2}\right)\right)a+b\left(\tanh^4\left(\frac{x}{2}\right)\right)-2a\left(\tanh^2\left(\frac{x}{2}\right)\right)+2\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b}-\frac{\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{x}{2}\right)\right)+2\sqrt{a}\right.}{2\left(a+b\right)^{\frac{3}{2}}\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^2)^2,x)
```

```
[Out] -2*(1/2*b/a/(a+b)*tanh(1/2*x)^3+1/2*b/a/(a+b)*tanh(1/2*x))/(tanh(1/2*x)^4*a
+b*tanh(1/2*x)^4-2*a*tanh(1/2*x)^2+2*tanh(1/2*x)^2*b+a+b)-1/2/(a+b)^(3/2)/a
^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))+1/2
/(a+b)^(3/2)/a^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+
b)^(1/2))-1/4/a^(3/2)/(a+b)^(3/2)*b*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)
*tanh(1/2*x)-(a+b)^(1/2))+1/4/a^(3/2)/(a+b)^(3/2)*b*ln((a+b)^(1/2)*tanh(1/2
*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))
```

maxima [B] time = 0.53, size = 134, normalized size = 2.06

$$-\frac{(2\,a+b)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}\left(a^2+ab\right)}-\frac{(2\,a+b)e^{(-2\,x)}+b}{a^2b+ab^2+2\left(2\,a^3+3\,a^2b+ab^2\right)e^{(-2\,x)}+\left(a^2b+ab^2\right)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a + b)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) +
 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + a*b)) - ((2*a + b)*e
^(-2*x) + b)/(a^2*b + a*b^2 + 2*(2*a^3 + 3*a^2*b + a*b^2)*e^(-2*x) + (a^2*b
 + a*b^2)*e^(-4*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\cosh(x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^2)^2,x)
```

```
[Out] int(1/(a + b*cosh(x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**2)**2,x)
```

```
[Out] Timed out
```

3.34 $\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{3b(2a+b)\sinh(x)\cosh(x)}{8a^2(a+b)^2\left(a+b\cosh^2(x)\right)}+\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(5/2)/(a+b)^
(5/2)-1/4*b*cosh(x)*sinh(x)/a/(a+b)/(a+b*cosh(x)^2)^2-3/8*b*(2*a+b)*cosh(x)
*sinh(x)/a^2/(a+b)^2/(a+b*cosh(x)^2)
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3184, 3173, 12, 3181, 208}

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{3b(2a+b)\sinh(x)\cosh(x)}{8a^2(a+b)^2\left(a+b\cosh^2(x)\right)}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^2)^(-3),x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(8*a^(5/2)
*(a + b)^(5/2)) - (b*Cosh[x]*Sinh[x])/(4*a*(a + b)*(a + b*Cosh[x]^2)^2) - (
3*b*(2*a + b)*Cosh[x]*Sinh[x])/(8*a^2*(a + b)^2*(a + b*Cosh[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx = -\frac{b\cosh(x)\sinh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2} - \frac{\int \frac{-4a-3b+2b\cosh^2(x)}{\left(a+b\cosh^2(x)\right)^2}\,dx}{4a(a+b)}$$

$$= -\frac{b\cosh(x)\sinh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2} - \frac{3b(2a+b)\cosh(x)\sinh(x)}{8a^2(a+b)^2\left(a+b\cosh^2(x)\right)} - \frac{\int \frac{-8a^2-8ab-3b^2}{a+b\cosh^2(x)}\,dx}{8a^2(a+b)^2}$$

$$= -\frac{b\cosh(x)\sinh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2} - \frac{3b(2a+b)\cosh(x)\sinh(x)}{8a^2(a+b)^2\left(a+b\cosh^2(x)\right)} + \frac{\left(8a^2+8ab+3b^2\right)\int \frac{}{a+}}{8a^2(a+b)^2}$$

$$= -\frac{b\cosh(x)\sinh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2} - \frac{3b(2a+b)\cosh(x)\sinh(x)}{8a^2(a+b)^2\left(a+b\cosh^2(x)\right)} + \frac{\left(8a^2+8ab+3b^2\right)\text{Sub}}{8a}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}} - \frac{b\cosh(x)\sinh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2} - \frac{3b(2a+b)\cos}{8a^2(a+b)^2\left(a-\right.}$$

Mathematica [A] time = 0.63, size = 106, normalized size = 0.99

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{\sqrt{a}\,b\sinh(2x)\left(16a^2+3b(2a+b)\cosh(2x)+16ab+3b^2\right)}{(a+b)^2(2a+b\cosh(2x)+b)^2}}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^2)^(-3),x]
```

```
[Out] (((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a + b)^(
5/2) - (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 + 3*b*(2*a + b)*Cosh[2*x])*Sinh[
2*x])/((a + b)^2*(2*a + b + b*Cosh[2*x])^2))/(8*a^(5/2))
```

fricas [B] time = 0.60, size = 5117, normalized size = 47.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^6 + 24*(8*a^
4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)*sinh(x)^5 + 4*(8*a^4*b + 1
6*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*sinh(x)^6 + 24*a^3*b^2 + 36*a^2*b^3 + 12*
a*b^4 + 12*(16*a^5 + 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 + 3*a*b^4)*cosh(x)^
4 + 12*(16*a^5 + 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 + 3*a*b^4 + 5*(8*a^4*b
+ 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^4 + 16*(5*(8*a^4*b
+ 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + 3*(16*a^5 + 40*a^4*b + 38*
a^3*b^2 + 17*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x)^3 + 4*(40*a^4*b + 80*a^3*b
^2 + 49*a^2*b^3 + 9*a*b^4)*cosh(x)^2 + 4*(40*a^4*b + 80*a^3*b^2 + 49*a^2*b^
3 + 9*a*b^4 + 15*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^4 +
18*(16*a^5 + 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 + 3*a*b^4)*cosh(x)^2)*sinh(
x)^2 + ((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^8 + 8*(8*a^2*b^2 + 8*a*b^3 +
```

3*b^4)*cosh(x)*sinh(x)^7 + (8*a^2*b^2 + 8*a*b^3 + 3*b^4)*sinh(x)^8 + 4*(16*
a^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^6 + 4*(16*a^3*b + 24*a^2*b^2
 + 14*a*b^3 + 3*b^4 + 7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^6
+ 8*(7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^3 + 3*(16*a^3*b + 24*a^2*b^2 +
 14*a*b^3 + 3*b^4)*cosh(x))*sinh(x)^5 + 2*(64*a^4 + 128*a^3*b + 112*a^2*b^2
 + 48*a*b^3 + 9*b^4)*cosh(x)^4 + 2*(35*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x
)^4 + 64*a^4 + 128*a^3*b + 112*a^2*b^2 + 48*a*b^3 + 9*b^4 + 30*(16*a^3*b +
24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^4 + 8*a^2*b^2 + 8*a*b^3 +
 3*b^4 + 8*(7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^5 + 10*(16*a^3*b + 24*a
^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^3 + (64*a^4 + 128*a^3*b + 112*a^2*b^2 +
48*a*b^3 + 9*b^4)*cosh(x))*sinh(x)^3 + 4*(16*a^3*b + 24*a^2*b^2 + 14*a*b^3
+ 3*b^4)*cosh(x)^2 + 4*(7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^6 + 15*(16*
a^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^4 + 16*a^3*b + 24*a^2*b^2 +
14*a*b^3 + 3*b^4 + 3*(64*a^4 + 128*a^3*b + 112*a^2*b^2 + 48*a*b^3 + 9*b^4)*
cosh(x)^2)*sinh(x)^2 + 8*((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^7 + 3*(16*a
^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^5 + (64*a^4 + 128*a^3*b + 112
*a^2*b^2 + 48*a*b^3 + 9*b^4)*cosh(x)^3 + (16*a^3*b + 24*a^2*b^2 + 14*a*b^3
+ 3*b^4)*cosh(x))*sinh(x))*sqrt(a^2 + a*b)*log((b^2*cosh(x)^4 + 4*b^2*cosh(
x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)
^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a
*b + b^2)*cosh(x))*sinh(x) - 4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(
x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*s
inh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 +
4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 8*(3*(8*a^4*b + 16*a^3*
b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^5 + 6*(16*a^5 + 40*a^4*b + 38*a^3*b^2 +
 17*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + (40*a^4*b + 80*a^3*b^2 + 49*a^2*b^3 + 9*
a*b^4)*cosh(x))*sinh(x))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(
x)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)*sinh(x)^7 + (a
^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*sinh(x)^8 + a^6*b^2 + 3*a^5*b^3 +
 3*a^4*b^4 + a^3*b^5 + 4*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3
*b^5)*cosh(x)^6 + 4*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5
+ 7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7
*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)^3 + 3*(2*a^7*b + 7*a^6
*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x))*sinh(x)^5 + 2*(8*a^8 + 32*
a^7*b + 51*a^6*b^2 + 41*a^5*b^3 + 17*a^4*b^4 + 3*a^3*b^5)*cosh(x)^4 + 2*(8*
a^8 + 32*a^7*b + 51*a^6*b^2 + 41*a^5*b^3 + 17*a^4*b^4 + 3*a^3*b^5 + 35*(a^6
*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)^4 + 30*(2*a^7*b + 7*a^6*b^2
 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)^5 + 10*(2*a^7*b + 7*a^6*b^2 + 9*a^
5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^3 + (8*a^8 + 32*a^7*b + 51*a^6*b^2 + 4
1*a^5*b^3 + 17*a^4*b^4 + 3*a^3*b^5)*cosh(x))*sinh(x)^3 + 4*(2*a^7*b + 7*a^6
*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^2 + 4*(2*a^7*b + 7*a^6*b^2
+ 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + 7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^
3*b^5)*cosh(x)^6 + 15*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^
5)*cosh(x)^4 + 3*(8*a^8 + 32*a^7*b + 51*a^6*b^2 + 41*a^5*b^3 + 17*a^4*b^4 +
 3*a^3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^
3*b^5)*cosh(x)^7 + 3*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5
)*cosh(x)^5 + (8*a^8 + 32*a^7*b + 51*a^6*b^2 + 41*a^5*b^3 + 17*a^4*b^4 + 3*
a^3*b^5)*cosh(x)^3 + (2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5
)*cosh(x))*sinh(x)), 1/8*(2*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*c
osh(x)^6 + 12*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)*sinh(x)
^5 + 2*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*sinh(x)^6 + 12*a^3*b^2
 + 18*a^2*b^3 + 6*a*b^4 + 6*(16*a^5 + 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 +
3*a*b^4)*cosh(x)^4 + 6*(16*a^5 + 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 + 3*a*b
^4 + 5*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^4 +
 8*(5*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + 3*(16*a^5 +
 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x)^3 + 2*(40*a
^4*b + 80*a^3*b^2 + 49*a^2*b^3 + 9*a*b^4)*cosh(x)^2 + 2*(40*a^4*b + 80*a^3*
b^2 + 49*a^2*b^3 + 9*a*b^4 + 15*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^

```
4)*cosh(x)^4 + 18*(16*a^5 + 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 + 3*a*b^4)*c
osh(x)^2)*sinh(x)^2 + ((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^8 + 8*(8*a^2*b
^2 + 8*a*b^3 + 3*b^4)*cosh(x)*sinh(x)^7 + (8*a^2*b^2 + 8*a*b^3 + 3*b^4)*sin
h(x)^8 + 4*(16*a^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^6 + 4*(16*a^3
*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4 + 7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x
)^2)*sinh(x)^6 + 8*(7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^3 + 3*(16*a^3*b
 + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x))*sinh(x)^5 + 2*(64*a^4 + 128*a^3*
b + 112*a^2*b^2 + 48*a*b^3 + 9*b^4)*cosh(x)^4 + 2*(35*(8*a^2*b^2 + 8*a*b^3
+ 3*b^4)*cosh(x)^4 + 64*a^4 + 128*a^3*b + 112*a^2*b^2 + 48*a*b^3 + 9*b^4 +
30*(16*a^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^4 + 8*a^2*
b^2 + 8*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh(x)^5 + 10*(
16*a^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^3 + (64*a^4 + 128*a^3*b +
 112*a^2*b^2 + 48*a*b^3 + 9*b^4)*cosh(x))*sinh(x)^3 + 4*(16*a^3*b + 24*a^2*
b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^2 + 4*(7*(8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh
(x)^6 + 15*(16*a^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^4 + 16*a^3*b
+ 24*a^2*b^2 + 14*a*b^3 + 3*b^4 + 3*(64*a^4 + 128*a^3*b + 112*a^2*b^2 + 48*
a*b^3 + 9*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cosh
(x)^7 + 3*(16*a^3*b + 24*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(x)^5 + (64*a^4 +
128*a^3*b + 112*a^2*b^2 + 48*a*b^3 + 9*b^4)*cosh(x)^3 + (16*a^3*b + 24*a^2*
b^2 + 14*a*b^3 + 3*b^4)*cosh(x))*sinh(x))*sqrt(-a^2 - a*b)*arctan(1/2*(b*co
sh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^
2 + a*b)) + 4*(3*(8*a^4*b + 16*a^3*b^2 + 11*a^2*b^3 + 3*a*b^4)*cosh(x)^5 +
6*(16*a^5 + 40*a^4*b + 38*a^3*b^2 + 17*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + (40*a
^4*b + 80*a^3*b^2 + 49*a^2*b^3 + 9*a*b^4)*cosh(x))*sinh(x))/((a^6*b^2 + 3*a
^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^
4 + a^3*b^5)*cosh(x)*sinh(x)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5
)*sinh(x)^8 + a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5 + 4*(2*a^7*b + 7*a^
6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^6 + 4*(2*a^7*b + 7*a^6*b^2
 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + 7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a
^3*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*
b^5)*cosh(x)^3 + 3*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*
cosh(x))*sinh(x)^5 + 2*(8*a^8 + 32*a^7*b + 51*a^6*b^2 + 41*a^5*b^3 + 17*a^4
*b^4 + 3*a^3*b^5)*cosh(x)^4 + 2*(8*a^8 + 32*a^7*b + 51*a^6*b^2 + 41*a^5*b^3
 + 17*a^4*b^4 + 3*a^3*b^5 + 35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*
cosh(x)^4 + 30*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh
(x)^2)*sinh(x)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)
^5 + 10*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^3 +
 (8*a^8 + 32*a^7*b + 51*a^6*b^2 + 41*a^5*b^3 + 17*a^4*b^4 + 3*a^3*b^5)*cosh
(x))*sinh(x)^3 + 4*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*
cosh(x)^2 + 4*(2*a^7*b + 7*a^6*b^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + 7*(a
^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)^6 + 15*(2*a^7*b + 7*a^6*b
^2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^4 + 3*(8*a^8 + 32*a^7*b + 51*
a^6*b^2 + 41*a^5*b^3 + 17*a^4*b^4 + 3*a^3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a
^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cosh(x)^7 + 3*(2*a^7*b + 7*a^6*b^
2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^5 + (8*a^8 + 32*a^7*b + 51*a^6
*b^2 + 41*a^5*b^3 + 17*a^4*b^4 + 3*a^3*b^5)*cosh(x)^3 + (2*a^7*b + 7*a^6*b^
2 + 9*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x))*sinh(x))]
```

giac [B] time = 0.68, size = 228, normalized size = 2.13

$$\frac{\left(8\,a^{2}+8\,ab+3\,b^{2}\right)\arctan\left(\frac{be^{(2\,x)}+2\,a+b}{2\,\sqrt{-a^{2}-ab}}\right)}{8\,\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)\sqrt{-a^{2}-ab}}+\frac{8\,a^{2}be^{(6\,x)}+8\,ab^{2}e^{(6\,x)}+3\,b^{3}e^{(6\,x)}+48\,a^{3}e^{(4\,x)}+72\,a^{2}be^{(4\,x)}+42\,ab^{2}e^{(4\,x}}{4\,\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)\left(be^{(4\,x)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*
b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(-a^2 - a*b)) + 1/4*(8*a^2*b*e^(6*x) + 8
```

```
*a*b^2*e^(6*x) + 3*b^3*e^(6*x) + 48*a^3*e^(4*x) + 72*a^2*b*e^(4*x) + 42*a*b
^2*e^(4*x) + 9*b^3*e^(4*x) + 40*a^2*b*e^(2*x) + 40*a*b^2*e^(2*x) + 9*b^3*e^
(2*x) + 6*a*b^2 + 3*b^3)/((a^4 + 2*a^3*b + a^2*b^2)*(b*e^(4*x) + 4*a*e^(2*x
) + 2*b*e^(2*x) + b)^2)
```

maple [B] time = 0.12, size = 477, normalized size = 4.46

$$-\frac{2\left(\frac{b(8a+3b)\left(\tanh^{7}\left(\frac{x}{2}\right)\right)}{8(a+b)a^{2}}-\frac{b\left(8a^{2}-13ab-9b^{2}\right)\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{8(a+b)^{2}a^{2}}-\frac{b\left(8a^{2}-13ab-9b^{2}\right)\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{8(a+b)^{2}a^{2}}+\frac{b(8a+3b)\tanh\left(\frac{x}{2}\right)}{8(a+b)a^{2}}\right)}{\left(\left(\tanh^{4}\left(\frac{x}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{x}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+2\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)^{2}}-\frac{\ln\left(-\sqrt{a+b}\ \left(\tanh\right.\right.}{2\left(a^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^2)^3,x)
```

```
[Out] -2*(1/8*b*(8*a+3*b)/(a+b)/a^2*tanh(1/2*x)^7-1/8*b*(8*a^2-13*a*b-9*b^2)/(a+b
)^2/a^2*tanh(1/2*x)^5-1/8*b*(8*a^2-13*a*b-9*b^2)/(a+b)^2/a^2*tanh(1/2*x)^3+
1/8*b*(8*a+3*b)/(a+b)/a^2*tanh(1/2*x))/(tanh(1/2*x)^4*a+b*tanh(1/2*x)^4-2*a
*tanh(1/2*x)^2+2*tanh(1/2*x)^2*b+a+b)^2-1/2/(a^2+2*a*b+b^2)/a^(1/2)/(a+b)^(
1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))-1/2/a
^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*
tanh(1/2*x)-(a+b)^(1/2))*b-3/16/a^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+
b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)-(a+b)^(1/2))*b^2+1/2/(a^2+2*a*
b+b^2)/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*
x)+(a+b)^(1/2))+1/2/a^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh
(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2))*b+3/16/a^(5/2)/(a^2+2*a*b+b^2)
/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*a^(1/2)*tanh(1/2*x)+(a+b)^(1/2)
)*b^2
```

maxima [B] time = 0.59, size = 344, normalized size = 3.21

$$-\frac{\left(8a^{2}+8ab+3b^{2}\right)\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{16\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\sqrt{(a+b)a}}-\frac{6ab^{2}+3b^{3}+\left(40a^{2}b\right.}{4\left(a^{4}b^{2}+2a^{3}b^{3}+a^{2}b^{4}+4\left(2a^{5}b+5a^{4}b^{2}+4a^{3}b^{3}+a^{2}b^{4}\right)e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(8*a^2 + 8*a*b + 3*b^2)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a)
)/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/((a^4 + 2*a^3*b + a^2*b^2)*sq
rt((a + b)*a)) - 1/4*(6*a*b^2 + 3*b^3 + (40*a^2*b + 40*a*b^2 + 9*b^3)*e^(-2
*x) + 3*(16*a^3 + 24*a^2*b + 14*a*b^2 + 3*b^3)*e^(-4*x) + (8*a^2*b + 8*a*b^
2 + 3*b^3)*e^(-6*x))/(a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(2*a^5*b + 5*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*e^(-2*x) + 2*(8*a^6 + 24*a^5*b + 27*a^4*b^2 + 14*a
^3*b^3 + 3*a^2*b^4)*e^(-4*x) + 4*(2*a^5*b + 5*a^4*b^2 + 4*a^3*b^3 + a^2*b^4
)*e^(-6*x) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cosh(x)^{2}+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^2)^3,x)
```

```
[Out] int(1/(a + b*cosh(x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**2)**3,x)
```

```
[Out] Timed out
```

3.35 $\int \frac{1}{1+\cosh^2(x)} \, dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)^(-1),x]
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\cosh^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{1-2x^2} \, dx, x, \coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\cosh^2(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cosh[x]^2)^(-1),x]
[Out] Integrate[(1 + Cosh[x]^2)^(-1), x]
```

fricas [B] time = 0.40, size = 66, normalized size = 4.40

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2+2\sqrt{2}-3}{\cosh(x)^2+\sinh(x)^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^
2 + 3))
```

giac [B] time = 0.14, size = 34, normalized size = 2.27

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3))
```

maple [B] time = 0.05, size = 86, normalized size = 5.73

$$\frac{\sqrt{2}\ \ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{8}-\frac{\sqrt{2}\ \ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^2),x)
```

```
[Out] 1/8*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2)
*tanh(1/2*x)+1))-1/8*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tanh
(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))
```

maxima [B] time = 0.59, size = 34, normalized size = 2.27

$$-\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3))
```

mupad [B] time = 0.14, size = 50, normalized size = 3.33

$$\frac{\sqrt{2}\left(\ln\left(-4\,e^{2\,x}-\frac{\sqrt{2}\left(12\,e^{2\,x}+4\right)}{4}\right)-\ln\left(\frac{\sqrt{2}\left(12\,e^{2\,x}+4\right)}{4}-4\,e^{2\,x}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 + 1),x)
```

```
[Out] (2^(1/2)*(log(- 4*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/4) - log((2^(1/2)*
(12*exp(2*x) + 4))/4 - 4*exp(2*x))))/4
```

sympy [B] time = 0.68, size = 60, normalized size = 4.00

$$-\frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{4}+\frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**2),x)
```

```
[Out] -sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/4 + sqrt(2)*log(4*ta
nh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/4
```

3.36 $\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx$

Optimal. Leaf size=35

$$\frac{3\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}$$

```
[Out] -1/4*cosh(x)*sinh(x)/(1+cosh(x)^2)+3/8*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3184, 12, 3181, 206}

$$\frac{3\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)^(-2),x]
```

```
[Out] (3*ArcTanh[Tanh[x]/Sqrt[2]])/(4*Sqrt[2]) - (Cosh[x]*Sinh[x])/(4*(1 + Cosh[x
]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx &= -\frac{\cosh(x)\sinh(x)}{4\left(1+\cosh^2(x)\right)} - \frac{1}{4}\int -\frac{3}{1+\cosh^2(x)}\,dx \\
&= -\frac{\cosh(x)\sinh(x)}{4\left(1+\cosh^2(x)\right)} + \frac{3}{4}\int \frac{1}{1+\cosh^2(x)}\,dx \\
&= -\frac{\cosh(x)\sinh(x)}{4\left(1+\cosh^2(x)\right)} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(x)\right) \\
&= \frac{3\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{\cosh(x)\sinh(x)}{4\left(1+\cosh^2(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 35, normalized size = 1.00

$$\frac{3\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{\sinh(2x)}{4(\cosh(2x)+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^2)^(-2),x]
```

```
[Out] (3*ArcTanh[Tanh[x]/Sqrt[2]])/(4*Sqrt[2]) - Sinh[2*x]/(4*(3 + Cosh[2*x]))
```

fricas [B] time = 0.53, size = 214, normalized size = 6.11

$$\frac{24\cosh(x)^2 + 3\left(\sqrt{2}\cosh(x)^4 + 4\sqrt{2}\cosh(x)\sinh(x)^3 + \sqrt{2}\sinh(x)^4 + 6\left(\sqrt{2}\cosh(x)^2 + \sqrt{2}\right)\sinh(x)^2 + 6 \cdots}{16\left(\cosh(x)^4 + 4\cosh(x)\sinh(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(24*cosh(x)^2 + 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + s
qrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 6*sqrt(2)*co
sh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(
-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sq
rt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 48*cos
h(x)*sinh(x) + 24*sinh(x)^2 + 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)
^4 + 6*(cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x))*
sinh(x) + 1)
```

giac [B] time = 0.13, size = 59, normalized size = 1.69

$$\frac{3}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2} - e^{(2x)} - 3}{2\sqrt{2} + e^{(2x)} + 3}\right) + \frac{3e^{(2x)} + 1}{2\left(e^{(4x)} + 6e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1/
2*(3*e^(2*x) + 1)/(e^(4*x) + 6*e^(2*x) + 1)
```

maple [B] time = 0.06, size = 113, normalized size = 3.23

$$-\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2} + \frac{\tanh\left(\frac{x}{2}\right)}{2}}{2\left(\tanh^4\left(\frac{x}{2}\right)+1\right)} + \frac{3\sqrt{2}\,\ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}\right)}{32} - \frac{3\sqrt{2}\,\ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^2)^2,x)
```

```
[Out] -1/2*(1/2*tanh(1/2*x)^3+1/2*tanh(1/2*x))/(tanh(1/2*x)^4+1)+3/32*2^(1/2)*ln(
(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1)
)-3/32*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1
/2)*tanh(1/2*x)+1))
```

maxima [B] time = 0.42, size = 59, normalized size = 1.69

$$-\frac{3}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)-\frac{3e^{(-2x)}+1}{2\left(6e^{(-2x)}+e^{(-4x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -3/16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
 1/2*(3*e^(-2*x) + 1)/(6*e^(-2*x) + e^(-4*x) + 1)
```

mupad [B] time = 1.05, size = 76, normalized size = 2.17

$$\frac{3\sqrt{2}\,\ln\left(-3\,e^{2x}-\frac{3\sqrt{2}\,\left(12\,e^{2x}+4\right)}{16}\right)}{16}-\frac{3\sqrt{2}\,\ln\left(\frac{3\sqrt{2}\,\left(12\,e^{2x}+4\right)}{16}-3\,e^{2x}\right)}{16}+\frac{\frac{3\,e^{2x}}{2}+\frac{1}{2}}{6\,e^{2x}+e^{4x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 + 1)^2,x)
```

```
[Out] (3*2^(1/2)*log(- 3*exp(2*x) - (3*2^(1/2)*(12*exp(2*x) + 4))/16))/16 - (3*2^
(1/2)*log((3*2^(1/2)*(12*exp(2*x) + 4))/16 - 3*exp(2*x)))/16 + ((3*exp(2*x)
)/2 + 1/2)/(6*exp(2*x) + exp(4*x) + 1)
```

sympy [B] time = 3.47, size = 211, normalized size = 6.03

$$-\frac{3\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)\tanh^{4}\left(\frac{x}{2}\right)}{16\tanh^{4}\left(\frac{x}{2}\right)+16}-\frac{3\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{16\tanh^{4}\left(\frac{x}{2}\right)+16}+\frac{3\sqrt{2}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**2)**2,x)
```

```
[Out] -3*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**4/(16*t
anh(x/2)**4 + 16) - 3*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)
/(16*tanh(x/2)**4 + 16) + 3*sqrt(2)*log(4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2
) + 4)*tanh(x/2)**4/(16*tanh(x/2)**4 + 16) + 3*sqrt(2)*log(4*tanh(x/2)**2 +
 4*sqrt(2)*tanh(x/2) + 4)/(16*tanh(x/2)**4 + 16) - 4*tanh(x/2)**3/(16*tanh(
x/2)**4 + 16) - 4*tanh(x/2)/(16*tanh(x/2)**4 + 16)
```

3.37 $\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx$

Optimal. Leaf size=51

$$\frac{19\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{9\sinh(x)\cosh(x)}{32\left(\cosh^2(x)+1\right)}-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

```
[Out] -1/8*cosh(x)*sinh(x)/(1+cosh(x)^2)^2-9/32*cosh(x)*sinh(x)/(1+cosh(x)^2)+19/
64*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3184, 3173, 12, 3181, 206}

$$\frac{19\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{9\sinh(x)\cosh(x)}{32\left(\cosh^2(x)+1\right)}-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)^(-3),x]
```

```
[Out] (19*ArcTanh[Tanh[x]/Sqrt[2]])/(32*Sqrt[2]) - (Cosh[x]*Sinh[x])/(8*(1 + Cosh
[x]^2)^2) - (9*Cosh[x]*Sinh[x])/(32*(1 + Cosh[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx &= -\frac{\cosh(x)\sinh(x)}{8\left(1+\cosh^2(x)\right)^2}-\frac{1}{8}\int \frac{-7+2\cosh^2(x)}{\left(1+\cosh^2(x)\right)^2}\,dx\\
&= -\frac{\cosh(x)\sinh(x)}{8\left(1+\cosh^2(x)\right)^2}-\frac{9\cosh(x)\sinh(x)}{32\left(1+\cosh^2(x)\right)}-\frac{1}{32}\int -\frac{19}{1+\cosh^2(x)}\,dx\\
&= -\frac{\cosh(x)\sinh(x)}{8\left(1+\cosh^2(x)\right)^2}-\frac{9\cosh(x)\sinh(x)}{32\left(1+\cosh^2(x)\right)}+\frac{19}{32}\int \frac{1}{1+\cosh^2(x)}\,dx\\
&= -\frac{\cosh(x)\sinh(x)}{8\left(1+\cosh^2(x)\right)^2}-\frac{9\cosh(x)\sinh(x)}{32\left(1+\cosh^2(x)\right)}+\frac{19}{32}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\coth(x)\right)\\
&= \frac{19\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{\cosh(x)\sinh(x)}{8\left(1+\cosh^2(x)\right)^2}-\frac{9\cosh(x)\sinh(x)}{32\left(1+\cosh^2(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 51, normalized size = 1.00

$$\frac{19\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{9\sinh(2x)}{32(\cosh(2x)+3)}-\frac{\sinh(2x)}{4(\cosh(2x)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^2)^(-3),x]

[Out] (19*ArcTanh[Tanh[x]/Sqrt[2]])/(32*Sqrt[2]) - Sinh[2*x]/(4*(3 + Cosh[2*x])^2
) - (9*Sinh[2*x])/(32*(3 + Cosh[2*x]))
```

fricas [B] time = 0.56, size = 575, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^3,x, algorithm="fricas")

[Out] 1/128*(152*cosh(x)^6 + 912*cosh(x)*sinh(x)^5 + 152*sinh(x)^6 + 456*(5*cosh(
x)^2 + 3)*sinh(x)^4 + 1368*cosh(x)^4 + 608*(5*cosh(x)^3 + 9*cosh(x))*sinh(x
)^3 + 8*(285*cosh(x)^4 + 1026*cosh(x)^2 + 89)*sinh(x)^2 + 712*cosh(x)^2 + 1
9*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*
(7*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^6 + 12*sqrt(2)*cosh(x)^6 + 8*(7*s
qrt(2)*cosh(x)^3 + 9*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 +
 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7
*sqrt(2)*cosh(x)^5 + 30*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 +
 4*(7*sqrt(2)*cosh(x)^6 + 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 + 3*s
qrt(2))*sinh(x)^2 + 12*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 + 9*sqrt(2)
*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*l
og(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2
*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 16*
(57*cosh(x)^5 + 342*cosh(x)^3 + 89*cosh(x))*sinh(x) + 72)/(cosh(x)^8 + 8*co
sh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 3)*sinh(x)^6 + 12*cosh(x)^6
+ 8*(7*cosh(x)^3 + 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 90*cosh(x)^2 +
19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 + 30*cosh(x)^3 + 19*cosh(x))*
```

```
sinh(x)^3 + 4*(7*cosh(x)^6 + 45*cosh(x)^4 + 57*cosh(x)^2 + 3)*sinh(x)^2 + 1
2*cosh(x)^2 + 8*(cosh(x)^7 + 9*cosh(x)^5 + 19*cosh(x)^3 + 3*cosh(x))*sinh(x
) + 1)
```

giac [A] time = 0.12, size = 71, normalized size = 1.39

$$\frac{19}{128}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+\frac{19e^{(6x)}+171e^{(4x)}+89e^{(2x)}+9}{16\left(e^{(4x)}+6e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 19/128*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) +
1/16*(19*e^(6*x) + 171*e^(4*x) + 89*e^(2*x) + 9)/(e^(4*x) + 6*e^(2*x) + 1)^
2
```

maple [B] time = 0.06, size = 129, normalized size = 2.53

$$-\frac{\frac{11\left(\tanh^7\left(\frac{x}{2}\right)\right)}{8}+\frac{7\left(\tanh^5\left(\frac{x}{2}\right)\right)}{8}+\frac{7\left(\tanh^3\left(\frac{x}{2}\right)\right)}{8}+\frac{11\tanh\left(\frac{x}{2}\right)}{8}}{4\left(\tanh^4\left(\frac{x}{2}\right)+1\right)^2}+\frac{19\sqrt{2}\ \ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{256}-\frac{19\sqrt{2}\ \ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\ \mathrm{ta}}{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\ \mathrm{ta}}\right.}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^2)^3,x)
```

```
[Out] -1/4*(11/8*tanh(1/2*x)^7+7/8*tanh(1/2*x)^5+7/8*tanh(1/2*x)^3+11/8*tanh(1/2*
x))/(tanh(1/2*x)^4+1)^2+19/256*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x
)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))-19/256*2^(1/2)*ln((tanh(1/2*x)^
2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))
```

maxima [A] time = 0.42, size = 83, normalized size = 1.63

$$-\frac{19}{128}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)-\frac{89e^{(-2x)}+171e^{(-4x)}+19e^{(-6x)}+9}{16\left(12e^{(-2x)}+38e^{(-4x)}+12e^{(-6x)}+e^{(-8x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -19/128*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3))
 - 1/16*(89*e^(-2*x) + 171*e^(-4*x) + 19*e^(-6*x) + 9)/(12*e^(-2*x) + 38*e^
(-4*x) + 12*e^(-6*x) + e^(-8*x) + 1)
```

mupad [B] time = 1.00, size = 112, normalized size = 2.20

$$\frac{19\sqrt{2}\ \ln\left(-\frac{19\,e^{2x}}{8}-\frac{19\sqrt{2}\left(12\,e^{2x}+4\right)}{128}\right)}{128}-\frac{17\,e^{2x}+3}{12\,e^{2x}+38\,e^{4x}+12\,e^{6x}+e^{8x}+1}-\frac{19\sqrt{2}\ \ln\left(\frac{19\sqrt{2}\left(12\,e^{2x}+4\right)}{128}-\frac{19\,e^{2x}}{8}\right)}{128}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 + 1)^3,x)
```

```
[Out] (19*2^(1/2)*log(- (19*exp(2*x))/8 - (19*2^(1/2)*(12*exp(2*x) + 4))/128))/12
8 - (17*exp(2*x) + 3)/(12*exp(2*x) + 38*exp(4*x) + 12*exp(6*x) + exp(8*x) +
 1) - (19*2^(1/2)*log((19*2^(1/2)*(12*exp(2*x) + 4))/128 - (19*exp(2*x))/8)
)/128 + ((19*exp(2*x))/16 + 57/16)/(6*exp(2*x) + exp(4*x) + 1)
```

sympy [B] time = 13.57, size = 428, normalized size = 8.39

$$-\frac{19\sqrt{2}\,\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+4\right)\tanh^{8}\left(\frac{x}{2}\right)}{128\tanh^{8}\left(\frac{x}{2}\right)+256\tanh^{4}\left(\frac{x}{2}\right)+128}-\frac{38\sqrt{2}\,\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+4\right)\tanh^{4}\left(\frac{x}{2}\right)}{128\tanh^{8}\left(\frac{x}{2}\right)+256\tanh^{4}\left(\frac{x}{2}\right)+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**2)**3,x)
```

```
[Out] -19*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**8/(128
*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 38*sqrt(2)*log(4*tanh(x/2)**2 - 4
*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**4/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 +
 128) - 19*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/(128*tanh(
x/2)**8 + 256*tanh(x/2)**4 + 128) + 19*sqrt(2)*log(4*tanh(x/2)**2 + 4*sqrt(
2)*tanh(x/2) + 4)*tanh(x/2)**8/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128)
+ 38*sqrt(2)*log(4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**4/(12
8*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) + 19*sqrt(2)*log(4*tanh(x/2)**2 +
4*sqrt(2)*tanh(x/2) + 4)/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 44*t
anh(x/2)**7/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 28*tanh(x/2)**5/(
128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 28*tanh(x/2)**3/(128*tanh(x/2)
**8 + 256*tanh(x/2)**4 + 128) - 44*tanh(x/2)/(128*tanh(x/2)**8 + 256*tanh(x
/2)**4 + 128)
```

3.38 $\int \frac{1}{1-\cosh^2(x)}\,dx$

Optimal. Leaf size=2

$$\coth(x)$$

```
[Out] coth(x)
```

Rubi [A] time = 0.02, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3175, 3767, 8}

$$\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^2)^(-1),x]
```

```
[Out] Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cosh^2(x)}\,dx &= -\int \operatorname{csch}^2(x)\,dx\\ &= i\,\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)\\ &= \coth(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^2)^(-1),x]
```

```
[Out] Coth[x]
```

fricas [B] time = 0.50, size = 20, normalized size = 10.00

$$\frac{2}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2),x, algorithm="fricas")
[Out] 2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [B] time = 0.11, size = 10, normalized size = 5.00

$$\frac{2}{e^{(2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2),x, algorithm="giac")
[Out] 2/(e^(2*x) - 1)
```

maple [B] time = 0.07, size = 16, normalized size = 8.00

$$\frac{\tanh\left(\frac{x}{2}\right)}{2} + \frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^2),x)
[Out] 1/2*tanh(1/2*x)+1/2/tanh(1/2*x)
```

maxima [B] time = 0.31, size = 10, normalized size = 5.00

$$-\frac{2}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2),x, algorithm="maxima")
[Out] -2/(e^(-2*x) - 1)
```

mupad [B] time = 0.06, size = 10, normalized size = 5.00

$$\frac{2}{e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^2 - 1),x)
[Out] 2/(exp(2*x) - 1)
```

sympy [B] time = 0.41, size = 14, normalized size = 7.00

$$\frac{\tanh\left(\frac{x}{2}\right)}{2} + \frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**2),x)
[Out] tanh(x/2)/2 + 1/(2*tanh(x/2))
```

3.39 $\int \frac{1}{\left(1-\cosh^2(x)\right)^2} dx$

Optimal. Leaf size=11

$$\coth(x) - \frac{\coth^3(x)}{3}$$

```
[Out] coth(x)-1/3*coth(x)^3
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3175, 3767}

$$\coth(x) - \frac{\coth^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^2)^(-2),x]
```

```
[Out] Coth[x] - Coth[x]^3/3
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-\cosh^2(x)\right)^2} dx &= \int \operatorname{csch}^4(x) \, dx \\
&= i \operatorname{Subst}\left(\int \left(1+x^2\right) dx, x, -i \coth(x)\right) \\
&= \coth(x) - \frac{\coth^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$\frac{2\coth(x)}{3} - \frac{1}{3}\coth(x)\operatorname{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^2)^(-2),x]
```

```
[Out] (2*Coth[x])/3 - (Coth[x]*Csch[x]^2)/3
```

fricas [B] time = 0.42, size = 84, normalized size = 7.64

$$-\frac{8\left(\cosh(x) + 2\sinh(x)\right)}{3\left(\cosh(x)^5 + 5\cosh(x)\sinh(x)^4 + \sinh(x)^5 + \left(10\cosh(x)^2 - 3\right)\sinh(x)^3 - 3\cosh(x)^3 + \left(10\cosh(x)^3 - 9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(x) + 2*sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 - 3)*sinh(x)^3 - 3*cosh(x)^3 + (10*cosh(x)^3 - 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 - 9*cosh(x)^2 + 4)*sinh(x) + 2*cosh(x))
```

giac [A] time = 0.14, size = 18, normalized size = 1.64

$$-\frac{4\left(3\,e^{(2\,x)}-1\right)}{3\left(e^{(2\,x)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*x) - 1)/(e^(2*x) - 1)^3
```

maple [B] time = 0.07, size = 32, normalized size = 2.91

$$-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24}+\frac{3\tanh\left(\frac{x}{2}\right)}{8}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}+\frac{3}{8\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^2)^2,x)
```

```
[Out] -1/24*tanh(1/2*x)^3+3/8*tanh(1/2*x)-1/24/tanh(1/2*x)^3+3/8/tanh(1/2*x)
```

maxima [B] time = 0.35, size = 49, normalized size = 4.45

$$\frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}-\frac{4}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e
^(-4*x) + e^(-6*x) - 1)
```

mupad [B] time = 0.98, size = 18, normalized size = 1.64

$$-\frac{4\left(3\,\mathrm{e}^{2\,x}-1\right)}{3\left(\mathrm{e}^{2\,x}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 - 1)^2,x)
```

```
[Out] -(4*(3*exp(2*x) - 1))/(3*(exp(2*x) - 1)^3)
```

sympy [B] time = 1.09, size = 34, normalized size = 3.09

$$-\frac{\tanh^{3}\left(\frac{x}{2}\right)}{24}+\frac{3\tanh\left(\frac{x}{2}\right)}{8}+\frac{3}{8\tanh\left(\frac{x}{2}\right)}-\frac{1}{24\tanh^{3}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**2)**2,x)
```

```
[Out] -tanh(x/2)**3/24 + 3*tanh(x/2)/8 + 3/(8*tanh(x/2)) - 1/(24*tanh(x/2)**3)
```

3.40 $\int \frac{1}{\left(1-\cosh^2(x)\right)^3} dx$

Optimal. Leaf size=19

$$\frac{\coth^5(x)}{5} - \frac{2\coth^3(x)}{3} + \coth(x)$$

```
[Out] coth(x)-2/3*coth(x)^3+1/5*coth(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3175, 3767}

$$\frac{\coth^5(x)}{5} - \frac{2\coth^3(x)}{3} + \coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^2)^(-3),x]
```

```
[Out] Coth[x] - (2*Coth[x]^3)/3 + Coth[x]^5/5
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-\cosh^2(x)\right)^3} dx &= -\int \operatorname{csch}^6(x)\, dx \\
&= i \operatorname{Subst}\left(\int \left(1+2x^2+x^4\right) dx, x, -i\coth(x)\right) \\
&= \coth(x) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.42

$$\frac{8\coth(x)}{15} + \frac{1}{5}\coth(x)\operatorname{csch}^4(x) - \frac{4}{15}\coth(x)\operatorname{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^2)^(-3),x]
```

```
[Out] (8*Coth[x])/15 - (4*Coth[x]*Csch[x]^2)/15 + (Coth[x]*Csch[x]^4)/5
```

fricas [B] time = 0.48, size = 185, normalized size = 9.74

$$\frac{}{15\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + \left(28\cosh(x)^2 - 5\right)\sinh(x)^6 - 5\cosh(x)^6 + 2\left(28\cosh(x)^3 - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 16/15*(11*cosh(x)^2 + 18*cosh(x)*sinh(x) + 11*sinh(x)^2 - 5)/(cosh(x)^8 + 8
*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 5)*sinh(x)^6 - 5*cosh(x)^6
 + 2*(28*cosh(x)^3 - 15*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 15*cosh(x)^2
 + 2)*sinh(x)^4 + 10*cosh(x)^4 + 4*(14*cosh(x)^5 - 25*cosh(x)^3 + 10*cosh(x
))*sinh(x)^3 + (28*cosh(x)^6 - 75*cosh(x)^4 + 60*cosh(x)^2 - 11)*sinh(x)^2
- 11*cosh(x)^2 + 2*(4*cosh(x)^7 - 15*cosh(x)^5 + 20*cosh(x)^3 - 9*cosh(x))*
sinh(x) + 5)
```

giac [A] time = 0.11, size = 24, normalized size = 1.26

$$\frac{16\left(10\,e^{(4\,x)}-5\,e^{(2\,x)}+1\right)}{15\left(e^{(2\,x)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(e^(2*x) - 1)^5
```

maple [B] time = 0.07, size = 48, normalized size = 2.53

$$\frac{\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{160}-\frac{5\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{96}+\frac{5\tanh\left(\frac{x}{2}\right)}{16}-\frac{5}{96\tanh\left(\frac{x}{2}\right)^{3}}+\frac{5}{16\tanh\left(\frac{x}{2}\right)}+\frac{1}{160\tanh\left(\frac{x}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^2)^3,x)
```

```
[Out] 1/160*tanh(1/2*x)^5-5/96*tanh(1/2*x)^3+5/16*tanh(1/2*x)-5/96/tanh(1/2*x)^3+
5/16/tanh(1/2*x)+1/160/tanh(1/2*x)^5
```

maxima [B] time = 0.35, size = 111, normalized size = 5.84

$$\frac{16\,e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}-\frac{32\,e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10
*x) - 1) - 32/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*
x) + e^(-10*x) - 1) - 16/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(
-8*x) + e^(-10*x) - 1)
```

mupad [B] time = 0.06, size = 24, normalized size = 1.26

$$\frac{16\,\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\,\left(\mathrm{e}^{2\,x}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^2 - 1)^3,x)
```

```
[Out] (16*(10*exp(4*x) - 5*exp(2*x) + 1))/(15*(exp(2*x) - 1)^5)
```

sympy [B] time = 2.78, size = 54, normalized size = 2.84

$$\frac{\tanh^{5}\left(\frac{x}{2}\right)}{160} - \frac{5\tanh^{3}\left(\frac{x}{2}\right)}{96} + \frac{5\tanh\left(\frac{x}{2}\right)}{16} + \frac{5}{16\tanh\left(\frac{x}{2}\right)} - \frac{5}{96\tanh^{3}\left(\frac{x}{2}\right)} + \frac{1}{160\tanh^{5}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**2)**3,x)
```

```
[Out] tanh(x/2)**5/160 - 5*tanh(x/2)**3/96 + 5*tanh(x/2)/16 + 5/(16*tanh(x/2)) -
5/(96*tanh(x/2)**3) + 1/(160*tanh(x/2)**5)
```

3.41 $\int \sqrt{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=49

$$-\frac{i\sqrt{a+b\cosh^2(x)}\,E\left(ix+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}$$

```
[Out] (-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),(-b/a)^(1/2))*(a+b*cosh(x)^2)^
(1/2)/(1+b*cosh(x)^2/a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\cosh^2(x)}\,E\left(ix+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[a + b*Cosh[x]^2]*EllipticE[Pi/2 + I*x, -(b/a)])/Sqrt[1 + (b*Cosh
[x]^2)/a]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cosh^2(x)}\,dx &= \frac{\sqrt{a+b\cosh^2(x)}\int \sqrt{1+\frac{b\cosh^2(x)}{a}}\,dx}{\sqrt{1+\frac{b\cosh^2(x)}{a}}}\\ &= -\frac{i\sqrt{a+b\cosh^2(x)}\,E\left(\frac{\pi}{2}+ix\middle|-\frac{b}{a}\right)}{\sqrt{1+\frac{b\cosh^2(x)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 1.08

$$-\frac{i\sqrt{2a+b\cosh(2x)+b}\,E\left(ix\middle|\frac{b}{a+b}\right)}{\sqrt{\frac{2a+b\cosh(2x)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]^2],x]
[Out] ((-I)*Sqrt[2*a + b + b*Cosh[2*x]]*EllipticE[I*x, b/(a + b)])/Sqrt[(2*a + b
+ b*Cosh[2*x])/(a + b)]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cosh(x)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cosh(x)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cosh(x)^2 + a), x)
```

maple [B] time = 0.28, size = 114, normalized size = 2.33

$$\frac{\sqrt{\frac{a+b\left(\cosh^2(x)\right)}{a}}\,\sqrt{-\left(\sinh^2(x)\right)}\,\left(a\,\mathrm{EllipticF}\left(\cosh(x)\sqrt{-\frac{b}{a}}, \sqrt{-\frac{a}{b}}\right) + b\,\mathrm{EllipticF}\left(\cosh(x)\sqrt{-\frac{b}{a}}, \sqrt{-\frac{a}{b}}\right) - b\,\mathrm{Ellip}\right.}{\sqrt{-\frac{b}{a}}\,\sinh(x)\sqrt{a+b\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)^2)^(1/2),x)
[Out] ((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*(a*EllipticF(cosh(x)*(-1/a*b)^
(1/2),(-a/b)^(1/2))+b*EllipticF(cosh(x)*(-1/a*b)^(1/2),(-a/b)^(1/2))-b*Elli
pticE(cosh(x)*(-1/a*b)^(1/2),(-a/b)^(1/2)))/(-1/a*b)^(1/2)/sinh(x)/(a+b*cos
h(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cosh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,{\cosh(x)}^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x)^2)^(1/2),x)
[Out] int((a + b*cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cosh(x)**2), x)
```

3.42 $\int \sqrt{1+\cosh^2(x)}\,dx$

Optimal. Leaf size=17

$$-iE\left(ix+\frac{\pi}{2}\middle|-1\right)$$

```
[Out] (-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),I)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3177}

$$-iE\left(ix+\frac{\pi}{2}\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Cosh[x]^2],x]
```

```
[Out] (-I)*EllipticE[Pi/2 + I*x, -1]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{1+\cosh^2(x)}\,dx=-iE\left(\frac{\pi}{2}+ix\middle|-1\right)$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.06

$$-i\sqrt{2}\,E\left(ix\middle|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Cosh[x]^2],x]
```

```
[Out] (-I)*Sqrt[2]*EllipticE[I*x, 1/2]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\cosh(x)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cosh(x)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(x)^2 + 1), x)
```

maple [B] time = 0.35, size = 58, normalized size = 3.41

$$-\frac{i\sqrt{\left(1+\cosh^2(x)\right)\left(\sinh^2(x)\right)}\,\sqrt{-\left(\sinh^2(x)\right)}\,\left(2\,\mathrm{EllipticF}\left(i\cosh(x),i\right)-\mathrm{EllipticE}\left(i\cosh(x),i\right)\right)}{\sqrt{\cosh^4(x)-1}\,\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cosh(x)^2)^(1/2),x)
```

```
[Out] -I*((1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)*(2*EllipticF(I*cosh(x
),I)-EllipticE(I*cosh(x),I))/(cosh(x)^4-1)^(1/2)/sinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cosh(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(x)^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int\sqrt{{\cosh(x)}^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2 + 1)^(1/2),x)
```

```
[Out] int((cosh(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cosh^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(x)**2 + 1), x)
```

3.43 $\int \sqrt{1-\cosh^2(x)}\,dx$

Optimal. Leaf size=13

$$\sqrt{-\sinh^2(x)}\,\coth(x)$$

```
[Out] coth(x)*(-sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3176, 3207, 2638}

$$\sqrt{-\sinh^2(x)}\,\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cosh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[-Sinh[x]^2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-\cosh^2(x)}\,dx &= \int \sqrt{-\sinh^2(x)}\,dx\\ &= \left(\operatorname{csch}(x)\sqrt{-\sinh^2(x)}\right)\int \sinh(x)\,dx\\ &= \coth(x)\sqrt{-\sinh^2(x)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\sqrt{-\sinh^2(x)}\,\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cosh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[-Sinh[x]^2]
```

fricas [A] time = 0.57, size = 1, normalized size = 0.08

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.13, size = 31, normalized size = 2.38

$$-\frac{1}{2} i e^{(-x)} \operatorname{sgn}\left(-e^{(3 x)} + e^{x}\right) - \frac{1}{2} i e^{x} \operatorname{sgn}\left(-e^{(3 x)} + e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*I*e^(-x)*sgn(-e^(3*x) + e^x) - 1/2*I*e^x*sgn(-e^(3*x) + e^x)
```

maple [A] time = 0.14, size = 15, normalized size = 1.15

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{-\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cosh(x)^2)^(1/2),x)
[Out] -sinh(x)*cosh(x)/(-sinh(x)^2)^(1/2)
```

maxima [C] time = 0.59, size = 11, normalized size = 0.85

$$-\frac{1}{2} i e^{(-x)} - \frac{1}{2} i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*I*e^(-x) - 1/2*I*e^x
```

mupad [B] time = 1.02, size = 13, normalized size = 1.00

$$\coth(x)\sqrt{1 - \cosh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cosh(x)^2)^(1/2),x)
[Out] coth(x)*(1 - cosh(x)^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \cosh^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)**2)**(1/2),x)
[Out] Integral(sqrt(1 - cosh(x)**2), x)
```

3.44 $\int \sqrt{-1+\cosh^2(x)}\, dx$

Optimal. Leaf size=11

$$\sqrt{\sinh^2(x)} \coth(x)$$

[Out] coth(x)*(sinh(x)^2)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3176, 3207, 2638}

$$\sqrt{\sinh^2(x)} \coth(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + Cosh[x]^2],x]

[Out] Coth[x]*Sqrt[Sinh[x]^2]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-1+\cosh^2(x)}\, dx &= \int \sqrt{\sinh^2(x)}\, dx \\
&= \left(\operatorname{csch}(x)\sqrt{\sinh^2(x)}\right) \int \sinh(x)\, dx \\
&= \coth(x)\sqrt{\sinh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\sqrt{\sinh^2(x)} \coth(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + Cosh[x]^2],x]

```
[Out] Coth[x]*Sqrt[Sinh[x]^2]
```

fricas [A] time = 0.39, size = 2, normalized size = 0.18

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)^2)^(1/2),x, algorithm="fricas")
[Out] cosh(x)
```

giac [B] time = 0.13, size = 31, normalized size = 2.82

$$\frac{1}{2}\,e^{(-x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)+\frac{1}{2}\,e^{x}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*e^(-x)*sgn(e^(3*x) - e^x) + 1/2*e^x*sgn(e^(3*x) - e^x)
```

maple [A] time = 0.16, size = 14, normalized size = 1.27

$$\frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cosh(x)^2)^(1/2),x)
[Out] (sinh(x)^2)^(1/2)*cosh(x)/sinh(x)
```

maxima [A] time = 1.42, size = 11, normalized size = 1.00

$$-\frac{1}{2}\,e^{(-x)}-\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*e^(-x) - 1/2*e^x
```

mupad [B] time = 0.95, size = 11, normalized size = 1.00

$$\coth(x)\,\sqrt{\cosh(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2 - 1)^(1/2),x)
[Out] coth(x)*(cosh(x)^2 - 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh^{2}(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)**2)**(1/2),x)
[Out] Integral(sqrt(cosh(x)**2 - 1), x)
```

3.45 $\int \sqrt{-1-\cosh^2(x)}\,dx$

Optimal. Leaf size=39

$$-\frac{i\sqrt{-\cosh^2(x)-1}\,E\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cosh^2(x)+1}}$$

```
[Out] (-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),I)*(-1-cosh(x)^2)^(1/2)/(1+cos
h(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3178, 3177}

$$-\frac{i\sqrt{-\cosh^2(x)-1}\,E\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cosh^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Cosh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[-1 - Cosh[x]^2]*EllipticE[Pi/2 + I*x, -1])/Sqrt[1 + Cosh[x]^2]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1-\cosh^2(x)}\,dx &= \frac{\sqrt{-1-\cosh^2(x)}\int \sqrt{1+\cosh^2(x)}\,dx}{\sqrt{1+\cosh^2(x)}}\\ &= -\frac{i\sqrt{-1-\cosh^2(x)}\,E\left(\frac{\pi}{2}+ix\middle|-1\right)}{\sqrt{1+\cosh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.03

$$\frac{i\sqrt{2}\sqrt{\cosh(2x)+3}\,E\left(ix\middle|\frac{1}{2}\right)}{\sqrt{-\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Cosh[x]^2],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[3 + Cosh[2*x]]*EllipticE[I*x, 1/2])/Sqrt[-3 - Cosh[2*x]]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\frac{2\left(e^{(2x)}-e^{x}\right)\mathrm{integral}\left(\frac{4\sqrt{-e^{(4x)}-6e^{(2x)}-1}\left(e^{(2x)}+1\right)}{e^{(6x)}-2e^{(5x)}+7e^{(4x)}-12e^{(3x)}+7e^{(2x)}-2e^{x}+1},x\right)+\sqrt{-e^{(4x)}-6e^{(2x)}-1}\left(e^{x}+1\right)}{2\left(e^{(2x)}-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(e^(2*x) - e^x)*integral(4*sqrt(-e^(4*x) - 6*e^(2*x) - 1)*(e^(2*x) +
 1)/(e^(6*x) - 2*e^(5*x) + 7*e^(4*x) - 12*e^(3*x) + 7*e^(2*x) - 2*e^x + 1),
 x) + sqrt(-e^(4*x) - 6*e^(2*x) - 1)*(e^x + 1))/(e^(2*x) - e^x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cosh(x)^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cosh(x)^2 - 1), x)
```

maple [A] time = 0.33, size = 62, normalized size = 1.59

$$-\frac{\sqrt{-\left(1+\cosh^{2}(x)\right)\left(\sinh^{2}(x)\right)}\,\sqrt{-\left(\sinh^{2}(x)\right)}\,\sqrt{1+\cosh^{2}(x)}\,\mathrm{EllipticE}\left(\cosh(x),i\right)}{\sqrt{1-\left(\cosh^{4}(x)\right)}\,\sinh(x)\sqrt{-1-\left(\cosh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cosh(x)^2)^(1/2),x)
```

```
[Out] -(-(1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)*(1+cosh(x)^2)^(1/2)*El
lipticE(cosh(x),I)/(1-cosh(x)^4)^(1/2)/sinh(x)/(-1-cosh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cosh(x)^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cosh(x)^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{-{\cosh(x)}^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cosh(x)^2 - 1)^(1/2),x)
```

```
[Out] int((- cosh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cosh^{2}(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-cosh(x)**2 - 1), x)
```

3.46 $\int \left(a+b\cosh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=133

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+\frac{ia(a+b)\sqrt{\frac{b\cosh^2(x)}{a}+1}\,F\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{3\sqrt{a+b\cosh^2(x)}}-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}\,E\left(ix+\right.}{3\sqrt{\frac{b\cosh^2(x)}{a}+1}}$$

```
[Out] 1/3*b*cosh(x)*sinh(x)*(a+b*cosh(x)^2)^(1/2)+2/3*(2*a+b)*(-sinh(x)^2)^(1/2)/
sinh(x)*EllipticE(cosh(x),(-b/a)^(1/2))*(a+b*cosh(x)^2)^(1/2)/(1+b*cosh(x)^
2/a)^(1/2)-1/3*a*(a+b)*(-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),(-b/a)^
(1/2))*(1+b*cosh(x)^2/a)^(1/2)/(a+b*cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+\frac{ia(a+b)\sqrt{\frac{b\cosh^2(x)}{a}+1}\,F\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{3\sqrt{a+b\cosh^2(x)}}-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}\,E\left(ix+\right.}{3\sqrt{\frac{b\cosh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^2)^(3/2),x]
```

```
[Out] (((-2*I)/3)*(2*a + b)*Sqrt[a + b*Cosh[x]^2]*EllipticE[Pi/2 + I*x, -(b/a)])/
Sqrt[1 + (b*Cosh[x]^2)/a] + ((I/3)*a*(a + b)*Sqrt[1 + (b*Cosh[x]^2)/a]*Elli
pticF[Pi/2 + I*x, -(b/a)])/Sqrt[a + b*Cosh[x]^2] + (b*Cosh[x]*Sqrt[a + b*Co
sh[x]^2]*Sinh[x])/3
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\cosh^2(x)\right)^{3/2} dx = \frac{1}{3}b\cosh(x)\sqrt{a+b\cosh^2(x)}\,\sinh(x)+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\cosh^2(x)}{\sqrt{a+b\cosh^2(x)}}\,dx$$

$$=\frac{1}{3}b\cosh(x)\sqrt{a+b\cosh^2(x)}\,\sinh(x)-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx+\frac{1}{3}(2($$

$$=\frac{1}{3}b\cosh(x)\sqrt{a+b\cosh^2(x)}\,\sinh(x)+\frac{\left(2(2a+b)\sqrt{a+b\cosh^2(x)}\right)\int \sqrt{1+\frac{b\cosh}{a}}}{3\sqrt{1+\frac{b\cosh^2(x)}{a}}}$$

$$=-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}\,E\left(\frac{\pi}{2}+ix\middle|-\frac{b}{a}\right)}{3\sqrt{1+\frac{b\cosh^2(x)}{a}}}+\frac{ia(a+b)\sqrt{1+\frac{b\cosh^2(x)}{a}}\,F\left(\frac{\pi}{2}+ix\middle|-\right.}{3\sqrt{a+b\cosh^2(x)}}$$

Mathematica [A] time = 0.48, size = 135, normalized size = 1.02

$$\frac{-8i\left(2a^2+3ab+b^2\right)\sqrt{\frac{2a+b\cosh(2x)+b}{a+b}}\,E\left(ix\middle|\frac{b}{a+b}\right)+\sqrt{2}\,b\sinh(2x)(2a+b\cosh(2x)+b)+4ia(a+b)\sqrt{\frac{2a+b\cosh(}{a+b}}}{12\sqrt{2a+b\cosh(2x)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^2)^(3/2),x]
```

```
[Out] ((-8*I)*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b + b*Cosh[2*x])/(a + b)]*Ellipti
cE[I*x, b/(a + b)] + (4*I)*a*(a + b)*Sqrt[(2*a + b + b*Cosh[2*x])/(a + b)]*
EllipticF[I*x, b/(a + b)] + Sqrt[2]*b*(2*a + b + b*Cosh[2*x])*Sinh[2*x])/(1
2*Sqrt[2*a + b + b*Cosh[2*x]])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cosh(x)^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(x)^2 + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cosh(x)^2+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x)^2 + a)^(3/2), x)
```

maple [B] time = 0.32, size = 321, normalized size = 2.41

$$\sqrt{-\frac{b}{a}}\, b^2\left(\cosh^5(x)\right)+\sqrt{-\frac{b}{a}}\, ab\left(\cosh^3(x)\right)-\sqrt{-\frac{b}{a}}\, b^2\left(\cosh^3(x)\right)+3a^2\sqrt{\frac{a+b\left(\cosh^2(x)\right)}{a}}\,\sqrt{-\left(\sinh^2(x)\right)}\ \mathrm{EllipticF}\left(\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*cosh(x)^5+(-1/a*b)^(1/2)*a*b*cosh(x)^3-(-1/a*b)^(1/
2)*b^2*cosh(x)^3+3*a^2*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*Ellipti
cF(cosh(x)*(-1/a*b)^(1/2),(-a/b)^(1/2))+5*a*b*((a+b*cosh(x)^2)/a)^(1/2)*(-s
inh(x)^2)^(1/2)*EllipticF(cosh(x)*(-1/a*b)^(1/2),(-a/b)^(1/2))+2*((a+b*cosh
(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*EllipticF(cosh(x)*(-1/a*b)^(1/2),(-a/b)^
(1/2))*b^2-4*a*b*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*EllipticE(cos
h(x)*(-1/a*b)^(1/2),(-a/b)^(1/2))-2*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^
(1/2)*EllipticE(cosh(x)*(-1/a*b)^(1/2),(-a/b)^(1/2))*b^2-(-1/a*b)^(1/2)*a*b
*cosh(x))/(-1/a*b)^(1/2)/sinh(x)/(a+b*cosh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cosh(x)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(b\cosh(x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x)^2)^(3/2),x)
```

```
[Out] int((a + b*cosh(x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.47 $\int \left(1+\cosh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=55

$$\frac{2}{3}iF\left(ix+\frac{\pi}{2}\middle|-1\right)-2iE\left(ix+\frac{\pi}{2}\middle|-1\right)+\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}$$

```
[Out] 2*(-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),I)-2/3*(-sinh(x)^2)^(1/2)/si
nh(x)*EllipticF(cosh(x),I)+1/3*cosh(x)*sinh(x)*(1+cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3180, 3172, 3177, 3182}

$$\frac{2}{3}iF\left(ix+\frac{\pi}{2}\middle|-1\right)-2iE\left(ix+\frac{\pi}{2}\middle|-1\right)+\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)^(3/2),x]
```

```
[Out] (-2*I)*EllipticE[Pi/2 + I*x, -1] + ((2*I)/3)*EllipticF[Pi/2 + I*x, -1] + (C
osh[x]*Sqrt[1 + Cosh[x]^2]*Sinh[x])/3
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(1+\cosh^2(x)\right)^{3/2} dx = \frac{1}{3}\cosh(x)\sqrt{1+\cosh^2(x)}\,\sinh(x) + \frac{1}{3}\int \frac{4+6\cosh^2(x)}{\sqrt{1+\cosh^2(x)}}\,dx$$
$$= \frac{1}{3}\cosh(x)\sqrt{1+\cosh^2(x)}\,\sinh(x) - \frac{2}{3}\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx + 2\int \sqrt{1+\cosh^2(x)}\,dx$$
$$= -2iE\left(\frac{\pi}{2}+ix\middle|-1\right) + \frac{2}{3}iF\left(\frac{\pi}{2}+ix\middle|-1\right) + \frac{1}{3}\cosh(x)\sqrt{1+\cosh^2(x)}\,\sinh(x)$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.93

$$\frac{4iF\left(ix\middle|\frac{1}{2}\right) - 24iE\left(ix\middle|\frac{1}{2}\right) + \sinh(2x)\sqrt{\cosh(2x)+3}}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^2)^(3/2),x]
[Out] ((-24*I)*EllipticE[I*x, 1/2] + (4*I)*EllipticF[I*x, 1/2] + Sqrt[3 + Cosh[2*
x]]*Sinh[2*x])/(6*Sqrt[2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cosh(x)^2+1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)^(3/2),x, algorithm="fricas")
[Out] integral((cosh(x)^2 + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cosh(x)^2+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((cosh(x)^2 + 1)^(3/2), x)
```

maple [A] time = 0.38, size = 99, normalized size = 1.80

$$-\frac{\sqrt{\left(1+\cosh^2(x)\right)\left(\sinh^2(x)\right)}\left(-\left(\cosh^5(x)\right)+10i\sqrt{1+\cosh^2(x)}\,\sqrt{-\left(\sinh^2(x)\right)}\ \text{EllipticF}\left(i\cosh(x), i\right) - 6i\sqrt{1}\right.}{3\sqrt{\cosh^4(x)-1}\ \sinh(x)\sqrt{1+\cosh^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cosh(x)^2)^(3/2),x)
[Out] -1/3*((1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-cosh(x)^5+10*I*(1+cosh(x)^2)^(1/2)*(
-sinh(x)^2)^(1/2)*EllipticF(I*cosh(x),I)-6*I*(1+cosh(x)^2)^(1/2)*(-sinh(x)^
2)^(1/2)*EllipticE(I*cosh(x),I)+cosh(x))/(cosh(x)^4-1)^(1/2)/sinh(x)/(1+cos
h(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cosh(x)^2 + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((cosh(x)^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\cosh(x)^2 + 1\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2 + 1)^(3/2),x)
[Out] int((cosh(x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cosh^2(x) + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)**2)**(3/2),x)
[Out] Integral((cosh(x)**2 + 1)**(3/2), x)
```

3.48 $\int \left(1-\cosh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)+\frac{2}{3}\sqrt{-\sinh^2(x)}\,\coth(x)$$

```
[Out] 1/3*coth(x)*(-sinh(x)^2)^(3/2)+2/3*coth(x)*(-sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3176, 3203, 3207, 2638}

$$\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)+\frac{2}{3}\sqrt{-\sinh^2(x)}\,\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^2)^(3/2),x]
```

```
[Out] (2*Coth[x]*Sqrt[-Sinh[x]^2])/3 + (Coth[x]*(-Sinh[x]^2)^(3/2))/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(1-\cosh^2(x)\right)^{3/2}\,dx &= \int \left(-\sinh^2(x)\right)^{3/2}\,dx\\
&= \frac{1}{3}\coth(x)\left(-\sinh^2(x)\right)^{3/2}+\frac{2}{3}\int \sqrt{-\sinh^2(x)}\,dx\\
&= \frac{1}{3}\coth(x)\left(-\sinh^2(x)\right)^{3/2}+\frac{1}{3}\left(2\operatorname{csch}(x)\sqrt{-\sinh^2(x)}\right)\int \sinh(x)\,dx\\
&= \frac{2}{3}\coth(x)\sqrt{-\sinh^2(x)}+\frac{1}{3}\coth(x)\left(-\sinh^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.76

$$-\frac{1}{12}\sqrt{-\sinh^2(x)}\,(\cosh(3x)-9\cosh(x))\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^2)^(3/2),x]
[Out] -1/12*((-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[-Sinh[x]^2])
```

fricas [A] time = 0.57, size = 1, normalized size = 0.03

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)^(3/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.15, size = 66, normalized size = 2.00

$$-\frac{1}{24}i\left(9\,e^{(2\,x)}\text{sgn}\left(-e^{(3\,x)}+e^{x}\right)-\text{sgn}\left(-e^{(3\,x)}+e^{x}\right)\right)e^{(-3\,x)}+\frac{1}{24}i\,e^{(3\,x)}\text{sgn}\left(-e^{(3\,x)}+e^{x}\right)-\frac{3}{8}i\,e^{x}\text{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/24*I*(9*e^(2*x)*sgn(-e^(3*x) + e^x) - sgn(-e^(3*x) + e^x))*e^(-3*x) + 1/
24*I*e^(3*x)*sgn(-e^(3*x) + e^x) - 3/8*I*e^x*sgn(-e^(3*x) + e^x)
```

maple [A] time = 0.20, size = 21, normalized size = 0.64

$$\frac{\sinh(x)\cosh(x)\left(\sinh^2(x)-2\right)}{3\sqrt{-\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cosh(x)^2)^(3/2),x)
[Out] 1/3*sinh(x)*cosh(x)*(sinh(x)^2-2)/(-sinh(x)^2)^(1/2)
```

maxima [C] time = 0.88, size = 23, normalized size = 0.70

$$\frac{1}{24}i\,e^{(3\,x)}-\frac{3}{8}i\,e^{(-x)}+\frac{1}{24}i\,e^{(-3\,x)}-\frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)^(3/2),x, algorithm="maxima")
[Out] 1/24*I*e^(3*x) - 3/8*I*e^(-x) + 1/24*I*e^(-3*x) - 3/8*I*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(1-\cosh(x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cosh(x)^2)^(3/2),x)
```

```
[Out] int((1 - cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1 - \cosh^2(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)**2)**(3/2),x)
[Out] Integral((1 - cosh(x)**2)**(3/2), x)
```

3.49 $\int \left(-1+\cosh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x)-\frac{2}{3}\sqrt{\sinh^2(x)}\,\coth(x)$$

[Out] 1/3*coth(x)*(sinh(x)^2)^(3/2)-2/3*coth(x)*(sinh(x)^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3176, 3203, 3207, 2638}

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x)-\frac{2}{3}\sqrt{\sinh^2(x)}\,\coth(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + Cosh[x]^2)^(3/2),x]

[Out] (-2*Coth[x]*Sqrt[Sinh[x]^2])/3 + (Coth[x]*(Sinh[x]^2)^(3/2))/3

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(-1+\cosh^2(x)\right)^{3/2}\,dx &= \int \sinh^2(x)^{3/2}\,dx\\ &= \frac{1}{3}\coth(x)\sinh^2(x)^{3/2}-\frac{2}{3}\int \sqrt{\sinh^2(x)}\,dx\\ &= \frac{1}{3}\coth(x)\sinh^2(x)^{3/2}-\frac{1}{3}\left(2\text{csch}(x)\sqrt{\sinh^2(x)}\right)\int \sinh(x)\,dx\\ &= -\frac{2}{3}\coth(x)\sqrt{\sinh^2(x)}+\frac{1}{3}\coth(x)\sinh^2(x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.79

$$\frac{1}{12}\sqrt{\sinh^2(x)}\,(\cosh(3x)-9\cosh(x))\mathrm{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Cosh[x]^2)^(3/2),x]
[Out] ((-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[Sinh[x]^2])/12
```

fricas [A] time = 0.48, size = 19, normalized size = 0.66

$$\frac{1}{12}\cosh(x)^3+\frac{1}{4}\cosh(x)\sinh(x)^2-\frac{3}{4}\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/12*cosh(x)^3 + 1/4*cosh(x)*sinh(x)^2 - 3/4*cosh(x)
```

giac [B] time = 0.14, size = 66, normalized size = 2.28

$$-\frac{1}{24}\left(9\,e^{(2\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)-\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right)e^{(-3\,x)}+\frac{1}{24}\,e^{(3\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)-\frac{3}{8}\,e^{x}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/24*(9*e^(2*x)*sgn(e^(3*x) - e^x) - sgn(e^(3*x) - e^x))*e^(-3*x) + 1/24*e
^(3*x)*sgn(e^(3*x) - e^x) - 3/8*e^x*sgn(e^(3*x) - e^x)
```

maple [A] time = 0.24, size = 21, normalized size = 0.72

$$\frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\cosh(x)\left(\cosh^2(x)-3\right)}{3\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cosh(x)^2)^(3/2),x)
[Out] 1/3*(sinh(x)^2)^(1/2)*cosh(x)*(cosh(x)^2-3)/sinh(x)
```

maxima [A] time = 0.43, size = 23, normalized size = 0.79

$$-\frac{1}{24}\,e^{(3\,x)}+\frac{3}{8}\,e^{(-x)}-\frac{1}{24}\,e^{(-3\,x)}+\frac{3}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)^2)^(3/2),x, algorithm="maxima")
[Out] -1/24*e^(3*x) + 3/8*e^(-x) - 1/24*e^(-3*x) + 3/8*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(\cosh(x)^2-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((cosh(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cosh^2(x) - 1\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cosh(x)**2)**(3/2),x)
```

```
[Out] Integral((cosh(x)**2 - 1)**(3/2), x)
```

3.50 $\int \left(-1-\cosh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+\frac{2i\sqrt{\cosh^2(x)+1}\,F\left(ix+\frac{\pi}{2}\middle|-1\right)}{3\sqrt{-\cosh^2(x)-1}}+\frac{2i\sqrt{-\cosh^2(x)-1}\,E\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cosh^2(x)+1}}$$

```
[Out] -1/3*cosh(x)*sinh(x)*(-1-cosh(x)^2)^(1/2)-2*(-sinh(x)^2)^(1/2)/sinh(x)*Elli
pticE(cosh(x),I)*(-1-cosh(x)^2)^(1/2)/(1+cosh(x)^2)^(1/2)-2/3*(-sinh(x)^2)^
(1/2)/sinh(x)*EllipticF(cosh(x),I)*(1+cosh(x)^2)^(1/2)/(-1-cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+\frac{2i\sqrt{\cosh^2(x)+1}\,F\left(ix+\frac{\pi}{2}\middle|-1\right)}{3\sqrt{-\cosh^2(x)-1}}+\frac{2i\sqrt{-\cosh^2(x)-1}\,E\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cosh^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Cosh[x]^2)^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[-1 - Cosh[x]^2]*EllipticE[Pi/2 + I*x, -1])/Sqrt[1 + Cosh[x]^2]
+ (((2*I)/3)*Sqrt[1 + Cosh[x]^2]*EllipticF[Pi/2 + I*x, -1])/Sqrt[-1 - Cosh[
x]^2] - (Cosh[x]*Sqrt[-1 - Cosh[x]^2]*Sinh[x])/3
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(-1-\cosh^2(x)\right)^{3/2}\,dx &= -\frac{1}{3}\cosh(x)\sqrt{-1-\cosh^2(x)}\,\sinh(x)+\frac{1}{3}\int \frac{4+6\cosh^2(x)}{\sqrt{-1-\cosh^2(x)}}\,dx\\
&= -\frac{1}{3}\cosh(x)\sqrt{-1-\cosh^2(x)}\,\sinh(x)-\frac{2}{3}\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx-2\int \sqrt{-1-\cos\\
&= -\frac{1}{3}\cosh(x)\sqrt{-1-\cosh^2(x)}\,\sinh(x)-\frac{\left(2\sqrt{-1-\cosh^2(x)}\right)\int \sqrt{1+\cosh^2(x)}\,dx}{\sqrt{1+\cosh^2(x)}}-\\
&= \frac{2i\sqrt{-1-\cosh^2(x)}\,E\left(\frac{\pi}{2}+ix\middle|-1\right)}{\sqrt{1+\cosh^2(x)}}+\frac{2i\sqrt{1+\cosh^2(x)}\,F\left(\frac{\pi}{2}+ix\middle|-1\right)}{3\sqrt{-1-\cosh^2(x)}}-\frac{1}{3}\cosh(x)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.77

$$\frac{6\sinh(2x)+\sinh(4x)+8i\sqrt{\cosh(2x)+3}\,F\left(ix\middle|\frac{1}{2}\right)-48i\sqrt{\cosh(2x)+3}\,E\left(ix\middle|\frac{1}{2}\right)}{12\sqrt{2}\sqrt{-\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Cosh[x]^2)^(3/2),x]
```

```
[Out] ((-48*I)*Sqrt[3 + Cosh[2*x]]*EllipticE[I*x, 1/2] + (8*I)*Sqrt[3 + Cosh[2*x]
]*EllipticF[I*x, 1/2] + 6*Sinh[2*x] + Sinh[4*x])/(12*Sqrt[2]*Sqrt[-3 - Cosh
[2*x]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\frac{24\left(e^{(4x)}-e^{(3x)}\right)\mathrm{integral}\left(-\frac{4\sqrt{-e^{(4x)}-6e^{(2x)}-1}\left(5e^{(2x)}+2e^{x}+5\right)}{3\left(e^{(6x)}-2e^{(5x)}+7e^{(4x)}-12e^{(3x)}+7e^{(2x)}-2e^{x}+1\right)},x\right)-\left(e^{(5x)}-e^{(4x)}+24e^{(3x)}+24e^{(2x)}-e^{x}\right.}{24\left(e^{(4x)}-e^{(3x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*(e^(4*x) - e^(3*x))*integral(-4/3*sqrt(-e^(4*x) - 6*e^(2*x) - 1)*(
5*e^(2*x) + 2*e^x + 5)/(e^(6*x) - 2*e^(5*x) + 7*e^(4*x) - 12*e^(3*x) + 7*e^
(2*x) - 2*e^x + 1), x) - (e^(5*x) - e^(4*x) + 24*e^(3*x) + 24*e^(2*x) - e^x
 + 1)*sqrt(-e^(4*x) - 6*e^(2*x) - 1))/(e^(4*x) - e^(3*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\cosh(x)^2-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cosh(x)^2 - 1)^(3/2), x)
```

maple [A] time = 0.42, size = 96, normalized size = 0.95

$$-\frac{\sqrt{-\left(1+\cosh^2(x)\right)\left(\sinh^2(x)\right)}\left(-\left(\cosh^5(x)\right)+2\sqrt{-\left(\sinh^2(x)\right)}\sqrt{1+\cosh^2(x)}\,\mathrm{EllipticF}(\cosh(x),i)-6\sqrt{-(s}\right.}{3\sqrt{1-\left(\cosh^4(x)\right)}\,\sinh(x)\sqrt{-1-\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cosh(x)^2)^(3/2),x)
```

```
[Out] -1/3*(-(1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-cosh(x)^5+2*(-sinh(x)^2)^(1/2)*(1+c
osh(x)^2)^(1/2)*EllipticF(cosh(x),I)-6*(-sinh(x)^2)^(1/2)*(1+cosh(x)^2)^(1/
2)*EllipticE(cosh(x),I)+cosh(x))/(1-cosh(x)^4)^(1/2)/sinh(x)/(-1-cosh(x)^2)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cosh(x)^2-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cosh(x)^2 - 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(-\cosh(x)^2-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cosh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((- cosh(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cosh^2(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cosh(x)**2)**(3/2),x)
```

```
[Out] Integral((-cosh(x)**2 - 1)**(3/2), x)
```

3.51 $\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx$

Optimal. Leaf size=49

$$-\frac{i\sqrt{\frac{b\cosh^2(x)}{a}+1}\,F\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{a+b\cosh^2(x)}}$$

```
[Out] (-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),(-b/a)^(1/2))*(1+b*cosh(x)^2/a
)^(1/2)/(a+b*cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\cosh^2(x)}{a}+1}\,F\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{a+b\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[1 + (b*Cosh[x]^2)/a]*EllipticF[Pi/2 + I*x, -(b/a)])/Sqrt[a + b*C
osh[x]^2]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx &= \frac{\sqrt{1+\frac{b\cosh^2(x)}{a}}\int \frac{1}{\sqrt{1+\frac{b\cosh^2(x)}{a}}}\,dx}{\sqrt{a+b\cosh^2(x)}}\\ &= -\frac{i\sqrt{1+\frac{b\cosh^2(x)}{a}}\,F\left(\frac{\pi}{2}+ix|-\frac{b}{a}\right)}{\sqrt{a+b\cosh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 1.08

$$-\frac{i\sqrt{\frac{2a+b\cosh(2x)+b}{a+b}}\,F\left(ix\middle|\frac{b}{a+b}\right)}{\sqrt{2a+b\cosh(2x)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cosh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a + b + b*Cosh[2*x])/(a + b)]*EllipticF[I*x, b/(a + b)])/Sqrt
[2*a + b + b*Cosh[2*x]]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cosh(x)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cosh(x)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cosh(x)^2 + a), x)
```

maple [A] time = 0.19, size = 66, normalized size = 1.35

$$\frac{\sqrt{\frac{a+b\left(\cosh^2(x)\right)}{a}}\,\sqrt{-\left(\sinh^2(x)\right)}\,\operatorname{EllipticF}\left(\cosh(x)\sqrt{-\frac{b}{a}},\sqrt{-\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\sinh(x)\sqrt{a+b\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*EllipticF(cos
h(x)*(-1/a*b)^(1/2),(-a/b)^(1/2))/sinh(x)/(a+b*cosh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cosh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,{\cosh(x)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cosh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(a + b*cosh(x)**2), x)
```

3.52 $\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx$

Optimal. Leaf size=17

$$-iF\left(ix+\frac{\pi}{2}\middle|-1\right)$$

```
[Out] (-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),I)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3182}

$$-iF\left(ix+\frac{\pi}{2}\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Cosh[x]^2],x]
[Out] (-I)*EllipticF[Pi/2 + I*x, -1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx=-iF\left(\frac{\pi}{2}+ix\middle|-1\right)$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.06

$$-\frac{iF\left(ix\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Cosh[x]^2],x]
[Out] ((-I)*EllipticF[I*x, 1/2])/Sqrt[2]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\cosh(x)^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(cosh(x)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cosh(x)^2 + 1), x)
```

maple [B] time = 0.29, size = 45, normalized size = 2.65

$$-\frac{i\sqrt{\left(1+\cosh^{2}(x)\right)\left(\sinh^{2}(x)\right)}\,\sqrt{-\left(\sinh^{2}(x)\right)}\,\operatorname{EllipticF}\left(i\cosh(x), i\right)}{\sqrt{\cosh^{4}(x)-1}\,\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^2)^(1/2),x)
```

```
[Out] -I*((1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)/(cosh(x)^4-1)^(1/2)*E
llipticF(I*cosh(x),I)/sinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cosh(x)^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{{\cosh(x)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 + 1)^(1/2),x)
```

```
[Out] int(1/(cosh(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh^{2}(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cosh(x)**2 + 1), x)
```

3.53 $\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{-\sinh^2(x)}}$$

```
[Out] -arctanh(cosh(x))*sinh(x)/(-sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3176, 3207, 3770}

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Cosh[x]^2],x]
```

```
[Out] -((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[-Sinh[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx &= \int \frac{1}{\sqrt{-\sinh^2(x)}}\,dx\\
&= \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{\sqrt{-\sinh^2(x)}}\\
&= -\frac{\tanh^{-1}(\cosh(x))\sinh(x)}{\sqrt{-\sinh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$\frac{\sinh(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Cosh[x]^2],x]
[Out] (Log[Tanh[x/2]]*Sinh[x])/Sqrt[-Sinh[x]^2]
```

fricas [A] time = 0.49, size = 1, normalized size = 0.06

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.15, size = 40, normalized size = 2.35

$$-\frac{i\log\left(e^x+1\right)}{\operatorname{sgn}\left(-e^{(3x)}+e^x\right)}+\frac{i\log\left(|e^x-1|\right)}{\operatorname{sgn}\left(-e^{(3x)}+e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^(1/2),x, algorithm="giac")
[Out] -I*log(e^x + 1)/sgn(-e^(3*x) + e^x) + I*log(abs(e^x - 1))/sgn(-e^(3*x) + e^
x)
```

maple [B] time = 0.14, size = 34, normalized size = 2.00

$$-\frac{\sinh(x)\sqrt{-\left(\cosh^2(x)\right)}\ \arctan\left(\frac{1}{\sqrt{-\left(\cosh^2(x)\right)}}\right)}{\cosh(x)\sqrt{-\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^2)^(1/2),x)
[Out] -sinh(x)*(-cosh(x)^2)^(1/2)*arctan(1/(-cosh(x)^2)^(1/2))/cosh(x)/(-sinh(x)^
2)^(1/2)
```

maxima [C] time = 0.82, size = 19, normalized size = 1.12

$$-i\log\left(e^{(-x)}+1\right)+i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{1-\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cosh(x)^2)^(1/2),x)
```

```
[Out] int(1/(1 - cosh(x)^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1 - \cosh^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - cosh(x)**2), x)
```

3.54 $\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx$

Optimal. Leaf size=15

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{\sinh^2(x)}}$$

```
[Out] -arctanh(cosh(x))*sinh(x)/(sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3176, 3207, 3770}

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Cosh[x]^2],x]
```

```
[Out] -((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[Sinh[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx &= \int \frac{1}{\sqrt{\sinh^2(x)}}\,dx\\ &= \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{\sqrt{\sinh^2(x)}}\\ &= -\frac{\tanh^{-1}(\cosh(x))\sinh(x)}{\sqrt{\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{\sinh(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Cosh[x]^2],x]
[Out] (Log[Tanh[x/2]]*Sinh[x])/Sqrt[Sinh[x]^2]
```

fricas [A] time = 0.78, size = 17, normalized size = 1.13

$$-\log\left(\cosh(x)+\sinh(x)+1\right)+\log\left(\cosh(x)+\sinh(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cosh(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

giac [B] time = 0.12, size = 39, normalized size = 2.60

$$-\frac{\log\left(e^x+1\right)}{\operatorname{sgn}\left(e^{(3x)}-e^x\right)}+\frac{\log\left(|e^x-1|\right)}{\operatorname{sgn}\left(e^{(3x)}-e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cosh(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(e^x + 1)/sgn(e^(3*x) - e^x) + log(abs(e^x - 1))/sgn(e^(3*x) - e^x)
```

maple [A] time = 0.17, size = 16, normalized size = 1.07

$$-\frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\operatorname{arctanh}\left(\cosh(x)\right)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+cosh(x)^2)^(1/2),x)
[Out] -(sinh(x)^2)^(1/2)*arctanh(cosh(x))/sinh(x)
```

maxima [A] time = 1.34, size = 17, normalized size = 1.13

$$\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cosh(x)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(-x) + 1) - log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{\cosh(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 - 1)^(1/2),x)
```

```
[Out] int(1/(cosh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh^2(x) - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cosh(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(cosh(x)**2 - 1), x)
```

3.55 $\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\, dx$

Optimal. Leaf size=39

$$-\frac{i\sqrt{\cosh^2(x)+1}\,F\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{-\cosh^2(x)-1}}$$

```
[Out] (-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),I)*(1+cosh(x)^2)^(1/2)/(-1-cos
h(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\cosh^2(x)+1}\,F\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{-\cosh^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Cosh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[1 + Cosh[x]^2]*EllipticF[Pi/2 + I*x, -1])/Sqrt[-1 - Cosh[x]^2]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\, dx &= \frac{\sqrt{1+\cosh^2(x)}\int \frac{1}{\sqrt{1+\cosh^2(x)}}\, dx}{\sqrt{-1-\cosh^2(x)}}\\ &= -\frac{i\sqrt{1+\cosh^2(x)}\,F\left(\frac{\pi}{2}+ix\middle|-1\right)}{\sqrt{-1-\cosh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 1.03

$$-\frac{i\sqrt{\cosh(2x)+3}\,F\left(ix\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Cosh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[3 + Cosh[2*x]]*EllipticF[I*x, 1/2])/(Sqrt[2]*Sqrt[-3 - Cosh[2*x]
])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{2\sqrt{-e^{(4x)} - 6e^{(2x)} - 1}}{e^{(4x)} + 6e^{(2x)} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*sqrt(-e^(4*x) - 6*e^(2*x) - 1)/(e^(4*x) + 6*e^(2*x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cosh(x)^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-cosh(x)^2 - 1), x)
```

maple [A] time = 0.34, size = 61, normalized size = 1.56

$$\frac{\sqrt{-\left(1+\cosh^2(x)\right)\left(\sinh^2(x)\right)}\,\sqrt{-\left(\sinh^2(x)\right)}\,\sqrt{1+\cosh^2(x)}\,\operatorname{EllipticF}(\cosh(x), i)}{\sqrt{1-\left(\cosh^4(x)\right)}\,\sinh(x)\sqrt{-1-\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-cosh(x)^2)^(1/2),x)
```

```
[Out] (-(1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)*(1+cosh(x)^2)^(1/2)/(1-
cosh(x)^4)^(1/2)*EllipticF(cosh(x),I)/sinh(x)/(-1-cosh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cosh(x)^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-cosh(x)^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-\cosh(x)^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- cosh(x)^2 - 1)^(1/2),x)
```

```
[Out] int(1/(- cosh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cosh^2(x) - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-cosh(x)**2 - 1), x)
```

3.56 $\int \frac{1}{a+b\cosh^3(x)}\,dx$

Optimal. Leaf size=288

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 2/3*arctanh((a^(1/3)-b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)+b^(1/3))^(1/2))/a^
(2/3)/(a^(1/3)-b^(1/3))^(1/2)/(a^(1/3)+b^(1/3))^(1/2)+2/3*arctanh((a^(1/3)+
(-1)^(1/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2))/a
^(2/3)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2
)+2/3*arctanh((a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)+(-1)^
(2/3)*b^(1/3))^(1/2))/a^(2/3)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)/(a^(1/3)+(
-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3213, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^3)^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) + b^(1/3)]])/(3
*a^(2/3)*Sqrt[a^(1/3) - b^(1/3)]*Sqrt[a^(1/3) + b^(1/3)]) + (2*ArcTanh[(Sqr
t[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3
)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^(1/3
)*b^(1/3)]) + (2*ArcTanh[(Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)]*Tanh[x/2])/Sqr
t[a^(1/3) + (-1)^(2/3)*b^(1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1
/3)]*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\cosh^3(x)}\,dx = \int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\cosh(x)\right)} - \frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\cosh(x)\right)} - \frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)\right.}\right.$$
$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\cosh(x)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\cosh(x)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\cosh(x)}\,dx}{3a^{2/3}}$$
$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}-\left(-\sqrt[3]{a}+\sqrt[3]{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{3a^{2/3}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}-\left(-\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\right.}\right.}{3a^{2/3}}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{2\tanh^{-}}{3a^{2/3}\sqrt{\sqrt[3]{a}-}}$$

Mathematica [C] time = 0.13, size = 105, normalized size = 0.36

$$\frac{2}{3}\mathrm{RootSum}\left[\#1^6 b+3\#1^4 b+8\#1^3 a+3\#1^2 b+b\&, \frac{\#1 x+2\#1\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\right.}{\#1^4 b+2\#1^2 b+4\#1 a+b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^3)^(-1),x]
```

```
[Out] (2*RootSum[b + 3*b*#1^2 + 8*a*#1^3 + 3*b*#1^4 + b*#1^6 & , (x*#1 + 2*Log[-C
osh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1)/(b + 4*a*#1 + 2*b*#
1^2 + b*#1^4) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cosh(x)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*cosh(x)^3 + a), x)
```

maple [C] time = 0.79, size = 100, normalized size = 0.35

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^6+(-3a-3b)\_Z^4+(3a-3b)\_Z^2-a-b\right)} \frac{\left(-\_R^4+2\_R^2-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^5 a-\_R^5 b-2\_R^3 a-2\_R^3 b+\_R a-\_R b}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^3),x)
```

```
[Out] 1/3*sum((-_R^4+2*_R^2-1)/(_R^5*a-_R^5*b-2*_R^3*a-2*_R^3*b+_R*a-_R*b)*ln(tan
h(1/2*x)-_R),_R=RootOf((a-b)*_Z^6+(-3*a-3*b)*_Z^4+(3*a-3*b)*_Z^2-a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cosh(x)^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cosh(x)^3 + a), x)
```

mupad [B] time = 5.16, size = 633, normalized size = 2.20

$$\sum_{k=1}^{6} \ln\left(-\frac{\left(-4\,e^{x} + \mathrm{root}\left(729\,a^4\,b^2\,d^6 - 729\,a^6\,d^6 + 243\,a^4\,d^4 - 27\,a^2\,d^2 + 1, d, k\right)\,b + \mathrm{root}\left(729\,a^4\,b^2\,d^6 - 729\,a^6\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^3),x)
```

```
[Out] symsum(log(-(24576*(root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a
^2*d^2 + 1, d, k)*b - 4*exp(x) + 54*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 24
3*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^3*a^2*b + 108*root(729*a^4*b^2*d^6 - 729*
a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^4*a^3*b + 81*root(729*a^4*b^2
*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^4*b + 24*root(
729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2*a^2*e
xp(x) + 216*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 +
 1, d, k)^3*a^3*exp(x) + 108*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d
^4 - 27*a^2*d^2 + 1, d, k)^4*a^4*exp(x) - 324*root(729*a^4*b^2*d^6 - 729*a^
6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^5*exp(x) + 12*root(729*a^4*
b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2*a*b - 20*root
(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)*a*exp(
x) + 27*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1,
d, k)^4*a^2*b^2*exp(x) + 405*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d
^4 - 27*a^2*d^2 + 1, d, k)^5*a^3*b^2*exp(x)))/b^5)*root(729*a^4*b^2*d^6 - 7
29*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**3),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{1}{a-b\cosh^3(x)}\,dx$

Optimal. Leaf size=288

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}}$$

```
[Out] 2/3*arctanh((a^(1/3)+b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)-b^(1/3))^(1/2))/a^
(2/3)/(a^(1/3)-b^(1/3))^(1/2)/(a^(1/3)+b^(1/3))^(1/2)+2/3*arctanh((a^(1/3)-
(-1)^(1/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2))/a
^(2/3)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2
)+2/3*arctanh((a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)-(-1)^
(2/3)*b^(1/3))^(1/2))/a^(2/3)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)/(a^(1/3)+(
-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3213, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cosh[x]^3)^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a^(1/3) + b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) - b^(1/3)]])/(3
*a^(2/3)*Sqrt[a^(1/3) - b^(1/3)]*Sqrt[a^(1/3) + b^(1/3)]) + (2*ArcTanh[(Sqr
t[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3
)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^(1/3
)*b^(1/3)]) + (2*ArcTanh[(Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)]*Tanh[x/2])/Sqr
t[a^(1/3) - (-1)^(2/3)*b^(1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1
/3)]*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a-b\cosh^3(x)}\,dx = \int \left(\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-\sqrt[3]{b}\cosh(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\cosh(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-(-1)^{2/3}\right.}\right.$$

$$=\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}\cosh(x)}\,dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\cosh(x)}\,dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\cosh(x)}\,dx}{3a^{2/3}}$$

$$=\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}-\left(\sqrt[3]{a}+\sqrt[3]{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{3a^{2/3}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}-\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\right)x^2}\right.}{3a^{2/3}}$$

$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\tan}{3a^{2/3}\sqrt{\sqrt[3]{a}}}$$

Mathematica [C] time = 0.09, size = 105, normalized size = 0.36

$$-\frac{2}{3}\text{RootSum}\left[\#1^6b+3\#1^4b-8\#1^3a+3\#1^2b+b\&,\frac{\#1x+2\#1\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-c\right.}{\#1^4b+2\#1^2b-4\#1a+b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cosh[x]^3)^(-1),x]
```

```
[Out] (-2*RootSum[b + 3*b*#1^2 - 8*a*#1^3 + 3*b*#1^4 + b*#1^6 & , (x*#1 + 2*Log[-
Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1)/(b - 4*a*#1 + 2*b*
#1^2 + b*#1^4) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{b\cosh(x)^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^3),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*cosh(x)^3 - a), x)
```

maple [C] time = 0.70, size = 94, normalized size = 0.33

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a+b)\_Z^6+(-3a+3b)\_Z^4+(3a+3b)\_Z^2-a+b\right)}\frac{\left(-\_R^4+2\_R^2-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^5a+\_R^5b-2\_R^3a+2\_R^3b+\_Ra+\_Rb}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cosh(x)^3),x)
```

```
[Out] 1/3*sum((-_R^4+2*_R^2-1)/(_R^5*a+_R^5*b-2*_R^3*a+2*_R^3*b+_R*a+_R*b)*ln(tan
h(1/2*x)-_R),_R=RootOf((a+b)*_Z^6+(-3*a+3*b)*_Z^4+(3*a+3*b)*_Z^2-a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\cosh(x)^3 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^3),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cosh(x)^3 - a), x)
```

mupad [B] time = 5.89, size = 633, normalized size = 2.20

$$\sum_{k=1}^{6} \ln\left(-\frac{\left(4\,e^x + \mathrm{root}\left(729\,a^4\,b^2\,d^6 - 729\,a^6\,d^6 + 243\,a^4\,d^4 - 27\,a^2\,d^2 + 1, d, k\right)\,b + \mathrm{root}\left(729\,a^4\,b^2\,d^6 - 729\,a^6\,d^6 +\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cosh(x)^3),x)
```

```
[Out] symsum(log(-(24576*(4*exp(x) + root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4
*d^4 - 27*a^2*d^2 + 1, d, k)*b + 54*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 24
3*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^3*a^2*b + 108*root(729*a^4*b^2*d^6 - 729*
a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^4*a^3*b + 81*root(729*a^4*b^2
*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^4*b - 24*root(
729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2*a^2*e
xp(x) - 216*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 +
 1, d, k)^3*a^3*exp(x) - 108*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d
^4 - 27*a^2*d^2 + 1, d, k)^4*a^4*exp(x) + 324*root(729*a^4*b^2*d^6 - 729*a^
6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^5*exp(x) + 12*root(729*a^4*
b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2*a*b + 20*root
(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)*a*exp(
x) - 27*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1,
d, k)^4*a^2*b^2*exp(x) - 405*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d
^4 - 27*a^2*d^2 + 1, d, k)^5*a^3*b^2*exp(x)))/b^5)*root(729*a^4*b^2*d^6 - 7
29*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)**3),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{1}{1+\cosh^3(x)}\,dx$

Optimal. Leaf size=91

$$-\frac{2\sqrt[4]{-\frac{1}{3}}\tanh^{-1}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1+(-1)^{2/3}\right)}-\frac{2\sqrt[4]{-\frac{1}{3}}\tan^{-1}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1-\sqrt[3]{-1}\right)}+\frac{\sinh(x)}{3(\cosh(x)+1)}$$

```
[Out] -2/9*(-1)^(1/4)*3^(3/4)*arctan((-1)^(3/4)*3^(1/4)*tanh(1/2*x))/(1-(-1)^(1/3
))-2/9*(-1)^(1/4)*3^(3/4)*arctanh((-1)^(3/4)*3^(1/4)*tanh(1/2*x))/(1+(-1)^(
2/3))+1/3*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3213, 2648, 2659, 205, 208}

$$-\frac{2\sqrt[4]{-\frac{1}{3}}\tanh^{-1}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1+(-1)^{2/3}\right)}-\frac{2\sqrt[4]{-\frac{1}{3}}\tan^{-1}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1-\sqrt[3]{-1}\right)}+\frac{\sinh(x)}{3(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^3)^(-1),x]
```

```
[Out] (-2*(-1/3)^(1/4)*ArcTan[(-1)^(3/4)*3^(1/4)*Tanh[x/2]])/(3*(1 - (-1)^(1/3)))
 - (2*(-1/3)^(1/4)*ArcTanh[(-1)^(3/4)*3^(1/4)*Tanh[x/2]])/(3*(1 + (-1)^(2/3
))) + Sinh[x]/(3*(1 + Cosh[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1+\cosh^3(x)}\,dx = \int \left(-\frac{1}{3(-1-\cosh(x))} - \frac{1}{3\left(-1+\sqrt[3]{-1}\cosh(x)\right)} - \frac{1}{3\left(-1-(-1)^{2/3}\cosh(x)\right)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{1}{-1-\cosh(x)}\,dx\right) - \frac{1}{3}\int \frac{1}{-1+\sqrt[3]{-1}\cosh(x)}\,dx - \frac{1}{3}\int \frac{1}{-1-(-1)^{2/3}\cosh(x)}\,dx$$
$$= \frac{\sinh(x)}{3(1+\cosh(x))} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{-1}-\left(-1-\sqrt[3]{-1}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - \frac{2}{3}\text{Subst}\left(\int\right.$$
$$= -\frac{2\sqrt[4]{-\frac{1}{3}}\tan^{-1}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1-\sqrt[3]{-1}\right)} - \frac{2\sqrt[4]{-\frac{1}{3}}\tanh^{-1}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1+(-1)^{2/3}\right)} + \frac{\sinh(x)}{3(1+\cosh(}$$

Mathematica [C] time = 0.99, size = 133, normalized size = 1.46

$$\frac{1}{18}\left(6\tanh\left(\frac{x}{2}\right) - \sqrt{6+2i\sqrt{3}}\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{\left(3+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{6-2i\sqrt{3}}}\right) - \sqrt{6-2i\sqrt{3}}\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{\left(3-i\sqrt{3}\right)}{\sqrt{6+2}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^3)^(-1),x]
```

```
[Out] (-(Sqrt[6 + (2*I)*Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((3 + I*Sqrt[3])*Tanh[x/2
])/Sqrt[6 - (2*I)*Sqrt[3]]]) - Sqrt[6 - (2*I)*Sqrt[3]]*(-3*I + Sqrt[3])*Arc
Tan[((3 - I*Sqrt[3])*Tanh[x/2])/Sqrt[6 + (2*I)*Sqrt[3]]] + 6*Tanh[x/2])/18
```

fricas [B] time = 0.54, size = 602, normalized size = 6.62

$$\frac{4\left(3^{\frac{3}{4}}e^x + 3^{\frac{3}{4}}\right)\sqrt{-4\sqrt{3}+8}\,\arctan\left(\frac{1}{12}\left(\sqrt{3}\left(\sqrt{3}+3\right)-3\sqrt{3}+9\right)e^x - \frac{1}{48}\left(2\sqrt{3}\left(\sqrt{3}+3\right) - \left(3^{\frac{3}{4}}\left(3\sqrt{3}+5\right)+3\cdot 3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(4*(3^(3/4)*e^x + 3^(3/4))*sqrt(-4*sqrt(3) + 8)*arctan(1/12*(sqrt(3)*(
sqrt(3) + 3) - 3*sqrt(3) + 9)*e^x - 1/48*(2*sqrt(3)*(sqrt(3) + 3) - (3^(3/4
)*(3*sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*sqrt(-4*sqrt(3) + 8) - 6*sqrt(
3) + 18)*sqrt(2*(3^(1/4)*(sqrt(3) + 2) - 3^(1/4)*e^x)*sqrt(-4*sqrt(3) + 8)
+ 4*sqrt(3) + 4*e^(2*x) - 4*e^x + 4) + 1/12*sqrt(3)*(sqrt(3) - 3) - 1/24*((
3^(3/4)*(3*sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*e^x - 3^(3/4)*(sqrt(3) +
 1) - 3*3^(1/4)*(sqrt(3) - 1))*sqrt(-4*sqrt(3) + 8) + 1/4*sqrt(3) - 1/4) +
4*(3^(3/4)*e^x + 3^(3/4))*sqrt(-4*sqrt(3) + 8)*arctan(-1/12*(sqrt(3)*(sqrt(
3) + 3) - 3*sqrt(3) + 9)*e^x + 1/48*(2*sqrt(3)*(sqrt(3) + 3) + (3^(3/4)*(3*
sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*sqrt(-4*sqrt(3) + 8) - 6*sqrt(3) +
18)*sqrt(-2*(3^(1/4)*(sqrt(3) + 2) - 3^(1/4)*e^x)*sqrt(-4*sqrt(3) + 8) + 4*
sqrt(3) + 4*e^(2*x) - 4*e^x + 4) - 1/12*sqrt(3)*(sqrt(3) - 3) - 1/24*((3^(3
/4)*(3*sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*e^x - 3^(3/4)*(sqrt(3) + 1)
- 3*3^(1/4)*(sqrt(3) - 1))*sqrt(-4*sqrt(3) + 8) - 1/4*sqrt(3) + 1/4) - (3^(
1/4)*(2*sqrt(3) + 3)*e^x + 3^(1/4)*(2*sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8)*lo
g(2*(3^(1/4)*(sqrt(3) + 2) - 3^(1/4)*e^x)*sqrt(-4*sqrt(3) + 8) + 4*sqrt(3)
+ 4*e^(2*x) - 4*e^x + 4) + (3^(1/4)*(2*sqrt(3) + 3)*e^x + 3^(1/4)*(2*sqrt(3
) + 3))*sqrt(-4*sqrt(3) + 8)*log(-2*(3^(1/4)*(sqrt(3) + 2) - 3^(1/4)*e^x)*s
qrt(-4*sqrt(3) + 8) + 4*sqrt(3) + 4*e^(2*x) - 4*e^x + 4) - 24)/(e^x + 1)
```

giac [B] time = 0.14, size = 275, normalized size = 3.02

$$\frac{1}{18}\sqrt{6\sqrt{3}+9}\log\left(4\left(2\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{6\sqrt{3}+9}+6e^{x}-3\right)^{2}+4\left(\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{3}\right)^{2}\right)-\frac{1}{18}\sqrt{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^3),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(6*sq
rt(3) + 9) + 6*e^x - 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(3))^2)
- 1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(6*
sqrt(3) + 9) - 6*e^x + 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) + 3*sqrt(3))^2
) - 1/9*sqrt(3)*sqrt(6*sqrt(3) + 9)*arctan(3*(sqrt(2*sqrt(3) - 3) + 2*e^x -
 1)/(sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(3)))/(2*sqrt(3) + 3) - 1/9*sqrt(3
)*sqrt(6*sqrt(3) + 9)*arctan(-3*(sqrt(2*sqrt(3) - 3) - 2*e^x + 1)/(sqrt(3)*
sqrt(6*sqrt(3) + 9) + 3*sqrt(3)))/(2*sqrt(3) + 3) - 2/3/(e^x + 1)
```

maple [B] time = 0.05, size = 216, normalized size = 2.37

$$\frac{\tanh\left(\frac{x}{2}\right)}{3}+\frac{3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)-1\right)}{18}+\frac{3^{\frac{3}{4}}\sqrt{2}\,\ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)+\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}+\frac{\sqrt{3}}{3}}{\tanh^{2}\left(\frac{x}{2}\right)-\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}+\frac{\sqrt{3}}{3}}\right)}{36}+\frac{3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\sqrt{2}\,3^{\frac{1}{4}}\,t\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^3),x)
```

```
[Out] 1/3*tanh(1/2*x)+1/18*3^(3/4)*2^(1/2)*arctan(2^(1/2)*3^(1/4)*tanh(1/2*x)-1)+
1/36*3^(3/4)*2^(1/2)*ln((tanh(1/2*x)^2+1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1/3*
3^(1/2))/(tanh(1/2*x)^2-1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1/3*3^(1/2)))+1/18*
3^(3/4)*2^(1/2)*arctan(2^(1/2)*3^(1/4)*tanh(1/2*x)+1)-1/12*2^(1/2)*3^(1/4)*
ln((tanh(1/2*x)^2-1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1/3*3^(1/2))/(tanh(1/2*x)
^2+1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1/3*3^(1/2)))-1/6*2^(1/2)*3^(1/4)*arctan
(2^(1/2)*3^(1/4)*tanh(1/2*x)-1)-1/6*2^(1/2)*3^(1/4)*arctan(2^(1/2)*3^(1/4)*
tanh(1/2*x)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{3\left(e^{x}+1\right)}-\int\frac{2\left(e^{(3x)}-4e^{(2x)}+e^{x}\right)}{3\left(e^{(4x)}-2e^{(3x)}+6e^{(2x)}-2e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^3),x, algorithm="maxima")
```

```
[Out] -2/3/(e^x + 1) - integrate(2/3*(e^(3*x) - 4*e^(2*x) + e^x)/(e^(4*x) - 2*e^(
3*x) + 6*e^(2*x) - 2*e^x + 1), x)
```

mupad [B] time = 3.43, size = 291, normalized size = 3.20

$$\ln\left(\frac{128}{9}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(\frac{160}{3}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(384\,\mathrm{e}^{x}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(1152\,\mathrm{e}^{x}-864\right)-192\right)-\frac{32\,\mathrm{e}^{x}}{3}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3 + 1),x)
```

```
[Out] log((1/18 - (3^(1/2)*1i)/54)^(1/2)*((1/18 - (3^(1/2)*1i)/54)^(1/2)*(384*exp
(x) + (1/18 - (3^(1/2)*1i)/54)^(1/2)*(1152*exp(x) - 864) - 192) - (32*exp(x
))/3 + 160/3) - (32*exp(x))/3 + 128/9)*(1/18 - (3^(1/2)*1i)/54)^(1/2) + log
(((3^(1/2)*1i)/54 + 1/18)^(1/2)*(((3^(1/2)*1i)/54 + 1/18)^(1/2)*(384*exp(x)
 + ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(1152*exp(x) - 864) - 192) - (32*exp(x))/
3 + 160/3) - (32*exp(x))/3 + 128/9)*((3^(1/2)*1i)/54 + 1/18)^(1/2) - log(12
8/9 - (1/18 - (3^(1/2)*1i)/54)^(1/2)*((1/18 - (3^(1/2)*1i)/54)^(1/2)*((1/18
 - (3^(1/2)*1i)/54)^(1/2)*(1152*exp(x) - 864) - 384*exp(x) + 192) - (32*exp
(x))/3 + 160/3) - (32*exp(x))/3)*(1/18 - (3^(1/2)*1i)/54)^(1/2) - log(128/9
 - ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(((3^(1/2)*1i)/54 + 1/18)^(1/2)*(((3^(1/2
)*1i)/54 + 1/18)^(1/2)*(1152*exp(x) - 864) - 384*exp(x) + 192) - (32*exp(x)
)/3 + 160/3) - (32*exp(x))/3)*((3^(1/2)*1i)/54 + 1/18)^(1/2) - 2/(3*(exp(x)
 + 1))
```

sympy [**B**] time = 3.26, size = 330, normalized size = 3.63

$$-\frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tanh^{2}\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{18+18\sqrt{3}}-\frac{3\sqrt{2}\sqrt[4]{3}\log\left(36\tanh^{2}\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+\right.}{18+18\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**3),x)
```

```
[Out] -2*sqrt(2)*3**(3/4)*log(36*tanh(x/2)**2 - 12*sqrt(2)*3**(3/4)*tanh(x/2) + 1
2*sqrt(3))/(18 + 18*sqrt(3)) - 3*sqrt(2)*3**(1/4)*log(36*tanh(x/2)**2 - 12*
sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/(18 + 18*sqrt(3)) + 3*sqrt(2)*3**(
1/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/(18
+ 18*sqrt(3)) + 2*sqrt(2)*3**(3/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3**(3/4
)*tanh(x/2) + 12*sqrt(3))/(18 + 18*sqrt(3)) + 6*tanh(x/2)/(18 + 18*sqrt(3))
 + 6*sqrt(3)*tanh(x/2)/(18 + 18*sqrt(3)) - 2*sqrt(2)*3**(3/4)*atan(sqrt(2)*
3**(1/4)*tanh(x/2) - 1)/(18 + 18*sqrt(3)) - 2*sqrt(2)*3**(3/4)*atan(sqrt(2)
*3**(1/4)*tanh(x/2) + 1)/(18 + 18*sqrt(3))
```

3.59 $\int \frac{1}{1-\cosh^3(x)}\,dx$

Optimal. Leaf size=95

$$-\frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1+\sqrt[3]{-1}\right)}-\frac{2\sqrt[4]{-1}\tan^{-1}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1-(-1)^{2/3}\right)}-\frac{\sinh(x)}{3(1-\cosh(x))}$$

```
[Out] -2/3*(-1)^(1/4)*arctan(1/3*(-1)^(3/4)*tanh(1/2*x)*3^(3/4))*3^(1/4)/(1-(-1)^
(2/3))-2/3*(-1)^(1/4)*arctanh(1/3*(-1)^(3/4)*tanh(1/2*x)*3^(3/4))*3^(1/4)/(
1+(-1)^(1/3))-1/3*sinh(x)/(1-cosh(x))
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3213, 2648, 2659, 208, 205}

$$-\frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1+\sqrt[3]{-1}\right)}-\frac{2\sqrt[4]{-1}\tan^{-1}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1-(-1)^{2/3}\right)}-\frac{\sinh(x)}{3(1-\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^3)^(-1),x]
```

```
[Out] (-2*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Tanh[x/2])/3^(1/4)])/(3^(3/4)*(1 - (-1)^(
2/3))) - (2*(-1)^(1/4)*ArcTanh[((-1)^(3/4)*Tanh[x/2])/3^(1/4)])/(3^(3/4)*(1
 + (-1)^(1/3))) - Sinh[x]/(3*(1 - Cosh[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1-\cosh^3(x)}\,dx = \int \left(\frac{1}{3(1-\cosh(x))} + \frac{1}{3\left(1+\sqrt[3]{-1}\,\cosh(x)\right)} + \frac{1}{3\left(1-(-1)^{2/3}\cosh(x)\right)}\right) dx$$
$$= \frac{1}{3}\int \frac{1}{1-\cosh(x)}\,dx + \frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\,\cosh(x)}\,dx + \frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\cosh(x)}\,dx$$
$$= -\frac{\sinh(x)}{3(1-\cosh(x))} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{-1}-\left(1-\sqrt[3]{-1}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) + \frac{2}{3}\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{2\sqrt[4]{-1}\,\tan^{-1}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1-(-1)^{2/3}\right)} - \frac{2\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1+\sqrt[3]{-1}\right)} - \frac{\sinh(x)}{3(1-\cosh(x))}$$

Mathematica [C] time = 0.61, size = 147, normalized size = 1.55

$$\frac{1}{3}\coth\left(\frac{x}{2}\right) + \frac{\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{\left(1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2\left(3-i\sqrt{3}\right)}}\right)}{3\sqrt{\frac{3}{2}\left(3-i\sqrt{3}\right)}} + \frac{\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{\left(1+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2\left(3+i\sqrt{3}\right)}}\right)}{3\sqrt{\frac{3}{2}\left(3+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^3)^(-1),x]
```

```
[Out] ((3*I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*Tanh[x/2])/Sqrt[2*(3 - I*Sqrt[3])]
])/(3*Sqrt[(3*(3 - I*Sqrt[3]))/2]) + ((-3*I + Sqrt[3])*ArcTan[((1 + I*Sqrt[
3])*Tanh[x/2])/Sqrt[2*(3 + I*Sqrt[3])]])/(3*Sqrt[(3*(3 + I*Sqrt[3]))/2]) +
Coth[x/2]/3
```

fricas [B] time = 0.57, size = 602, normalized size = 6.34

$$-\frac{4\left(3^{\frac{3}{4}}e^x - 3^{\frac{3}{4}}\right)\sqrt{-4\sqrt{3}+8}\,\arctan\left(\frac{1}{12}\left(\sqrt{3}\left(\sqrt{3}+3\right)-3\sqrt{3}+9\right)e^x - \frac{1}{48}\left(2\sqrt{3}\left(\sqrt{3}+3\right)-\left(3^{\frac{3}{4}}\left(3\sqrt{3}+5\right)+3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(4*(3^(3/4)*e^x - 3^(3/4))*sqrt(-4*sqrt(3) + 8)*arctan(1/12*(sqrt(3)*
(sqrt(3) + 3) - 3*sqrt(3) + 9)*e^x - 1/48*(2*sqrt(3)*(sqrt(3) + 3) - (3^(3/
4)*(3*sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*sqrt(-4*sqrt(3) + 8) - 6*sqrt
(3) + 18)*sqrt(2*(3^(1/4)*(sqrt(3) + 2) + 3^(1/4)*e^x)*sqrt(-4*sqrt(3) + 8)
 + 4*sqrt(3) + 4*e^(2*x) + 4*e^x + 4) - 1/12*sqrt(3)*(sqrt(3) - 3) - 1/24*(
(3^(3/4)*(3*sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*e^x + 3^(3/4)*(sqrt(3)
+ 1) + 3*3^(1/4)*(sqrt(3) - 1))*sqrt(-4*sqrt(3) + 8) - 1/4*sqrt(3) + 1/4) +
 4*(3^(3/4)*e^x - 3^(3/4))*sqrt(-4*sqrt(3) + 8)*arctan(-1/12*(sqrt(3)*(sqrt
(3) + 3) - 3*sqrt(3) + 9)*e^x + 1/48*(2*sqrt(3)*(sqrt(3) + 3) + (3^(3/4)*(3
*sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*sqrt(-4*sqrt(3) + 8) - 6*sqrt(3) +
 18)*sqrt(-2*(3^(1/4)*(sqrt(3) + 2) + 3^(1/4)*e^x)*sqrt(-4*sqrt(3) + 8) + 4
*sqrt(3) + 4*e^(2*x) + 4*e^x + 4) + 1/12*sqrt(3)*(sqrt(3) - 3) - 1/24*((3^(
3/4)*(3*sqrt(3) + 5) + 3*3^(1/4)*(sqrt(3) + 1))*e^x + 3^(3/4)*(sqrt(3) + 1)
 + 3*3^(1/4)*(sqrt(3) - 1))*sqrt(-4*sqrt(3) + 8) + 1/4*sqrt(3) - 1/4) + (3^
(1/4)*(2*sqrt(3) + 3)*e^x - 3^(1/4)*(2*sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8)*l
og(2*(3^(1/4)*(sqrt(3) + 2) + 3^(1/4)*e^x)*sqrt(-4*sqrt(3) + 8) + 4*sqrt(3)
 + 4*e^(2*x) + 4*e^x + 4) - (3^(1/4)*(2*sqrt(3) + 3)*e^x - 3^(1/4)*(2*sqrt(
3) + 3))*sqrt(-4*sqrt(3) + 8)*log(-2*(3^(1/4)*(sqrt(3) + 2) + 3^(1/4)*e^x)*
sqrt(-4*sqrt(3) + 8) + 4*sqrt(3) + 4*e^(2*x) + 4*e^x + 4) - 24)/(e^x - 1)
```

giac [B] time = 0.17, size = 275, normalized size = 2.89

$$-\frac{1}{18}\sqrt{6\sqrt{3}+9}\log\left(4\left(2\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{6\sqrt{3}+9}+6e^{x}+3\right)^{2}+4\left(\sqrt{3}\sqrt{6\sqrt{3}+9}+3\sqrt{3}\right)^{2}\right)+\frac{1}{18}\sqrt{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^3),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(6*s
qrt(3) + 9) + 6*e^x + 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) + 3*sqrt(3))^2)
 + 1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(6
*sqrt(3) + 9) - 6*e^x - 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(3))^
2) + 1/9*sqrt(3)*sqrt(6*sqrt(3) + 9)*arctan(3*(sqrt(2*sqrt(3) - 3) + 2*e^x
+ 1)/(sqrt(3)*sqrt(6*sqrt(3) + 9) + 3*sqrt(3)))/(2*sqrt(3) + 3) + 1/9*sqrt(
3)*sqrt(6*sqrt(3) + 9)*arctan(-3*(sqrt(2*sqrt(3) - 3) - 2*e^x - 1)/(sqrt(3)
*sqrt(6*sqrt(3) + 9) - 3*sqrt(3)))/(2*sqrt(3) + 3) + 2/3/(e^x - 1)
```

maple [B] time = 0.07, size = 212, normalized size = 2.23

$$\frac{3^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}+1\right)}{6}+\frac{3^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}-1\right)}{6}+\frac{3^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)+\sqrt{3}}{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)+\sqrt{3}}\right)}{12}-\frac{3^{\frac{3}{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^3),x)
```

```
[Out] 1/6*3^(1/4)*2^(1/2)*arctan(1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1)+1/6*3^(1/4)*2
^(1/2)*arctan(1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)-1)+1/12*3^(1/4)*2^(1/2)*ln((t
anh(1/2*x)^2+2^(1/2)*3^(1/4)*tanh(1/2*x)+3^(1/2))/(tanh(1/2*x)^2-2^(1/2)*3^
(1/4)*tanh(1/2*x)+3^(1/2)))-1/18*3^(3/4)*2^(1/2)*arctan(1/3*3^(3/4)*tanh(1/
2*x)*2^(1/2)+1)-1/18*3^(3/4)*2^(1/2)*arctan(1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)
-1)-1/36*3^(3/4)*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*3^(1/4)*tanh(1/2*x)+3^(1
/2))/(tanh(1/2*x)^2+2^(1/2)*3^(1/4)*tanh(1/2*x)+3^(1/2)))+1/3/tanh(1/2*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3\,(e^{x}-1)}+\int\frac{2\left(e^{(3x)}+4\,e^{(2x)}+e^{x}\right)}{3\left(e^{(4x)}+2\,e^{(3x)}+6\,e^{(2x)}+2\,e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/3/(e^x - 1) + integrate(2/3*(e^(3*x) + 4*e^(2*x) + e^x)/(e^(4*x) + 2*e^(3
*x) + 6*e^(2*x) + 2*e^x + 1), x)
```

mupad [B] time = 3.42, size = 295, normalized size = 3.11

$$\ln\left(\frac{32\,e^{x}}{3}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1i}{54}}\,\left(\frac{32\,e^{x}}{3}-\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1i}{54}}\,\left(384\,e^{x}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1i}{54}}\,(1152\,e^{x}+864)+192\right)+\frac{160}{3}\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^3 - 1),x)
```

```
[Out] log((32*exp(x))/3 + (1/18 - (3^(1/2)*1i)/54)^(1/2)*((32*exp(x))/3 - (1/18 -
 (3^(1/2)*1i)/54)^(1/2)*(384*exp(x) + (1/18 - (3^(1/2)*1i)/54)^(1/2)*(1152*
exp(x) + 864) + 192) + 160/3) + 128/9)*(1/18 - (3^(1/2)*1i)/54)^(1/2) + log
((32*exp(x))/3 + ((3^(1/2)*1i)/54 + 1/18)^(1/2)*((32*exp(x))/3 - ((3^(1/2)*
1i)/54 + 1/18)^(1/2)*(384*exp(x) + ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(1152*exp
(x) + 864) + 192) + 160/3) + 128/9)*((3^(1/2)*1i)/54 + 1/18)^(1/2) - log((3
2*exp(x))/3 - (1/18 - (3^(1/2)*1i)/54)^(1/2)*((32*exp(x))/3 + (1/18 - (3^(1
/2)*1i)/54)^(1/2)*(384*exp(x) - (1/18 - (3^(1/2)*1i)/54)^(1/2)*(1152*exp(x)
 + 864) + 192) + 160/3) + 128/9)*(1/18 - (3^(1/2)*1i)/54)^(1/2) - log((32*e
xp(x))/3 - ((3^(1/2)*1i)/54 + 1/18)^(1/2)*((32*exp(x))/3 + ((3^(1/2)*1i)/54
 + 1/18)^(1/2)*(384*exp(x) - ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(1152*exp(x) +
864) + 192) + 160/3) + 128/9)*((3^(1/2)*1i)/54 + 1/18)^(1/2) + 2/(3*(exp(x)
 - 1))
```

sympy [**B**] time = 3.72, size = 405, normalized size = 4.26

$$\frac{\sqrt{2}\,\sqrt[4]{3}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\,\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{-18\tanh\left(\frac{x}{2}\right)+6\sqrt{3}\tanh\left(\frac{x}{2}\right)}-\frac{\sqrt{2}\,\sqrt[4]{3}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\,\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\right.}{-18\tanh\left(\frac{x}{2}\right)+6\sqrt{3}\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**3),x)
```

```
[Out] sqrt(2)*3**(1/4)*log(4*tanh(x/2)**2 - 4*sqrt(2)*3**(1/4)*tanh(x/2) + 4*sqrt
(3))*tanh(x/2)/(-18*tanh(x/2) + 6*sqrt(3)*tanh(x/2)) - sqrt(2)*3**(1/4)*log
(4*tanh(x/2)**2 + 4*sqrt(2)*3**(1/4)*tanh(x/2) + 4*sqrt(3))*tanh(x/2)/(-18*
tanh(x/2) + 6*sqrt(3)*tanh(x/2)) - 4*sqrt(2)*3**(1/4)*tanh(x/2)*atan(sqrt(2
)*3**(3/4)*tanh(x/2)/3 - 1)/(-18*tanh(x/2) + 6*sqrt(3)*tanh(x/2)) + 2*sqrt(
2)*3**(3/4)*tanh(x/2)*atan(sqrt(2)*3**(3/4)*tanh(x/2)/3 - 1)/(-18*tanh(x/2)
 + 6*sqrt(3)*tanh(x/2)) - 4*sqrt(2)*3**(1/4)*tanh(x/2)*atan(sqrt(2)*3**(3/4
)*tanh(x/2)/3 + 1)/(-18*tanh(x/2) + 6*sqrt(3)*tanh(x/2)) + 2*sqrt(2)*3**(3/
4)*tanh(x/2)*atan(sqrt(2)*3**(3/4)*tanh(x/2)/3 + 1)/(-18*tanh(x/2) + 6*sqrt
(3)*tanh(x/2)) - 6/(-18*tanh(x/2) + 6*sqrt(3)*tanh(x/2)) + 2*sqrt(3)/(-18*t
anh(x/2) + 6*sqrt(3)*tanh(x/2))
```

3.60 $\int \frac{1}{a+b\cosh^4(x)}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a+b}+\sqrt{a}}-\sqrt{2}\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+b}}-\frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a+b}+\sqrt{a}}+\sqrt{2}\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+b}}-\frac{\sqrt{\sqrt{a}}}{}$$

```
[Out] 1/4*arctanh(((a^(1/2)+(a+b)^(1/2))^(1/2)-a^(1/4)*2^(1/2)*tanh(x))/(a^(1/2)-
(a+b)^(1/2))^(1/2))*(a^(1/2)-(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^(1/2)
-1/4*arctanh(((a^(1/2)+(a+b)^(1/2))^(1/2)+a^(1/4)*2^(1/2)*tanh(x))/(a^(1/2)
-(a+b)^(1/2))^(1/2))*(a^(1/2)-(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^(1/2
)-1/8*ln((a+b)^(1/2)-a^(1/4)*2^(1/2)*(a^(1/2)+(a+b)^(1/2))^(1/2)*tanh(x)+a^
(1/2)*tanh(x)^2)*(a^(1/2)+(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^(1/2)+1/
8*ln((a+b)^(1/2)+a^(1/4)*2^(1/2)*(a^(1/2)+(a+b)^(1/2))^(1/2)*tanh(x)+a^(1/2
)*tanh(x)^2)*(a^(1/2)+(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 485, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\log\left((a+b)^{3/4}\coth^2(x)-\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\coth(x)+\sqrt{a}\sqrt[4]{a+b}\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a+b}+\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^4)^(-1),x]
```

```
[Out] ((Sqrt[a] - Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]
 - Sqrt[2]*(a + b)^(3/4)*Coth[x])/(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
)])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - (
(Sqrt[a] - Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]] +
 Sqrt[2]*(a + b)^(3/4)*Coth[x])/(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])
])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - ((
Sqrt[a] + Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a +
 b + Sqrt[a]*Sqrt[a + b]]*Coth[x] + (a + b)^(3/4)*Coth[x]^2])/(4*Sqrt[2]*a^
(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + ((Sqrt[a] + Sqrt[a
 + b])*Log[Sqrt[a]*(a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqr
t[a + b]]*Coth[x] + (a + b)^(3/4)*Coth[x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^(1
/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^4(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a+b)x^4}\,dx, x, \coth(x)\right)$$

$$= \frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\left(1+\frac{\sqrt{a}}{\sqrt{a+b}}\right)x}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \coth(x)\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} + \frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\cdots}}{(a+\cdots}}{\frac{\sqrt{a}}{\sqrt{a+b}}+\cdots}\cdots\right.}{2\sqrt{2}\,a^{3/4}\sqrt{\cdots}}$$

$$= -\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \coth(x)\right)}{4\sqrt{a}\,(a+b)} - \frac{\left(\sqrt{a}-\sqrt{a+b}\right)\text{S}\cdots}{\cdots}$$

$$= -\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}\,\coth(x)+(a+b)^{3/4}\,\text{co}\cdots\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}$$

$$= \frac{\left(\sqrt{a}-\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\coth(x)\right)}{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}} - \frac{\left(\sqrt{a}-\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\cdots}{\cdots}\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a\cdots}}$$

Mathematica [C] time = 0.24, size = 121, normalized size = 0.34

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+i\sqrt{a}\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{-a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+i\sqrt{a}\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[-a + I*Sqrt[a]*Sqrt[b]]]/(Sqrt[a]*Sqrt[-
a + I*Sqrt[a]*Sqrt[b]]) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + I*Sqrt[a]*Sqrt
[b]]]/(2*Sqrt[a]*Sqrt[a + I*Sqrt[a]*Sqrt[b]])
```

fricas [B] time = 0.91, size = 771, normalized size = 2.14

$$-\frac{1}{4}\sqrt{\frac{\left(a^2+ab\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\left(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x)^2+2\left(ab+\left(a^4+a^3b\right)\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))
*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*(a*b + (a^4 + a^3*
b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) + 2*(a^3 + a^2*b)*sqrt(-b/(a^5 + 2*a^4*b
 + a^3*b^2)) + b) + 1/4*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)
) + 1)/(a^2 + a*b))*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - 2
*(a*b + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))*sqrt(((a^2 + a*b)
*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) + 2*(a^3 + a^2*b)*sqr
t(-b/(a^5 + 2*a^4*b + a^3*b^2)) + b) - 1/4*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5
+ 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(
x) + b*sinh(x)^2 + 2*(a*b - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)
))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))
- 2*(a^3 + a^2*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + b) + 1/4*sqrt(-((a^2
 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - 2*(a*b - (a^4 + a^3*b)*sqrt(-b/(a^5
 + 2*a^4*b + a^3*b^2)))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2
)) - 1)/(a^2 + a*b)) - 2*(a^3 + a^2*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) +
 b)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-36,9]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[95,47]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.The choice was done assuming [a,b]=[56,74]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [a,b]=[10,-88]8*(sqrt(1/1024)*sqrt(1/1048576*(1048576*
```

```
a^2-1048576*a*sqrt(-a*b))/(a^4+a^3*b))*ln(abs(60*a^4*b*exp(x)^2+6*a^4*b-24*
a^4*sqrt(-a*b)*exp(x)^2+68*a^3*b^2*exp(x)^2+2*a^3*b^2+16*a^3*b*sqrt(-a*b)*e
xp(x)^2+12*a^3*b*sqrt(-a*b)+48*a^3*b*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2+6*a^3*
b*sqrt(a^2+a*sqrt(-a*b))-24*a^3*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2-
16*a^2*b^3*exp(x)^2-8*a^2*b^3+64*a^2*b^2*sqrt(-a*b)*exp(x)^2+16*a^2*b^2*sqr
t(-a*b)+61*a^2*b^2*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2+5*a^2*b^2*sqrt(a^2+a*sqr
t(-a*b))-5*a^2*b*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2+9*a^2*b*sqrt(-a
*b)*sqrt(a^2+a*sqrt(-a*b))-4*a*b^3*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2-4*a*b^3*
sqrt(a^2+a*sqrt(-a*b))+36*a*b^2*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2+
12*a*b^2*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))))-sqrt(1/1024)*sqrt(1/1048576*(1
048576*a^2-1048576*a*sqrt(-a*b))/(a^4+a^3*b))*ln(abs(60*a^4*b*exp(x)^2+6*a^
4*b-24*a^4*sqrt(-a*b)*exp(x)^2+68*a^3*b^2*exp(x)^2+2*a^3*b^2+16*a^3*b*sqrt(
-a*b)*exp(x)^2+12*a^3*b*sqrt(-a*b)-48*a^3*b*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2
-6*a^3*b*sqrt(a^2+a*sqrt(-a*b))+24*a^3*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))*ex
p(x)^2-16*a^2*b^3*exp(x)^2-8*a^2*b^3+64*a^2*b^2*sqrt(-a*b)*exp(x)^2+16*a^2*
b^2*sqrt(-a*b)-61*a^2*b^2*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2-5*a^2*b^2*sqrt(a^
2+a*sqrt(-a*b))+5*a^2*b*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2-9*a^2*b*
sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))+4*a*b^3*sqrt(a^2+a*sqrt(-a*b))*exp(x)^2+4
*a*b^3*sqrt(a^2+a*sqrt(-a*b))-36*a*b^2*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))*ex
p(x)^2-12*a*b^2*sqrt(-a*b)*sqrt(a^2+a*sqrt(-a*b))))+sqrt(1/1024)*sqrt(1/104
8576*(1048576*a^2+1048576*a*sqrt(-a*b))/(a^4+a^3*b))*ln(abs(60*a^4*b*exp(x)
^2+6*a^4*b+24*a^4*sqrt(-a*b)*exp(x)^2+68*a^3*b^2*exp(x)^2+2*a^3*b^2-16*a^3*
b*sqrt(-a*b)*exp(x)^2-12*a^3*b*sqrt(-a*b)+48*a^3*b*sqrt(a^2-a*sqrt(-a*b))*e
xp(x)^2+6*a^3*b*sqrt(a^2-a*sqrt(-a*b))+24*a^3*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a
*b))*exp(x)^2-16*a^2*b^3*exp(x)^2-8*a^2*b^3-64*a^2*b^2*sqrt(-a*b)*exp(x)^2-
16*a^2*b^2*sqrt(-a*b)+61*a^2*b^2*sqrt(a^2-a*sqrt(-a*b))*exp(x)^2+5*a^2*b^2*
sqrt(a^2-a*sqrt(-a*b))+5*a^2*b*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a*b))*exp(x)^2-9
*a^2*b*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a*b))-4*a*b^3*sqrt(a^2-a*sqrt(-a*b))*exp
(x)^2-4*a*b^3*sqrt(a^2-a*sqrt(-a*b))-36*a*b^2*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a
*b))*exp(x)^2-12*a*b^2*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a*b))))-sqrt(1/1024)*sqr
t(1/1048576*(1048576*a^2+1048576*a*sqrt(-a*b))/(a^4+a^3*b))*ln(abs(60*a^4*b
*exp(x)^2+6*a^4*b+24*a^4*sqrt(-a*b)*exp(x)^2+68*a^3*b^2*exp(x)^2+2*a^3*b^2-
16*a^3*b*sqrt(-a*b)*exp(x)^2-12*a^3*b*sqrt(-a*b)-48*a^3*b*sqrt(a^2-a*sqrt(-
a*b))*exp(x)^2-6*a^3*b*sqrt(a^2-a*sqrt(-a*b))-24*a^3*sqrt(-a*b)*sqrt(a^2-a*
sqrt(-a*b))*exp(x)^2-16*a^2*b^3*exp(x)^2-8*a^2*b^3-64*a^2*b^2*sqrt(-a*b)*ex
p(x)^2-16*a^2*b^2*sqrt(-a*b)-61*a^2*b^2*sqrt(a^2-a*sqrt(-a*b))*exp(x)^2-5*a
^2*b^2*sqrt(a^2-a*sqrt(-a*b))-5*a^2*b*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a*b))*exp
(x)^2+9*a^2*b*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a*b))+4*a*b^3*sqrt(a^2-a*sqrt(-a*
b))*exp(x)^2+4*a*b^3*sqrt(a^2-a*sqrt(-a*b))+36*a*b^2*sqrt(-a*b)*sqrt(a^2-a*
sqrt(-a*b))*exp(x)^2+12*a*b^2*sqrt(-a*b)*sqrt(a^2-a*sqrt(-a*b)))))
```

maple [C] time = 0.09, size = 121, normalized size = 0.34

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^8+(-4a+4b)\_Z^6+(6a+6b)\_Z^4+(-4a+4b)\_Z^2+a+b\right)} \frac{\left(-\_R^6+3\_R^4-3\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^7a+\_R^7b-3\_R^5a+3\_R^5b+3\_R^3a+3\_R^3b-\_Ra+\_Rb}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^4),x)
```

```
[Out] 1/4*sum((-_R^6+3*_R^4-3*_R^2+1)/(_R^7*a+_R^7*b-3*_R^5*a+3*_R^5*b+3*_R^3*a+3
*_R^3*b-_R*a+_R*b)*ln(tanh(1/2*x)-_R),_R=RootOf((a+b)*_Z^8+(-4*a+4*b)*_Z^6+
(6*a+6*b)*_Z^4+(-4*a+4*b)*_Z^2+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cosh(x)^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cosh(x)^4 + a), x)
```

mupad [B] time = 7.55, size = 1563, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^4),x)
```

```
[Out] log((524288*(1024*a^3*exp(2*x) - 35*b^3*exp(2*x) + 185*a*b^2 + 464*a^2*b +
256*a^3 - 24*b^3 + 988*a*b^2*exp(2*x) + 2048*a^2*b*exp(2*x)))/(a*b^6*(a + b
)^2) - (((((4194304*(253*a*b^3 - b^4*exp(2*x) + 1184*a^3*b + 512*a^4 - b^4
+ 930*a^2*b^2 + 1392*a^2*b^2*exp(2*x) + 627*a*b^3*exp(2*x) + 768*a^3*b*exp(
2*x)))/(b^6*(a + b)^2) + (8388608*a*((a^2 + (-a^3*b)^(1/2))/(a^3*(a + b)))^
(1/2)*(512*a^3*exp(2*x) - 6*b^3*exp(2*x) + 181*a*b^2 + 432*a^2*b + 256*a^3
- 5*b^3 + 622*a*b^2*exp(2*x) + 1152*a^2*b*exp(2*x)))/(b^6*(a + b)))*((a^2 +
 (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4 - (2097152*(176*a*b + 1536*a^2*exp
(2*x) - 134*b^2*exp(2*x) + 256*a^2 - 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a +
 b)))*((a^2 + (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4)*((a^2 + (-a^3*b)^(1/
2))/(16*(a^3*b + a^4)))^(1/2) - log((524288*(1024*a^3*exp(2*x) - 35*b^3*exp
(2*x) + 185*a*b^2 + 464*a^2*b + 256*a^3 - 24*b^3 + 988*a*b^2*exp(2*x) + 204
8*a^2*b*exp(2*x)))/(a*b^6*(a + b)^2) - (((((4194304*(253*a*b^3 - b^4*exp(2*
x) + 1184*a^3*b + 512*a^4 - b^4 + 930*a^2*b^2 + 1392*a^2*b^2*exp(2*x) + 627
*a*b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a + b)^2) - (8388608*a*((a^2 +
 (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2)*(512*a^3*exp(2*x) - 6*b^3*exp(2*x) +
181*a*b^2 + 432*a^2*b + 256*a^3 - 5*b^3 + 622*a*b^2*exp(2*x) + 1152*a^2*b*e
xp(2*x)))/(b^6*(a + b)))*((a^2 + (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4 +
(2097152*(176*a*b + 1536*a^2*exp(2*x) - 134*b^2*exp(2*x) + 256*a^2 - 75*b^2
 + 1408*a*b*exp(2*x)))/(b^6*(a + b)))*((a^2 + (-a^3*b)^(1/2))/(a^3*(a + b))
)^(1/2))/4)*((a^2 + (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2) - log((524288
*(1024*a^3*exp(2*x) - 35*b^3*exp(2*x) + 185*a*b^2 + 464*a^2*b + 256*a^3 - 2
4*b^3 + 988*a*b^2*exp(2*x) + 2048*a^2*b*exp(2*x)))/(a*b^6*(a + b)^2) - ((((
(4194304*(253*a*b^3 - b^4*exp(2*x) + 1184*a^3*b + 512*a^4 - b^4 + 930*a^2*b
^2 + 1392*a^2*b^2*exp(2*x) + 627*a*b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6
*(a + b)^2) - (8388608*a*((a^2 - (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2)*(512*
a^3*exp(2*x) - 6*b^3*exp(2*x) + 181*a*b^2 + 432*a^2*b + 256*a^3 - 5*b^3 + 6
22*a*b^2*exp(2*x) + 1152*a^2*b*exp(2*x)))/(b^6*(a + b)))*((a^2 - (-a^3*b)^(
1/2))/(a^3*(a + b)))^(1/2))/4 + (2097152*(176*a*b + 1536*a^2*exp(2*x) - 134
*b^2*exp(2*x) + 256*a^2 - 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a + b)))*((a^2
 - (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4)*((a^2 - (-a^3*b)^(1/2))/(16*(a^
3*b + a^4)))^(1/2) + log((524288*(1024*a^3*exp(2*x) - 35*b^3*exp(2*x) + 185
*a*b^2 + 464*a^2*b + 256*a^3 - 24*b^3 + 988*a*b^2*exp(2*x) + 2048*a^2*b*exp
(2*x)))/(a*b^6*(a + b)^2) - (((((4194304*(253*a*b^3 - b^4*exp(2*x) + 1184*a
^3*b + 512*a^4 - b^4 + 930*a^2*b^2 + 1392*a^2*b^2*exp(2*x) + 627*a*b^3*exp(
2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a + b)^2) + (8388608*a*((a^2 - (-a^3*b)^(
1/2))/(a^3*(a + b)))^(1/2)*(512*a^3*exp(2*x) - 6*b^3*exp(2*x) + 181*a*b^2 +
 432*a^2*b + 256*a^3 - 5*b^3 + 622*a*b^2*exp(2*x) + 1152*a^2*b*exp(2*x)))/(
b^6*(a + b)))*((a^2 - (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4 - (2097152*(1
76*a*b + 1536*a^2*exp(2*x) - 134*b^2*exp(2*x) + 256*a^2 - 75*b^2 + 1408*a*b
*exp(2*x)))/(b^6*(a + b)))*((a^2 - (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4)
*((a^2 - (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**4),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{1}{a-b\cosh^4(x)}\,dx$

Optimal. Leaf size=101

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] 1/2*arctanh(a^(1/4)*tanh(x)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/4)/(a^(1/2)-b^(1/
2))^(1/2)+1/2*arctanh(a^(1/4)*tanh(x)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/4)/(a^(
1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3209, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cosh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[(a^(1/4)*Tanh[x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(3/4)*Sqrt[Sqrt[a]
- Sqrt[b]]) + ArcTanh[(a^(1/4)*Tanh[x])/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*a^(3/4)
*Sqrt[Sqrt[a] + Sqrt[b]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a-b\cosh^4(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \coth(x)\right)$$
$$= \frac{1}{2}\left(-1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \coth(x)\right) + \frac{1}{2}\left(-1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\Bigg(\int$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Mathematica [**A**] time = 0.20, size = 109, normalized size = 1.08

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cosh[x]^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(Sqrt[a]*Sqrt[-a
+ Sqrt[a]*Sqrt[b]]) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/
(2*Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]])
```

fricas [**B**] time = 0.51, size = 779, normalized size = 7.71

$$-\frac{1}{4}\sqrt{\frac{\left(a^2-ab\right)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}+1}{a^2-ab}}\log\Bigg(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x)^2+2\Bigg(ab-\left(a^4-a^3b\right)\sqrt{\frac{}{a^5-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b))*
log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*(a*b - (a^4 - a^3*b
)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*
b + a^3*b^2)) + 1)/(a^2 - a*b)) - 2*(a^3 - a^2*b)*sqrt(b/(a^5 - 2*a^4*b + a
^3*b^2)) + b) + 1/4*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1
)/(a^2 - a*b))*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - 2*(a*b
 - (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))*sqrt(((a^2 - a*b)*sqrt(
b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)) - 2*(a^3 - a^2*b)*sqrt(b/(a^
5 - 2*a^4*b + a^3*b^2)) + b) - 1/4*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b
 + a^3*b^2)) - 1)/(a^2 - a*b))*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + 2*(a*b + (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))*sqrt(-(
(a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b)) + 2*(a^3 -
a^2*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + b) + 1/4*sqrt(-((a^2 - a*b)*sqrt
(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))*log(b*cosh(x)^2 + 2*b*cosh(
x)*sinh(x) + b*sinh(x)^2 - 2*(a*b + (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a
^3*b^2)))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 -
a*b)) + 2*(a^3 - a^2*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + b)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[34,61]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[95,-54]Precision problem choo
sing root in common_EXT, current precision 14Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.The ch
oice was done assuming [a,b]=[-81,-38]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done assuming [a,b]=[-70,-54]8*(sqrt(1/1024)*sqrt(1/1048576*(1048576*a^2+
1048576*a*sqrt(a*b))/(a^4-a^3*b))*ln(abs(60*a^4*b*exp(x)^2+6*a^4*b-24*a^4*s
qrt(a*b)*exp(x)^2-68*a^3*b^2*exp(x)^2-2*a^3*b^2-16*a^3*b*sqrt(a*b)*exp(x)^2
-12*a^3*b*sqrt(a*b)+48*a^3*b*sqrt(a^2-a*sqrt(a*b))*exp(x)^2+6*a^3*b*sqrt(a^
2-a*sqrt(a*b))-24*a^3*sqrt(a*b)*sqrt(a^2-a*sqrt(a*b))*exp(x)^2-16*a^2*b^3*e
xp(x)^2-8*a^2*b^3+64*a^2*b^2*sqrt(a*b)*exp(x)^2+16*a^2*b^2*sqrt(a*b)-61*a^2
*b^2*sqrt(a^2-a*sqrt(a*b))*exp(x)^2-5*a^2*b^2*sqrt(a^2-a*sqrt(a*b))+5*a^2*b
*sqrt(a*b)*sqrt(a^2-a*sqrt(a*b))*exp(x)^2-9*a^2*b*sqrt(a*b)*sqrt(a^2-a*sqrt
(a*b))-4*a*b^3*sqrt(a^2-a*sqrt(a*b))*exp(x)^2-4*a*b^3*sqrt(a^2-a*sqrt(a*b))
+36*a*b^2*sqrt(a*b)*sqrt(a^2-a*sqrt(a*b))*exp(x)^2+12*a*b^2*sqrt(a*b)*sqrt(
a^2-a*sqrt(a*b))))-sqrt(1/1024)*sqrt(1/1048576*(1048576*a^2+1048576*a*sqrt(
a*b))/(a^4-a^3*b))*ln(abs(60*a^4*b*exp(x)^2+6*a^4*b-24*a^4*sqrt(a*b)*exp(x)
^2-68*a^3*b^2*exp(x)^2-2*a^3*b^2-16*a^3*b*sqrt(a*b)*exp(x)^2-12*a^3*b*sqrt(
a*b)-48*a^3*b*sqrt(a^2-a*sqrt(a*b))*exp(x)^2-6*a^3*b*sqrt(a^2-a*sqrt(a*b))+
24*a^3*sqrt(a*b)*sqrt(a^2-a*sqrt(a*b))*exp(x)^2-16*a^2*b^3*exp(x)^2-8*a^2*b
^3+64*a^2*b^2*sqrt(a*b)*exp(x)^2+16*a^2*b^2*sqrt(a*b)+61*a^2*b^2*sqrt(a^2-a
*sqrt(a*b))*exp(x)^2+5*a^2*b^2*sqrt(a^2-a*sqrt(a*b))-5*a^2*b*sqrt(a*b)*sqrt
(a^2-a*sqrt(a*b))*exp(x)^2+9*a^2*b*sqrt(a*b)*sqrt(a^2-a*sqrt(a*b))+4*a*b^3*
sqrt(a^2-a*sqrt(a*b))*exp(x)^2+4*a*b^3*sqrt(a^2-a*sqrt(a*b))-36*a*b^2*sqrt(
a*b)*sqrt(a^2-a*sqrt(a*b))*exp(x)^2-12*a*b^2*sqrt(a*b)*sqrt(a^2-a*sqrt(a*b)
)))+sqrt(1/1024)*sqrt(1/1048576*(1048576*a^2-1048576*a*sqrt(a*b))/(a^4-a^3*
b))*ln(abs(60*a^4*b*exp(x)^2+6*a^4*b+24*a^4*sqrt(a*b)*exp(x)^2-68*a^3*b^2*e
xp(x)^2-2*a^3*b^2+16*a^3*b*sqrt(a*b)*exp(x)^2+12*a^3*b*sqrt(a*b)+48*a^3*b*s
qrt(a^2+a*sqrt(a*b))*exp(x)^2+6*a^3*b*sqrt(a^2+a*sqrt(a*b))+24*a^3*sqrt(a*b
)*sqrt(a^2+a*sqrt(a*b))*exp(x)^2-16*a^2*b^3*exp(x)^2-8*a^2*b^3-64*a^2*b^2*s
qrt(a*b)*exp(x)^2-16*a^2*b^2*sqrt(a*b)-61*a^2*b^2*sqrt(a^2+a*sqrt(a*b))*exp
(x)^2-5*a^2*b^2*sqrt(a^2+a*sqrt(a*b))-5*a^2*b*sqrt(a*b)*sqrt(a^2+a*sqrt(a*b
))*exp(x)^2+9*a^2*b*sqrt(a*b)*sqrt(a^2+a*sqrt(a*b))-4*a*b^3*sqrt(a^2+a*sqrt
(a*b))*exp(x)^2-4*a*b^3*sqrt(a^2+a*sqrt(a*b))-36*a*b^2*sqrt(a*b)*sqrt(a^2+a
*sqrt(a*b))*exp(x)^2-12*a*b^2*sqrt(a*b)*sqrt(a^2+a*sqrt(a*b))))-sqrt(1/1024
)*sqrt(1/1048576*(1048576*a^2-1048576*a*sqrt(a*b))/(a^4-a^3*b))*ln(abs(60*a
^4*b*exp(x)^2+6*a^4*b+24*a^4*sqrt(a*b)*exp(x)^2-68*a^3*b^2*exp(x)^2-2*a^3*b
^2+16*a^3*b*sqrt(a*b)*exp(x)^2+12*a^3*b*sqrt(a*b)-48*a^3*b*sqrt(a^2+a*sqrt(
a*b))*exp(x)^2-6*a^3*b*sqrt(a^2+a*sqrt(a*b))-24*a^3*sqrt(a*b)*sqrt(a^2+a*sq
rt(a*b))*exp(x)^2-16*a^2*b^3*exp(x)^2-8*a^2*b^3-64*a^2*b^2*sqrt(a*b)*exp(x)
^2-16*a^2*b^2*sqrt(a*b)+61*a^2*b^2*sqrt(a^2+a*sqrt(a*b))*exp(x)^2+5*a^2*b^2
*sqrt(a^2+a*sqrt(a*b))+5*a^2*b*sqrt(a*b)*sqrt(a^2+a*sqrt(a*b))*exp(x)^2-9*a
^2*b*sqrt(a*b)*sqrt(a^2+a*sqrt(a*b))+4*a*b^3*sqrt(a^2+a*sqrt(a*b))*exp(x)^2
+4*a*b^3*sqrt(a^2+a*sqrt(a*b))+36*a*b^2*sqrt(a*b)*sqrt(a^2+a*sqrt(a*b))*exp
(x)^2+12*a*b^2*sqrt(a*b)*sqrt(a^2+a*sqrt(a*b)))))
```

maple [C] time = 0.09, size = 127, normalized size = 1.26

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{8}+(-4a-4b)\_Z^{6}+(6a-6b)\_Z^{4}+(-4a-4b)\_Z^{2}+a-b\right)}\frac{\left(-\_R^{6}+3\_R^{4}-3\_R^{2}+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^{7}a-\_R^{7}b-3\_R^{5}a-3\_R^{5}b+3\_R^{3}a-3\_R^{3}b-\_Ra-\_Rb}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cosh(x)^4),x)
```

```
[Out] 1/4*sum((-_R^6+3*_R^4-3*_R^2+1)/(_R^7*a-_R^7*b-3*_R^5*a-3*_R^5*b+3*_R^3*a-3
*_R^3*b-_R*a-_R*b)*ln(tanh(1/2*x)-_R),_R=RootOf((a-b)*_Z^8+(-4*a-4*b)*_Z^6+
(6*a-6*b)*_Z^4+(-4*a-4*b)*_Z^2+a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\cosh(x)^4 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cosh(x)^4 - a), x)
```

mupad [B] time = 8.90, size = 1487, normalized size = 14.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cosh(x)^4),x)
```

```
[Out] log(((((1/(a^2 - (a^3*b)^(1/2)))^(1/2)*((4194304*(b^4*exp(2*x) + 253*a*b^3
+ 1184*a^3*b - 512*a^4 + b^4 - 930*a^2*b^2 - 1392*a^2*b^2*exp(2*x) + 627*a*
b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a - b)^2) - (8388608*a*(1/(a^2 -
(a^3*b)^(1/2)))^(1/2)*(512*a^3*exp(2*x) + 6*b^3*exp(2*x) + 181*a*b^2 - 432*
a^2*b + 256*a^3 + 5*b^3 + 622*a*b^2*exp(2*x) - 1152*a^2*b*exp(2*x)))/(b^6*(
a - b))))/4 - (2097152*(176*a*b - 1536*a^2*exp(2*x) + 134*b^2*exp(2*x) - 25
6*a^2 + 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a - b)))*(1/(a^2 - (a^3*b)^(1/2)
))^(1/2))/4 + (524288*(1024*a^3*exp(2*x) + 35*b^3*exp(2*x) + 185*a*b^2 - 46
4*a^2*b + 256*a^3 + 24*b^3 + 988*a*b^2*exp(2*x) - 2048*a^2*b*exp(2*x)))/(a*
b^6*(a - b)^2))*(-(a^2 + (a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2) - log((((
(1/(a^2 - (a^3*b)^(1/2)))^(1/2)*((4194304*(b^4*exp(2*x) + 253*a*b^3 + 1184*
a^3*b - 512*a^4 + b^4 - 930*a^2*b^2 - 1392*a^2*b^2*exp(2*x) + 627*a*b^3*exp
(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a - b)^2) + (8388608*a*(1/(a^2 - (a^3*b)
^(1/2)))^(1/2)*(512*a^3*exp(2*x) + 6*b^3*exp(2*x) + 181*a*b^2 - 432*a^2*b +
 256*a^3 + 5*b^3 + 622*a*b^2*exp(2*x) - 1152*a^2*b*exp(2*x)))/(b^6*(a - b))
))/4 + (2097152*(176*a*b - 1536*a^2*exp(2*x) + 134*b^2*exp(2*x) - 256*a^2 +
 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a - b)))*(1/(a^2 - (a^3*b)^(1/2)))^(1/2
))/4 + (524288*(1024*a^3*exp(2*x) + 35*b^3*exp(2*x) + 185*a*b^2 - 464*a^2*b
 + 256*a^3 + 24*b^3 + 988*a*b^2*exp(2*x) - 2048*a^2*b*exp(2*x)))/(a*b^6*(a
- b)^2))*(-(a^2 + (a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2) + log(((1/(a^2 +
 (a^3*b)^(1/2)))^(1/2)*(((1/(a^2 + (a^3*b)^(1/2)))^(1/2)*((4194304*(b^4*exp
(2*x) + 253*a*b^3 + 1184*a^3*b - 512*a^4 + b^4 - 930*a^2*b^2 - 1392*a^2*b^2
*exp(2*x) + 627*a*b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a - b)^2) - (83
88608*a*(1/(a^2 + (a^3*b)^(1/2)))^(1/2)*(512*a^3*exp(2*x) + 6*b^3*exp(2*x)
+ 181*a*b^2 - 432*a^2*b + 256*a^3 + 5*b^3 + 622*a*b^2*exp(2*x) - 1152*a^2*b
*exp(2*x)))/(b^6*(a - b))))/4 - (2097152*(176*a*b - 1536*a^2*exp(2*x) + 134
*b^2*exp(2*x) - 256*a^2 + 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a - b))))/4 +
(524288*(1024*a^3*exp(2*x) + 35*b^3*exp(2*x) + 185*a*b^2 - 464*a^2*b + 256*
a^3 + 24*b^3 + 988*a*b^2*exp(2*x) - 2048*a^2*b*exp(2*x)))/(a*b^6*(a - b)^2)
)*(-(a^2 - (a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2) - log(((1/(a^2 + (a^3*b
)^(1/2)))^(1/2)*(((1/(a^2 + (a^3*b)^(1/2)))^(1/2)*((4194304*(b^4*exp(2*x) +
 253*a*b^3 + 1184*a^3*b - 512*a^4 + b^4 - 930*a^2*b^2 - 1392*a^2*b^2*exp(2*
x) + 627*a*b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a - b)^2) + (8388608*a
*(1/(a^2 + (a^3*b)^(1/2)))^(1/2)*(512*a^3*exp(2*x) + 6*b^3*exp(2*x) + 181*a
*b^2 - 432*a^2*b + 256*a^3 + 5*b^3 + 622*a*b^2*exp(2*x) - 1152*a^2*b*exp(2*
```

```
x)))/(b^6*(a - b))))/4 + (2097152*(176*a*b - 1536*a^2*exp(2*x) + 134*b^2*ex
p(2*x) - 256*a^2 + 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a - b))))/4 + (524288
*(1024*a^3*exp(2*x) + 35*b^3*exp(2*x) + 185*a*b^2 - 464*a^2*b + 256*a^3 + 2
4*b^3 + 988*a*b^2*exp(2*x) - 2048*a^2*b*exp(2*x)))/(a*b^6*(a - b)^2))*(-(a^
2 - (a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)**4),x)
```

```
[Out] Timed out
```

3.62 $\int \frac{1}{1+\cosh^4(x)}\,dx$

Optimal. Leaf size=176

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(2\coth^2(x)-2\sqrt{1+\sqrt{2}}\,\coth(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(\sqrt{2}\coth^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\,\coth(\right.$$

```
[Out] -1/4*arctan((-2*coth(x)+(1+2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2))/(1+2^(1/2))^(
1/2)+1/4*arctan((2*coth(x)+(1+2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2))/(1+2^(1/2)
)^(1/2)-1/8*ln(2*coth(x)^2+2^(1/2)-2*coth(x)*(1+2^(1/2))^(1/2))*(1+2^(1/2))
^(1/2)+1/8*ln(1+coth(x)^2*2^(1/2)+coth(x)*(2+2*2^(1/2))^(1/2))*(1+2^(1/2))^
(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(2\coth^2(x)-2\sqrt{1+\sqrt{2}}\,\coth(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(\sqrt{2}\coth^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\,\coth(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[1 + Sqrt[2]] - 2*Coth[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqr
t[2]]) + ArcTan[(Sqrt[1 + Sqrt[2]] + 2*Coth[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt
[1 + Sqrt[2]]) - (Sqrt[1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Coth[
x] + 2*Coth[x]^2])/8 + (Sqrt[1 + Sqrt[2]]*Log[1 + Sqrt[2*(1 + Sqrt[2])]*Cot
h[x] + Sqrt[2]*Coth[x]^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\cosh^4(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{1-2x^2+2x^4}\,dx, x, \coth(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}-\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \coth(x)\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}+\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}+\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \coth(x)\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\\
&= \frac{1}{8}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \coth(x)\right)+\frac{1}{8}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int\right.\\
&= -\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(\sqrt{2}-2\sqrt{1+\sqrt{2}}\,\coth(x)+2\coth^2(x)\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(1+\sqrt{2(}\right.\\
&= -\frac{1}{4}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}-2\coth(x)}{\sqrt{-1+\sqrt{2}}}\right)+\frac{1}{4}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}+2\text{cot}}{\sqrt{-1+\sqrt{2}}}\right.\end{aligned}$$

Mathematica [C] time = 0.08, size = 45, normalized size = 0.26

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{2\sqrt{1-i}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[1 - I]]/(2*Sqrt[1 - I]) + ArcTanh[Tanh[x]/Sqrt[1 + I]]
/(2*Sqrt[1 + I])
```

fricas [B] time = 0.48, size = 590, normalized size = 3.35

$$-\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\,\log\left(\left(2^{\frac{3}{4}}e^{(2x)}+2^{\frac{1}{4}}\left(3\sqrt{2}+4\right)\right)\sqrt{-2\sqrt{2}+4}+4\sqrt{2}+e^{(4x)}+2e^{(2x)}+5\right)+\frac{1}{16}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^4),x, algorithm="fricas")
```

```
[Out] -1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log((2^(3/4)*e^(2*x) + 2^(
1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) + 2*e^(2*x
) + 5) + 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2
*x) + 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) +
 2*e^(2*x) + 5) + 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt(2)*(5*
sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) - (
2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4)
+ 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x) + 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*
sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) + 2*e^(2*x) + 5) + 1/14*sqrt(2)*(3*sqrt(
2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2
*x) + 2^(3/4)*(2*sqrt(2) + 1) + 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2)
+ 4) + 1/7*sqrt(2) - 3/7) + 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/14*(
sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) + 1/28*(2*sqrt(2)*(5*sqrt(
2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sq
rt(2) + 4) + 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) + 2^(1/4)*(3*sqrt(2) +
4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) + 2*e^(2*x) + 5) - 1/14*sqrt
(2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2
) + 6))*e^(2*x) + 2^(3/4)*(2*sqrt(2) + 1) + 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt
(-2*sqrt(2) + 4) - 1/7*sqrt(2) + 3/7)
```

giac [C] time = 0.21, size = 281, normalized size = 1.60

$$-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}-2}\left(-\frac{i}{\sqrt{2}-1}+1\right)\log\left((20i+10)\sqrt{2}e^{(2x)}+10\sqrt{2}\sqrt{10\sqrt{2}+14}+50\sqrt{2}-(2i-14)\sqrt{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^4),x, algorithm="giac")
```

```
[Out] -(1/16*I + 1/16)*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10)
*sqrt(2)*e^(2*x) + 10*sqrt(2)*sqrt(10*sqrt(2) + 14) + 50*sqrt(2) - (2*I - 1
4)*sqrt(10*sqrt(2) + 14) + (28*I + 14)*e^(2*x) + 70) + (1/16*I + 1/16)*sqrt
(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10)*sqrt(2)*e^(2*x) - 10
*sqrt(2)*sqrt(10*sqrt(2) + 14) + 50*sqrt(2) + (2*I - 14)*sqrt(10*sqrt(2) +
14) + (28*I + 14)*e^(2*x) + 70) - (1/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(-I/(
sqrt(2) + 1) + 1)*log(2*sqrt(2)*e^(2*x) + 2*sqrt(2)*sqrt(2*sqrt(2) - 2) + (
4*I + 2)*sqrt(2) + (2*I - 2)*sqrt(2*sqrt(2) - 2) - 2*e^(2*x) - 4*I - 2) + (
1/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(-I/(sqrt(2) + 1) + 1)*log(2*sqrt(2)*e^(
2*x) - 2*sqrt(2)*sqrt(2*sqrt(2) - 2) + (4*I + 2)*sqrt(2) - (2*I - 2)*sqrt(2
*sqrt(2) - 2) - 2*e^(2*x) - 4*I - 2)
```

maple [C] time = 0.07, size = 37, normalized size = 0.21

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-2\_Z^2+1\right)}\_R\ln\left(2\tanh\left(\frac{x}{2}\right)\_R+\tanh^2\left(\frac{x}{2}\right)+1\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^4),x)
```

```
[Out] 1/4*sum(_R*ln(2*tanh(1/2*x)*_R+tanh(1/2*x)^2+1),_R=RootOf(2*_Z^4-2*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\cosh(x)^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(cosh(x)^4 + 1), x)
```

mupad [B] time = 1.01, size = 205, normalized size = 1.16

$$\frac{\sqrt{2}\ \sqrt{1-\mathrm{i}}\ \ln\left(\mathrm{e}^{2\,x}\ (436273152+91291648\mathrm{i})+\sqrt{2}\ \sqrt{1-\mathrm{i}}\ (-9830400+56623104\mathrm{i})+\sqrt{2}\ \sqrt{1-\mathrm{i}}\ \mathrm{e}^{2\,x}\ (21889\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4 + 1),x)
```

```
[Out] (2^(1/2)*(1 - 1i)^(1/2)*log(exp(2*x)*(436273152 + 91291648i) - 2^(1/2)*(1 -
 1i)^(1/2)*(9830400 - 56623104i) + 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(2188902
40 + 149422080i) + (21168128 + 94306304i)))/8 - (2^(1/2)*(1 - 1i)^(1/2)*log
(exp(2*x)*(436273152 + 91291648i) + 2^(1/2)*(1 - 1i)^(1/2)*(9830400 - 56623
104i) - 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(218890240 + 149422080i) + (2116812
8 + 94306304i)))/8 + (2^(1/2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152 - 9129
1648i) - 2^(1/2)*(1 + 1i)^(1/2)*(9830400 + 56623104i) + 2^(1/2)*(1 + 1i)^(1
/2)*exp(2*x)*(218890240 - 149422080i) + (21168128 - 94306304i)))/8 - (2^(1/
2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152 - 91291648i) + 2^(1/2)*(1 + 1i)^(
1/2)*(9830400 + 56623104i) - 2^(1/2)*(1 + 1i)^(1/2)*exp(2*x)*(218890240 - 1
49422080i) + (21168128 - 94306304i)))/8
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**4),x)
```

```
[Out] Timed out
```

3.63 $\int \frac{1}{1-\cosh^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\coth(x)}{2}$$

[Out] 1/2*coth(x)+1/4*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3209, 388, 206}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\coth(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - Cosh[x]^4)^(-1),x]

[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(2*Sqrt[2]) + Coth[x]/2

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cosh^4(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{1-2x^2}\,dx,x,\coth(x)\right)\\ &= \frac{\coth(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\coth(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.11, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)+2\coth(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]] + 2*Coth[x])/4
```

fricas [B] time = 0.63, size = 115, normalized size = 4.60

$$\frac{\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 - \sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2 - 4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x) + 3\left(2\right.}{\cosh(x)^2+\sinh(x)^2+3}\right.}{8\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - s
qrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(
x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^2 +
3)) + 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [B] time = 0.12, size = 43, normalized size = 1.72

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\sqrt{2} - e^{(2x)} - 3}{2\sqrt{2} + e^{(2x)} + 3}\right) + \frac{1}{e^{(2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^4),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1/(
e^(2*x) - 1)
```

maple [B] time = 0.08, size = 100, normalized size = 4.00

$$\frac{\tanh\left(\frac{x}{2}\right)}{4} + \frac{1}{4\tanh\left(\frac{x}{2}\right)} + \frac{\sqrt{2}\ \ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{16} - \frac{\sqrt{2}\ \ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^4),x)
```

```
[Out] 1/4*tanh(1/2*x)+1/4/tanh(1/2*x)+1/16*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh
(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))-1/16*2^(1/2)*ln((tanh(1/2
*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))
```

maxima [B] time = 0.44, size = 45, normalized size = 1.80

$$-\frac{1}{8}\sqrt{2}\log\left(-\frac{2\sqrt{2} - e^{(-2x)} - 3}{2\sqrt{2} + e^{(-2x)} + 3}\right) - \frac{1}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^4),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
1/(e^(-2*x) - 1)
```

mupad [B] time = 0.12, size = 61, normalized size = 2.44

$$\frac{\sqrt{2}\ \ln\left(-2\,e^{2x} - \frac{\sqrt{2}\,\left(12\,e^{2x}+4\right)}{8}\right)}{8} - \frac{\sqrt{2}\ \ln\left(\frac{\sqrt{2}\,\left(12\,e^{2x}+4\right)}{8} - 2\,e^{2x}\right)}{8} + \frac{1}{e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^4 - 1),x)
```

```
[Out] (2^(1/2)*log(- 2*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/8))/8 - (2^(1/2)*lo
g((2^(1/2)*(12*exp(2*x) + 4))/8 - 2*exp(2*x)))/8 + 1/(exp(2*x) - 1)
```

sympy [B] time = 1.87, size = 75, normalized size = 3.00

$$-\frac{\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{8}+\frac{\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)}{4}+\frac{1}{4\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**4),x)
```

```
[Out] -sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/8 + sqrt(2)*log(4*ta
nh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/8 + tanh(x/2)/4 + 1/(4*tanh(x/2))
```

3.64 $\int \frac{1}{a+b\cosh^5(x)}\,dx$

Optimal. Leaf size=494

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+(-1}}$$

```
[Out] 2/5*arctanh((a^(1/5)-b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+b^(1/5))^(1/2))/a^
(4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)+
(-1)^(1/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2))/a
^(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2
)+2/5*arctanh((a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+(-1)^
(2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(
-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)*tan
h(1/2*x)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^
(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)-(-1)^(
4/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)
/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3213, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+(-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^5)^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a^(1/5) - b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) + b^(1/5)]])/(5
*a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTanh[(Sqr
t[a^(1/5) + (-1)^(1/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5
)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5
)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*Tanh[x/2])/Sqr
t[a^(1/5) + (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1
/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5) + (-1)^
(3/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*S
qrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2*
ArcTanh[(Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) + (-1)^
(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5)
 + (-1)^(4/5)*b^(1/5)])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e, x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\cosh^5(x)}\,dx = \int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\cosh(x)\right)} - \frac{1}{5a^{4/5}\left(-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cosh(x)\right)} - \frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)\right.}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}\cosh(x)}\,dx}{5a^{4/5}} - \frac{\int \frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cosh(x)}\,dx}{5a^{4/5}} - \frac{\int \frac{1}{-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cosh(x)}\,dx}{5a^{4/5}} - \frac{\int \frac{}{-\sqrt[5]{a}+(-1}}{5}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}-\left(-\sqrt[5]{a}+\sqrt[5]{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{5a^{4/5}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}-\left(-\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}\right.}\right.}{5a^{4/5}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}} + \frac{2\tanh^{-}}{5a^{4/5}\sqrt{\sqrt[5]{a}-}}$$

Mathematica [C] time = 0.29, size = 139, normalized size = 0.28

$$\frac{8}{5}\text{RootSum}\left[\#1^{10}b+5\#1^8b+10\#1^6b+32\#1^5a+10\#1^4b+5\#1^2b+b\&, \frac{\#1^3x+2\#1^3\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cos\right.}{\#1^8b+4\#1^6b+6\#1^4b+16\#}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^5)^(-1),x]
```

```
[Out] (8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b*
#1^10 & , (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]
*#1]*#1^3)/(b + 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])/5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cosh(x)^5+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*cosh(x)^5 + a), x)
```

maple [C] time = 0.11, size = 156, normalized size = 0.32

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{10}+(-5a-5b)\_Z^{8}+(10a-10b)\_Z^{6}+(-10a-10b)\_Z^{4}+(5a-5b)\_Z^{2}-a-b\right)}\frac{\left(-\_R^{8}+4\_R^{6}-6\_R^{4}+4\_R^{2}-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right.}{\_R^{9}a-\_R^{9}b-4\_R^{7}a-4\_R^{7}b+6\_R^{5}a-6\_R^{5}b-4\_R^{3}a-4}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^5),x)
```

```
[Out] 1/5*sum((-_R^8+4*_R^6-6*_R^4+4*_R^2-1)/(_R^9*a-_R^9*b-4*_R^7*a-4*_R^7*b+6*_
R^5*a-6*_R^5*b-4*_R^3*a-4*_R^3*b+_R*a-_R*b)*ln(tanh(1/2*x)-_R),_R=RootOf((a
-b)*_Z^10+(-5*a-5*b)*_Z^8+(10*a-10*b)*_Z^6+(-10*a-10*b)*_Z^4+(5*a-5*b)*_Z^2
-a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cosh(x)^5 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cosh(x)^5 + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^5),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cosh^{5}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**5),x)
```

```
[Out] Integral(1/(a + b*cosh(x)**5), x)
```

3.65 $\int \frac{1}{a+b\cosh^6(x)}\,dx$

Optimal. Leaf size=171

$$\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)+b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)+b^(1/
3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2))/a
^(5/6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a^(1
/3)+(-1)^(2/3)*b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[(a^(1/6)*Tanh[x])/Sqrt[a^(1/3) + b^(1/3)]]/(3*a^(5/6)*Sqrt[a^(1/3)
+ b^(1/3)]) + ArcTanh[(a^(1/6)*Tanh[x])/Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]]
/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTanh[(a^(1/6)*Tanh[x])
/Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)]]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b
^(1/3)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1+\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \coth(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \coth(x)\right)}{3a} + \frac{\text{Subs}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.22, size = 132, normalized size = 0.77

$$\frac{16}{3}\text{RootSum}\left[\#1^6 b + 6\#1^5 b + 15\#1^4 b + 64\#1^3 a + 20\#1^3 b + 15\#1^2 b + 6\#1 b + b\&, \frac{\#1^2 x + \#1^2\log(-\#1\sinh(x)}{\#1^5 b + 5\#1^4 b + 10\#1^3 b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^6)^(-1),x]
```

```
[Out] (16*RootSum[b + 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6*
b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]
*#1]*#1^2)/(b + 5*b*#1 + 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 + 5*b*#1^4 + b*#
1^5) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.32, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^6),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.10, size = 177, normalized size = 1.04

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a+b)\_Z^{12}+(-6a+6b)\_Z^{10}+(15a+15b)\_Z^{8}+(-20a+20b)\_Z^{6}+(15a+15b)\_Z^{4}+(-6a+6b)\_Z^{2}+a+b\right)} \frac{\left(-\_R^{10}+5\_R^{8}-\right.}{\_R^{11}a+\_R^{11}b-5\_R^{9}a+5\_R^{9}b+10}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^6),x)
```

```
[Out] 1/6*sum((-_R^10+5*_R^8-10*_R^6+10*_R^4-5*_R^2+1)/(_R^11*a+_R^11*b-5*_R^9*a+
5*_R^9*b+10*_R^7*a+10*_R^7*b-10*_R^5*a+10*_R^5*b+5*_R^3*a+5*_R^3*b-_R*a+_R*
b)*ln(tanh(1/2*x)-_R),_R=RootOf((a+b)*_Z^12+(-6*a+6*b)*_Z^10+(15*a+15*b)*_Z
^8+(-20*a+20*b)*_Z^6+(15*a+15*b)*_Z^4+(-6*a+6*b)*_Z^2+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cosh(x)^6 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cosh(x)^6 + a), x)
```

mupad [B] time = 58.39, size = 844, normalized size = 4.94

$$\sum_{k=1}^{6} \ln\left(\mathrm{root}\left(46656\,a^5\,b\,d^6 + 46656\,a^6\,d^6 - 3888\,a^4\,d^4 + 108\,a^2\,d^2 - 1, d, k\right)\left(\mathrm{root}\left(46656\,a^5\,b\,d^6 + 46656\,a^6\,d^6 - 3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^6),x)
```

```
[Out] symsum(log(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4 + 108*a^2*d^
2 - 1, d, k)*(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4 + 108*a^2
*d^2 - 1, d, k)*(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4 + 108*
a^2*d^2 - 1, d, k)*(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4 + 1
08*a^2*d^2 - 1, d, k)*((1459166279268040704*(327680*a^7*exp(2*x) + 298496*a
^6*b + 65536*a^7 + 158*a^2*b^5 + 91315*a^3*b^4 + 348176*a^4*b^3 + 489952*a^
5*b^2 + 196*a^2*b^5*exp(2*x) + 274019*a^3*b^4*exp(2*x) + 1132876*a^4*b^3*ex
p(2*x) + 1770440*a^5*b^2*exp(2*x) + 1239040*a^6*b*exp(2*x)))/(b^10*(a + b)^
3) + (17509995351216488448*root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*
d^4 + 108*a^2*d^2 - 1, d, k)*(262144*a^7*exp(2*x) + 203520*a^6*b + 65536*a^
7 + 453*a^3*b^4 + 72022*a^4*b^3 + 209472*a^5*b^2 + 630*a^3*b^4*exp(2*x) + 2
54512*a^4*b^3*exp(2*x) + 767508*a^5*b^2*exp(2*x) + 775680*a^6*b*exp(2*x)))/
(b^10*(a + b)^2)) - (486388759756013568*(655360*a^5*exp(2*x) - 9*a*b^4 + 37
0176*a^4*b + 196608*a^5 - 24408*a^2*b^3 + 149088*a^3*b^2 - 63676*a^2*b^3*ex
p(2*x) + 526248*a^3*b^2*exp(2*x) - 10*a*b^4*exp(2*x) + 1245184*a^4*b*exp(2*
x)))/(b^10*(a + b)^2)) - (40532396646334464*(655360*a^5*exp(2*x) - b^5*exp(
2*x) - 24677*a*b^4 + 773120*a^4*b + 262144*a^5 - b^5 + 198071*a^2*b^3 + 733
696*a^3*b^2 + 477713*a^2*b^3*exp(2*x) + 1770640*a^3*b^2*exp(2*x) - 53861*a*
b^4*exp(2*x) + 1894400*a^4*b*exp(2*x)))/(b^10*(a + b)^3)) + (13510798882111
488*(655360*a^3*exp(2*x) + 11382*b^3*exp(2*x) + 144416*a*b^2 + 269056*a^2*b
 + 131072*a^3 + 6459*b^3 + 677524*a*b^2*exp(2*x) + 1321472*a^2*b*exp(2*x)))
/(b^10*(a + b)^2)) + (1125899906842624*(851968*a^4*exp(2*x) + 6006*b^4*exp(
2*x) + 211497*a*b^3 + 597504*a^3*b + 196608*a^4 + 3840*b^4 + 608544*a^2*b^2
 + 2562504*a^2*b^2*exp(2*x) + 864565*a*b^3*exp(2*x) + 2555904*a^3*b*exp(2*x
)))/(b^10*(a + b)^2*(a*b + a^2)))*root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 38
88*a^4*d^4 + 108*a^2*d^2 - 1, d, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cosh^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**6),x)
```

```
[Out] Integral(1/(a + b*cosh(x)**6), x)
```

3.66 $\int \frac{1}{a+b\cosh^8(x)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] -1/4*arctanh((-a)^(1/8)*tanh(x)/((-a)^(1/4)-b^(1/4))^(1/2))/(-a)^(7/8)/((-a
)^(1/4)-b^(1/4))^(1/2)-1/4*arctanh((-a)^(1/8)*tanh(x)/((-a)^(1/4)-I*b^(1/4)
)^(1/2))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)-1/4*arctanh((-a)^(1/8)*tan
h(x)/((-a)^(1/4)+I*b^(1/4))^(1/2))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)-
1/4*arctanh((-a)^(1/8)*tanh(x)/((-a)^(1/4)+b^(1/4))^(1/2))/(-a)^(7/8)/((-a)
^(1/4)+b^(1/4))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{(-a)^{5/8}\tanh(x)}{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x]^8)^(-1),x]
```

```
[Out] -ArcTanh[((-a)^(1/8)*Tanh[x])/Sqrt[(-a)^(1/4) - I*b^(1/4)]]/(4*(-a)^(7/8)*S
qrt[(-a)^(1/4) - I*b^(1/4)]) - ArcTanh[((-a)^(1/8)*Tanh[x])/Sqrt[(-a)^(1/4)
 + I*b^(1/4)]]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ArcTanh[((-a)^
(1/8)*Tanh[x])/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) +
b^(1/4)]) - ArcTanh[((-a)^(5/8)*Tanh[x])/Sqrt[(-a)^(5/4) + a*b^(1/4)]]/(4*(
-a)^(3/8)*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \coth(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \coth(x)\right)}{4a} + \text{Subst}\left(\int\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\right.}{4(-a)^{3/8}\sqrt{(}}$$

Mathematica [C] time = 0.28, size = 158, normalized size = 0.64

$$16\text{RootSum}\left[\#1^8b + 8\#1^7b + 28\#1^6b + 56\#1^5b + 256\#1^4a + 70\#1^4b + 56\#1^3b + 28\#1^2b + 8\#1b + b\&, \frac{\#1^3}{\#1^7b + 7\#}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 + 56
*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x
] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 + 128*a*#1^3 + 3
5*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.84, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.10, size = 233, normalized size = 0.95

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a+b)\_Z^{16}+(-8a+8b)\_Z^{14}+(28a+28b)\_Z^{12}+(-56a+56b)\_Z^{10}+(70a+70b)\_Z^{8}+(-56a+56b)\_Z^{6}+(28a+28b)\_Z^{4}+(-8a+8b)\_Z^{2}+a+b\right)}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)^8),x)
```

```
[Out] 1/8*sum((-_R^14+7*_R^12-21*_R^10+35*_R^8-35*_R^6+21*_R^4-7*_R^2+1)/(_R^15*a
+_R^15*b-7*_R^13*a+7*_R^13*b+21*_R^11*a+21*_R^11*b-35*_R^9*a+35*_R^9*b+35*_
R^7*a+35*_R^7*b-21*_R^5*a+21*_R^5*b+7*_R^3*a+7*_R^3*b-_R*a+_R*b)*ln(tanh(1/
2*x)-_R),_R=RootOf((a+b)*_Z^16+(-8*a+8*b)*_Z^14+(28*a+28*b)*_Z^12+(-56*a+56
*b)*_Z^10+(70*a+70*b)*_Z^8+(-56*a+56*b)*_Z^6+(28*a+28*b)*_Z^4+(-8*a+8*b)*_Z
^2+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cosh(x)^8 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cosh(x)^8 + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)^8),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cosh^8(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)**8),x)
```

```
[Out] Integral(1/(a + b*cosh(x)**8), x)
```

3.67 $\int \frac{1}{a-b\cosh^5(x)}\,dx$

Optimal. Leaf size=494

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+(-1)^{2/5}}}$$

```
[Out] 2/5*arctanh((a^(1/5)+b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-b^(1/5))^(1/2))/a^
(4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)-
(-1)^(1/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2))/a
^(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2
)+2/5*arctanh((a^(1/5)+(-1)^(2/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-(-1)^
(2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(
-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2)*tan
h(1/2*x)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^
(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)+(-1)^(
4/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)
/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3213, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+(-1)^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cosh[x]^5)^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a^(1/5) + b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) - b^(1/5)]])/(5
*a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTanh[(Sqr
t[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) + (-1)^(1/5)*b^(1/5
)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5
)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]*Tanh[x/2])/Sqr
t[a^(1/5) - (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1
/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5) - (-1)^
(3/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*S
qrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2*
ArcTanh[(Sqrt[a^(1/5) + (-1)^(4/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) - (-1)^
(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5)
 + (-1)^(4/5)*b^(1/5)])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a-b\cosh^5(x)}\,dx = \int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\cosh(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cosh(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\cdots\right.}\right.$$

$$= \frac{\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}\cosh(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cosh(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cosh(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+(-1)^{3/5}\cdots}}{5a\cdots}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}-\left(\sqrt[5]{a}+\sqrt[5]{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}-\left(\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}\right)x^2\cdots}\right.}{5a^{4/5}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\tan\cdots}{5a^{4/5}\sqrt{\sqrt[5]{a}\cdots}}$$

Mathematica [C] time = 0.27, size = 139, normalized size = 0.28

$$-\frac{8}{5}\text{RootSum}\left[\#1^{10}b+5\#1^8b+10\#1^6b-32\#1^5a+10\#1^4b+5\#1^2b+b\&,\frac{\#1^3x+2\#1^3\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cdots\right.}{\#1^8b+4\#1^6b+6\#1^4b-\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cosh[x]^5)^(-1),x]
```

```
[Out] (-8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 - 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b
*#1^10 & , (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2
]*#1]*#1^3)/(b + 4*b*#1^2 - 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])/
5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{b\cosh(x)^5-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*cosh(x)^5 - a), x)
```

maple [C] time = 0.09, size = 150, normalized size = 0.30

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{10}+(-5a+5b)\_Z^{8}+(10a+10b)\_Z^{6}+(-10a+10b)\_Z^{4}+(5a+5b)\_Z^{2}-a+b\right)}\frac{\left(-\_R^{8}+4\_R^{6}-6\_R^{4}+4\_R^{2}-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_\right.}{\_R^{9}a+\_R^{9}b-4\_R^{7}a+4\_R^{7}b+6\_R^{5}a+6\_R^{5}b-4\_R^{3}a+4\_R}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cosh(x)^5),x)
```

```
[Out] 1/5*sum((-_R^8+4*_R^6-6*_R^4+4*_R^2-1)/(_R^9*a+_R^9*b-4*_R^7*a+4*_R^7*b+6*_
R^5*a+6*_R^5*b-4*_R^3*a+4*_R^3*b+_R*a+_R*b)*ln(tanh(1/2*x)-_R),_R=RootOf((a
+b)*_Z^10+(-5*a+5*b)*_Z^8+(10*a+10*b)*_Z^6+(-10*a+10*b)*_Z^4+(5*a+5*b)*_Z^2
-a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{b\cosh(x)^5-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^5),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cosh(x)^5 - a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cosh(x)^5),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{a-b\cosh^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)**5),x)
```

```
[Out] Integral(1/(a - b*cosh(x)**5), x)
```

3.68 $\int \frac{1}{a-b\cosh^6(x)}\,dx$

Optimal. Leaf size=175

$$\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)-b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)-b^(1/
3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2))/a
^(5/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a^(1
/3)-(-1)^(2/3)*b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cosh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[(a^(1/6)*Tanh[x])/Sqrt[a^(1/3) - b^(1/3)]]/(3*a^(5/6)*Sqrt[a^(1/3)
- b^(1/3)]) + ArcTanh[(a^(1/6)*Tanh[x])/Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]]
/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(a^(1/6)*Tanh[x])
/Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)]]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b
^(1/3)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\cosh^6(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \coth(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \coth(x)\right)}{3a} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.17, size = 132, normalized size = 0.75

$$-\frac{16}{3}\text{RootSum}\left[\#1^6b+6\#1^5b+15\#1^4b-64\#1^3a+20\#1^3b+15\#1^2b+6\#1b+b\&, \frac{\#1^2x+\#1^2\log(-\#1\sinh(x)+}{\#1^5b+5\#1^4b+10\#1^3b-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cosh[x]^6)^(-1),x]
```

```
[Out] (-16*RootSum[b + 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6
*b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x
]*#1]*#1^2)/(b + 5*b*#1 - 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 + 5*b*#1^4 + b*
#1^5) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^6),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.10, size = 183, normalized size = 1.05

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a-b)\_Z^{12}+(-6a-6b)\_Z^{10}+(15a-15b)\_Z^8+(-20a-20b)\_Z^6+(15a-15b)\_Z^4+(-6a-6b)\_Z^2+a-b\right)} \frac{\left(-\_R^{10}+5\_R^8-10\_R^6\right.}{\_R^{11}a-\_R^{11}b-5\_R^9a-5\_R^9b+10\_R^7a}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cosh(x)^6),x)
```

```
[Out] 1/6*sum((-_R^10+5*_R^8-10*_R^6+10*_R^4-5*_R^2+1)/(_R^11*a-_R^11*b-5*_R^9*a-
5*_R^9*b+10*_R^7*a-10*_R^7*b-10*_R^5*a-10*_R^5*b+5*_R^3*a-5*_R^3*b-_R*a-_R*
b)*ln(tanh(1/2*x)-_R),_R=RootOf((a-b)*_Z^12+(-6*a-6*b)*_Z^10+(15*a-15*b)*_Z
^8+(-20*a-20*b)*_Z^6+(15*a-15*b)*_Z^4+(-6*a-6*b)*_Z^2+a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b \cosh(x)^6 - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^6),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cosh(x)^6 - a), x)
```

mupad [B] time = 57.40, size = 855, normalized size = 4.89

$$\sum_{k=1}^{6} \ln\left(\mathrm{root}\left(46656\, a^5\, b\, d^6 - 46656\, a^6\, d^6 + 3888\, a^4\, d^4 - 108\, a^2\, d^2 + 1, d, k\right) \left(\mathrm{root}\left(46656\, a^5\, b\, d^6 - 46656\, a^6\, d^6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cosh(x)^6),x)
```

```
[Out] symsum(log(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a^2*d^
2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a^2
*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*
a^2*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 1
08*a^2*d^2 + 1, d, k)*((1459166279268040704*(327680*a^7*exp(2*x) - 298496*a
^6*b + 65536*a^7 - 158*a^2*b^5 + 91315*a^3*b^4 - 348176*a^4*b^3 + 489952*a^
5*b^2 - 196*a^2*b^5*exp(2*x) + 274019*a^3*b^4*exp(2*x) - 1132876*a^4*b^3*ex
p(2*x) + 1770440*a^5*b^2*exp(2*x) - 1239040*a^6*b*exp(2*x)))/(b^10*(a - b)^
3) + (17509995351216488448*root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*
d^4 - 108*a^2*d^2 + 1, d, k)*(262144*a^7*exp(2*x) - 203520*a^6*b + 65536*a^
7 + 453*a^3*b^4 - 72022*a^4*b^3 + 209472*a^5*b^2 + 630*a^3*b^4*exp(2*x) - 2
54512*a^4*b^3*exp(2*x) + 767508*a^5*b^2*exp(2*x) - 775680*a^6*b*exp(2*x)))/
(b^10*(a - b)^2)) - (486388759756013568*(655360*a^5*exp(2*x) - 9*a*b^4 - 37
0176*a^4*b + 196608*a^5 + 24408*a^2*b^3 + 149088*a^3*b^2 + 63676*a^2*b^3*ex
p(2*x) + 526248*a^3*b^2*exp(2*x) - 10*a*b^4*exp(2*x) - 1245184*a^4*b*exp(2*
x)))/(b^10*(a - b)^2)) - (40532396646334464*(655360*a^5*exp(2*x) + b^5*exp(
2*x) - 24677*a*b^4 - 773120*a^4*b + 262144*a^5 + b^5 - 198071*a^2*b^3 + 733
696*a^3*b^2 - 477713*a^2*b^3*exp(2*x) + 1770640*a^3*b^2*exp(2*x) - 53861*a*
b^4*exp(2*x) - 1894400*a^4*b*exp(2*x)))/(b^10*(a - b)^3)) + (13510798882111
488*(655360*a^3*exp(2*x) - 11382*b^3*exp(2*x) + 144416*a*b^2 - 269056*a^2*b
 + 131072*a^3 - 6459*b^3 + 677524*a*b^2*exp(2*x) - 1321472*a^2*b*exp(2*x)))
/(b^10*(a - b)^2)) - (1125899906842624*(851968*a^4*exp(2*x) + 6006*b^4*exp(
2*x) - 211497*a*b^3 - 597504*a^3*b + 196608*a^4 + 3840*b^4 + 608544*a^2*b^2
 + 2562504*a^2*b^2*exp(2*x) - 864565*a*b^3*exp(2*x) - 2555904*a^3*b*exp(2*x
)))/(b^10*(a - b)^2*(a*b - a^2)))*root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 38
88*a^4*d^4 - 108*a^2*d^2 + 1, d, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b \cosh^6(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)**6),x)
```

```
[Out] Integral(1/(a - b*cosh(x)**6), x)
```

3.69 $\int \frac{1}{a-b\cosh^8(x)}\,dx$

Optimal. Leaf size=213

$$\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

```
[Out] 1/4*arctanh(a^(1/8)*tanh(x)/(a^(1/4)-b^(1/4))^(1/2))/a^(7/8)/(a^(1/4)-b^(1/
4))^(1/2)+1/4*arctanh(a^(1/8)*tanh(x)/(a^(1/4)-I*b^(1/4))^(1/2))/a^(7/8)/(a
^(1/4)-I*b^(1/4))^(1/2)+1/4*arctanh(a^(1/8)*tanh(x)/(a^(1/4)+I*b^(1/4))^(1/
2))/a^(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)+1/4*arctanh(a^(1/8)*tanh(x)/(a^(1/4)+
b^(1/4))^(1/2))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cosh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[(a^(1/8)*Tanh[x])/Sqrt[a^(1/4) - b^(1/4)]]/(4*a^(7/8)*Sqrt[a^(1/4)
- b^(1/4)]) + ArcTanh[(a^(1/8)*Tanh[x])/Sqrt[a^(1/4) - I*b^(1/4)]]/(4*a^(7/
8)*Sqrt[a^(1/4) - I*b^(1/4)]) + ArcTanh[(a^(1/8)*Tanh[x])/Sqrt[a^(1/4) + I*
b^(1/4)]]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTanh[(a^(1/8)*Tanh[x])
/Sqrt[a^(1/4) + b^(1/4)]]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\cosh^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx, x, \coth(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx, x, \coth(x)\right)}{4a} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}\right.}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Mathematica [C] time = 0.22, size = 158, normalized size = 0.74

$$-16\text{RootSum}\left[\#1^8b+8\#1^7b+28\#1^6b+56\#1^5b-256\#1^4a+70\#1^4b+56\#1^3b+28\#1^2b+8\#1b+b\&, \frac{}{\#1^7b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cosh[x]^8)^(-1),x]
```

```
[Out] -16*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 + 5
6*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[
x] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 - 128*a*#1^3 +
35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.83, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.10, size = 239, normalized size = 1.12

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a-b)\_Z^{16}+(-8a-8b)\_Z^{14}+(28a-28b)\_Z^{12}+(-56a-56b)\_Z^{10}+(70a-70b)\_Z^{8}+(-56a-56b)\_Z^{6}+(28a-28b)\_Z^{4}+(-8a-8b)\_Z^{2}+a-b\right)}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cosh(x)^8),x)
```

```
[Out] 1/8*sum((-_R^14+7*_R^12-21*_R^10+35*_R^8-35*_R^6+21*_R^4-7*_R^2+1)/(_R^15*a
-_R^15*b-7*_R^13*a-7*_R^13*b+21*_R^11*a-21*_R^11*b-35*_R^9*a-35*_R^9*b+35*_
R^7*a-35*_R^7*b-21*_R^5*a-21*_R^5*b+7*_R^3*a-7*_R^3*b-_R*a-_R*b)*ln(tanh(1/
2*x)-_R),_R=RootOf((a-b)*_Z^16+(-8*a-8*b)*_Z^14+(28*a-28*b)*_Z^12+(-56*a-56
*b)*_Z^10+(70*a-70*b)*_Z^8+(-56*a-56*b)*_Z^6+(28*a-28*b)*_Z^4+(-8*a-8*b)*_Z
^2+a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\cosh(x)^8 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)^8),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cosh(x)^8 - a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cosh(x)^8),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\cosh^8(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cosh(x)**8),x)
```

```
[Out] Integral(1/(a - b*cosh(x)**8), x)
```

3.70 $\int \frac{1}{1+\cosh^5(x)}\,dx$

Optimal. Leaf size=223

$$\frac{2\tanh^{-1}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tanh^{-1}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan^{-1}\left(\ldots\right.}{5\left(1+\ldots\right.}$$

```
[Out] 1/5*sinh(x)/(1+cosh(x))-2/5*arctan(tanh(1/2*x)/((-1+(-1)^(1/5))/(1+(-1)^(1/
5)))^(1/2))/(-1+(-1)^(2/5))^(1/2)+2/5*arctanh(((1-(-1)^(4/5))/(1+(-1)^(4/5)
))^(1/2)*tanh(1/2*x))/(1+(-1)^(3/5))^(1/2)-2/5*arctan(((-1-(-1)^(3/5))/(1-(
-1)^(3/5)))^(1/2)*tanh(1/2*x))*((-1-(-1)^(3/5))/(1-(-1)^(3/5)))^(1/2)/(1+(-
1)^(3/5))+2/5*arctanh(((1-(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2)*tanh(1/2*x))/(1
-(-1)^(4/5))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3213, 2648, 2659, 205, 208}

$$\frac{2\tanh^{-1}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tanh^{-1}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan^{-1}\left(\ldots\right.}{5\left(1+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[Tanh[x/2]/Sqrt[-((1 - (-1)^(1/5))/(1 + (-1)^(1/5)))]])/(5*Sqrt[-
1 + (-1)^(2/5)]) - (2*Sqrt[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*ArcTan[Sqr
t[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*Tanh[x/2]])/(5*(1 + (-1)^(3/5))) +
(2*ArcTanh[Sqrt[(1 - (-1)^(2/5))/(1 + (-1)^(2/5))]*Tanh[x/2]])/(5*Sqrt[1 -
(-1)^(4/5)]) + (2*ArcTanh[Sqrt[(1 - (-1)^(4/5))/(1 + (-1)^(4/5))]*Tanh[x/2]
])/(5*Sqrt[1 + (-1)^(3/5)]) + Sinh[x]/(5*(1 + Cosh[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1+\cosh^5(x)}\,dx = \int \left(-\frac{1}{5(-1-\cosh(x))} - \frac{1}{5\left(-1+\sqrt[5]{-1}\cosh(x)\right)} - \frac{1}{5\left(-1-(-1)^{2/5}\cosh(x)\right)} - \frac{1}{5\left(-1+(-1)^{3}\right.}\right.$$

$$= -\left(\frac{1}{5}\int \frac{1}{-1-\cosh(x)}\,dx\right) - \frac{1}{5}\int \frac{1}{-1+\sqrt[5]{-1}\cosh(x)}\,dx - \frac{1}{5}\int \frac{1}{-1-(-1)^{2/5}\cosh(x)}\,dx - \frac{1}{5}$$

$$= \frac{\sinh(x)}{5(1+\cosh(x))} - \frac{2}{5}\mathrm{Subst}\left(\int \frac{1}{-1+\sqrt[5]{-1}-\left(-1-\sqrt[5]{-1}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - \frac{2}{5}\mathrm{Subst}\left(\int\right.$$

$$= -\frac{2\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{-1+(-1)^{2/5}}} - \frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan^{-1}\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)} + \frac{2\tanh^{-1}\left(\sqrt{\frac{1-(-1)^{2/}}{1+(-1)^{2/}}}\right.}{5\sqrt{1-(-1)}}$$

Mathematica [C] time = 0.10, size = 445, normalized size = 2.00

$$\frac{1}{5}\tanh\left(\frac{x}{2}\right) - \frac{1}{10}\mathrm{RootSum}\left[\#1^8 - 2\#1^7 + 8\#1^6 - 14\#1^5 + 30\#1^4 - 14\#1^3 + 8\#1^2 - 2\#1 + 1\&, \frac{\#1^6 x + 2\#1^6\log\left(-\#\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^5)^(-1),x]
```

```
[Out] -1/10*RootSum[1 - 2*#1 + 8*#1^2 - 14*#1^3 + 30*#1^4 - 14*#1^5 + 8*#1^6 - 2*
#1^7 + #1^8 & , (x + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2
]*#1] - 4*x*#1 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1
]*#1 + 15*x*#1^2 + 30*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]
*#1]*#1^2 - 40*x*#1^3 - 80*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh
[x/2]*#1]*#1^3 + 15*x*#1^4 + 30*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 -
 Sinh[x/2]*#1]*#1^4 - 4*x*#1^5 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#
1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*
#1 - Sinh[x/2]*#1]*#1^6)/(-1 + 8*#1 - 21*#1^2 + 60*#1^3 - 35*#1^4 + 24*#1^5
 - 7*#1^6 + 4*#1^7) & ] + Tanh[x/2]/5
```

fricas [B] time = 1.05, size = 3228, normalized size = 14.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^5),x, algorithm="fricas")
```

```
[Out] -1/8000*(8*sqrt(10)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*
sqrt(5) + 60)*((sqrt(5) - 1)*e^x + sqrt(5) - 1)*(40*sqrt(5) + 200)^(1/4)*sq
rt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*arctan(1/40*sqrt(10)*((sqrt(5) - 5)*e^x
 + sqrt(5) + 1)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 1/400*sqrt(10)*(sqr
```

```
t(10)*((sqrt(5) - 5)*e^x + 2*sqrt(5))*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)
 + 10*((sqrt(5) - 1)*e^x - sqrt(5) - 1)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) +
 5) - 1/64000*(80*sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) -
 5) + 8*sqrt(10)*(sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) -
 5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 1))*sqrt(sqrt(5) + 5) - sqrt(2*sqrt
(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*((sqrt(10)*sqrt(2
*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) - 3) + 2*(3*sqrt(5) - 7)*sqrt(2*sq
rt(5) + 5))*(40*sqrt(5) + 200)^(3/4) + 4*(sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt
(sqrt(5) + 5)*(sqrt(5) - 5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 1))*(40*sqr
t(5) + 200)^(1/4)) + 1600*sqrt(2*sqrt(5) + 5))*sqrt(-20*sqrt(10)*sqrt(sqrt(
5) + 5)*(sqrt(5) - 5) - 200*(sqrt(5) + 1)*e^x - 2*(sqrt(10)*(sqrt(5)*e^x +
sqrt(5) - 5)*sqrt(sqrt(5) + 5) + 5*sqrt(5) - 25)*sqrt(2*sqrt(10)*(2*sqrt(5)
 - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5) + 200)^(1/4) + 400*e
^(2*x) + 400) - 1/16000*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) -
 20*sqrt(5) + 60)*((sqrt(10)*(5*(sqrt(5) - 3)*e^x + 2*sqrt(5))*sqrt(2*sqrt(
5) + 5)*sqrt(sqrt(5) + 5) + 10*((3*sqrt(5) - 7)*e^x - 2)*sqrt(2*sqrt(5) + 5
))*(40*sqrt(5) + 200)^(3/4) + 20*(sqrt(10)*((sqrt(5) - 5)*e^x + 2*sqrt(5))*
sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 10*((sqrt(5) - 1)*e^x - sqrt(5) - 1
)*sqrt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(1/4)) - 1/4*sqrt(2*sqrt(5) + 5)*
(sqrt(5) - 2*e^x + 1)) + 8*sqrt(10)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sq
rt(5) + 5) - 20*sqrt(5) + 60)*((sqrt(5) - 1)*e^x + sqrt(5) - 1)*(40*sqrt(5)
 + 200)^(1/4)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*arctan(-1/40*sqrt(10)*(
(sqrt(5) - 5)*e^x + sqrt(5) + 1)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) - 1/
400*sqrt(10)*(sqrt(10)*((sqrt(5) - 5)*e^x + 2*sqrt(5))*sqrt(2*sqrt(5) + 5)*
sqrt(sqrt(5) + 5) + 10*((sqrt(5) - 1)*e^x - sqrt(5) - 1)*sqrt(2*sqrt(5) + 5
))*sqrt(sqrt(5) + 5) + 1/64000*(80*sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5
) + 5)*(sqrt(5) - 5) + 8*sqrt(10)*(sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5
) + 5)*(sqrt(5) - 5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 1))*sqrt(sqrt(5) +
 5) + sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*
((sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) - 3) + 2*(3*sqrt(
5) - 7)*sqrt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(3/4) + 4*(sqrt(10)*sqrt(2*
sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt
(5) - 1))*(40*sqrt(5) + 200)^(1/4)) + 1600*sqrt(2*sqrt(5) + 5))*sqrt(-20*sq
rt(10)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) - 200*(sqrt(5) + 1)*e^x + 2*(sqrt(10
)*(sqrt(5)*e^x + sqrt(5) - 5)*sqrt(sqrt(5) + 5) + 5*sqrt(5) - 25)*sqrt(2*sq
rt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5) + 2
00)^(1/4) + 400*e^(2*x) + 400) - 1/16000*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sq
rt(sqrt(5) + 5) - 20*sqrt(5) + 60)*((sqrt(10)*(5*(sqrt(5) - 3)*e^x + 2*sqrt
(5))*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 10*((3*sqrt(5) - 7)*e^x - 2)*s
qrt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(3/4) + 20*(sqrt(10)*((sqrt(5) - 5)*
e^x + 2*sqrt(5))*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 10*((sqrt(5) - 1)*
e^x - sqrt(5) - 1)*sqrt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(1/4)) + 1/4*sqr
t(2*sqrt(5) + 5)*(sqrt(5) - 2*e^x + 1)) + 4*((sqrt(5) + 1)*e^x + sqrt(5) +
1)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-2
*sqrt(5) + 5)*(-40*sqrt(5) + 200)^(3/4)*arctan(-1/32000*((20*(3*sqrt(5) + 7
)*e^x + (5*(sqrt(5) + 3)*e^x + 2*sqrt(5))*sqrt(-40*sqrt(5) + 200) + 40)*(-4
0*sqrt(5) + 200)^(3/4) + 20*(20*(sqrt(5) + 1)*e^x + ((sqrt(5) + 5)*e^x + 2*
sqrt(5))*sqrt(-40*sqrt(5) + 200) - 20*sqrt(5) + 20)*(-40*sqrt(5) + 200)^(1/
4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-
2*sqrt(5) + 5) + 1/128000*((((sqrt(5) + 3)*sqrt(-40*sqrt(5) + 200) + 12*sqr
t(5) + 28)*(-40*sqrt(5) + 200)^(3/4) + 4*((sqrt(5) + 5)*sqrt(-40*sqrt(5) +
200) + 20*sqrt(5) + 20)*(-40*sqrt(5) + 200)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sq
rt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-2*sqrt(5) + 5) - 4*(((sqrt(5
) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 20)*sqrt(-40*sqrt(5) + 200) +
 20*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) - 800)*sqrt(-2*sqrt(5) + 5))*sqrt
(200*(sqrt(5) - 1)*e^x + ((sqrt(5)*e^x + sqrt(5) + 5)*sqrt(-40*sqrt(5) + 20
0) + 10*sqrt(5) + 50)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sq
rt(5) + 60)*(-40*sqrt(5) + 200)^(1/4) + 10*(sqrt(5) + 5)*sqrt(-40*sqrt(5) +
 200) + 400*e^(2*x) + 400) + 1/1600*(20*((sqrt(5) + 5)*e^x + sqrt(5) - 1)*s
```

```
qrt(-40*sqrt(5) + 200) + (20*(sqrt(5) + 1)*e^x + ((sqrt(5) + 5)*e^x + 2*sqr
t(5))*sqrt(-40*sqrt(5) + 200) - 20*sqrt(5) + 20)*sqrt(-40*sqrt(5) + 200) -
400*sqrt(5) - 800*e^x + 400)*sqrt(-2*sqrt(5) + 5)) + 4*((sqrt(5) + 1)*e^x +
 sqrt(5) + 1)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) +
60)*sqrt(-2*sqrt(5) + 5)*(-40*sqrt(5) + 200)^(3/4)*arctan(-1/32000*((20*(3*
sqrt(5) + 7)*e^x + (5*(sqrt(5) + 3)*e^x + 2*sqrt(5))*sqrt(-40*sqrt(5) + 200
) + 40)*(-40*sqrt(5) + 200)^(3/4) + 20*(20*(sqrt(5) + 1)*e^x + ((sqrt(5) +
5)*e^x + 2*sqrt(5))*sqrt(-40*sqrt(5) + 200) - 20*sqrt(5) + 20)*(-40*sqrt(5)
 + 200)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) +
 60)*sqrt(-2*sqrt(5) + 5) + 1/128000*((((sqrt(5) + 3)*sqrt(-40*sqrt(5) + 20
0) + 12*sqrt(5) + 28)*(-40*sqrt(5) + 200)^(3/4) + 4*((sqrt(5) + 5)*sqrt(-40
*sqrt(5) + 200) + 20*sqrt(5) + 20)*(-40*sqrt(5) + 200)^(1/4))*sqrt(-(2*sqrt
(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-2*sqrt(5) + 5) +
4*(((sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 20)*sqrt(-40*sqrt(
5) + 200) + 20*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) - 800)*sqrt(-2*sqrt(5)
 + 5))*sqrt(200*(sqrt(5) - 1)*e^x - ((sqrt(5)*e^x + sqrt(5) + 5)*sqrt(-40*s
qrt(5) + 200) + 10*sqrt(5) + 50)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 2
00) + 20*sqrt(5) + 60)*(-40*sqrt(5) + 200)^(1/4) + 10*(sqrt(5) + 5)*sqrt(-4
0*sqrt(5) + 200) + 400*e^(2*x) + 400) - 1/1600*(20*((sqrt(5) + 5)*e^x + sqr
t(5) - 1)*sqrt(-40*sqrt(5) + 200) + (20*(sqrt(5) + 1)*e^x + ((sqrt(5) + 5)*
e^x + 2*sqrt(5))*sqrt(-40*sqrt(5) + 200) - 20*sqrt(5) + 20)*sqrt(-40*sqrt(5
) + 200) - 400*sqrt(5) - 800*e^x + 400)*sqrt(-2*sqrt(5) + 5)) + 2*(sqrt(10)
*((sqrt(5) - 5)*e^x + sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 40*e^x - 40)*sqrt(2*
sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5) +
 200)^(1/4)*log(-20*sqrt(10)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) - 200*(sqrt(5)
 + 1)*e^x + 2*(sqrt(10)*(sqrt(5)*e^x + sqrt(5) - 5)*sqrt(sqrt(5) + 5) + 5*s
qrt(5) - 25)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5)
 + 60)*(40*sqrt(5) + 200)^(1/4) + 400*e^(2*x) + 400) - 2*(sqrt(10)*((sqrt(5
) - 5)*e^x + sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 40*e^x - 40)*sqrt(2*sqrt(10)*
(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5) + 200)^(1/
4)*log(-20*sqrt(10)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) - 200*(sqrt(5) + 1)*e^x
 - 2*(sqrt(10)*(sqrt(5)*e^x + sqrt(5) - 5)*sqrt(sqrt(5) + 5) + 5*sqrt(5) -
25)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(4
0*sqrt(5) + 200)^(1/4) + 400*e^(2*x) + 400) + (((sqrt(5) + 5)*e^x + sqrt(5)
 + 5)*sqrt(-40*sqrt(5) + 200) + 80*e^x + 80)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40
*sqrt(5) + 200) + 20*sqrt(5) + 60)*(-40*sqrt(5) + 200)^(1/4)*log(200*(sqrt(
5) - 1)*e^x + ((sqrt(5)*e^x + sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 10*sqr
t(5) + 50)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)
*(-40*sqrt(5) + 200)^(1/4) + 10*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 400
*e^(2*x) + 400) - (((sqrt(5) + 5)*e^x + sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200
) + 80*e^x + 80)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5)
 + 60)*(-40*sqrt(5) + 200)^(1/4)*log(200*(sqrt(5) - 1)*e^x - ((sqrt(5)*e^x
+ sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 10*sqrt(5) + 50)*sqrt(-(2*sqrt(5)
+ 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*(-40*sqrt(5) + 200)^(1/4) +
 10*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 400*e^(2*x) + 400) + 3200)/(e^x
 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.07, size = 62, normalized size = 0.28

$$\frac{\tanh\left(\frac{x}{2}\right)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^8+10\_Z^4+1\right)}\frac{\left(-5\_R^6+5\_R^4-5\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^7+\_R^3}\right)}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^5),x)
```

```
[Out] 1/5*tanh(1/2*x)+1/50*sum((-5*_R^6+5*_R^4-5*_R^2+1)/(_R^7+_R^3)*ln(tanh(1/2*
x)-_R),_R=RootOf(5*_Z^8+10*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{5(e^x+1)}-\int \frac{2\left(e^{(7x)}-4e^{(6x)}+15e^{(5x)}-40e^{(4x)}+15e^{(3x)}-4e^{(2x)}+e^x\right)}{5\left(e^{(8x)}-2e^{(7x)}+8e^{(6x)}-14e^{(5x)}+30e^{(4x)}-14e^{(3x)}+8e^{(2x)}-2e^x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^5),x, algorithm="maxima")
```

```
[Out] -2/5/(e^x + 1) - integrate(2/5*(e^(7*x) - 4*e^(6*x) + 15*e^(5*x) - 40*e^(4*
x) + 15*e^(3*x) - 4*e^(2*x) + e^x)/(e^(8*x) - 2*e^(7*x) + 8*e^(6*x) - 14*e^
(5*x) + 30*e^(4*x) - 14*e^(3*x) + 8*e^(2*x) - 2*e^x + 1), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5 + 1),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**5),x)
```

```
[Out] Timed out
```

3.71 $\int \frac{1}{1+\cosh^6(x)}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}$$

```
[Out] 1/6*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)+1/3*arctanh(tanh(x)/(1-(-1)^(1/3))
^(1/2))/(1-(-1)^(1/3))^(1/2)+1/3*arctanh(tanh(x)/(1+(-1)^(2/3))^(1/2))/(1+(
-1)^(2/3))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(3*Sqrt[2]) + ArcTanh[Tanh[x]/Sqrt[1 - (-1)^(1/3)]
]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Tanh[x]/Sqrt[1 + (-1)^(2/3)]]/(3*Sqrt[
1 + (-1)^(2/3)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\cosh^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1+\cosh^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\,\cosh^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\cosh^2(x)}\,dx$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(x)\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[3]{-1}\right)x^2}\,dx, x, \coth(x)\right) + \frac{1}{3}$$

$$= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}$$

Mathematica [C] time = 0.46, size = 68, normalized size = 0.82

$$\frac{1}{6}\left(\sqrt{2}\,\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right) + i\sqrt{3}\left(\tan^{-1}\left(\frac{1-2i\tanh(x)}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1+2i\tanh(x)}{\sqrt{3}}\right)\right) + \tan^{-1}(\text{csch}(x)\text{sech}(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^6)^(-1),x]
```

```
[Out] (ArcTan[Csch[x]*Sech[x]] + I*Sqrt[3]*(ArcTan[(1 - (2*I)*Tanh[x])/Sqrt[3]] -
 ArcTan[(1 + (2*I)*Tanh[x])/Sqrt[3]]) + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]])/6
```

fricas [B] time = 1.59, size = 158, normalized size = 1.90

$$-\frac{1}{12}\sqrt{3}\log\left(16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{3}\log\left(-16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\left(2\sqrt{2}-3\right)e^{(2x)}+12}{e^{(4x)}+6e^{(2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(16*sqrt(3) + 4*e^(4*x) + 28) + 1/12*sqrt(3)*log(-16*sqrt(
3) + 4*e^(4*x) + 28) + 1/12*sqrt(2)*log(-(2*(2*sqrt(2) - 3)*e^(2*x) + 12*sq
rt(2) - e^(4*x) - 17)/(e^(4*x) + 6*e^(2*x) + 1)) + 1/3*arctan(-(sqrt(3) + 2
)*e^(2*x) + 1/2*(sqrt(3) + 2)*sqrt(-16*sqrt(3) + 4*e^(4*x) + 28)) - 1/3*arc
tan(-(sqrt(3) - 2)*e^(2*x) + sqrt(4*sqrt(3) + e^(4*x) + 7)*(sqrt(3) - 2))
```

giac [B] time = 0.14, size = 140, normalized size = 1.69

$$\frac{1}{36}\left(\left(2\sqrt{3}-3\right)e^{(4x)}+2\sqrt{3}-3\right)\arctan\left(\frac{e^{(2x)}}{\sqrt{3}+2}\right)-\frac{1}{36}\left(\left(2\sqrt{3}+3\right)e^{(4x)}+2\sqrt{3}+3\right)\arctan\left(-\frac{e^{(2x)}}{\sqrt{3}-2}\right)-\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^6),x, algorithm="giac")
```

```
[Out] 1/36*((2*sqrt(3) - 3)*e^(4*x) + 2*sqrt(3) - 3)*arctan(e^(2*x)/(sqrt(3) + 2)
) - 1/36*((2*sqrt(3) + 3)*e^(4*x) + 2*sqrt(3) + 3)*arctan(-e^(2*x)/(sqrt(3)
 - 2)) - 1/12*sqrt(3)*log((sqrt(3) + 2)^2 + e^(4*x)) + 1/12*sqrt(3)*log((sq
rt(3) - 2)^2 + e^(4*x)) + 1/12*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sq
rt(2) + e^(2*x) + 3))
```

maple [C] time = 0.09, size = 208, normalized size = 2.51

$$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^3+2\_Z^2-2\_Z+1\right)}\frac{\left(-\_R^2-4\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3+3\_R^2+2\_R-1}\right)}{6}+\frac{\sqrt{2}\,\ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}\right)}{24}-\frac{\sqrt{2}\,\ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)-}{\tanh^2\left(\frac{x}{2}\right)+}\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^6),x)
```

```
[Out] 1/6*sum((-_R^2-4*_R+1)/(2*_R^3+3*_R^2+2*_R-1)*ln(tanh(1/2*x)-_R),_R=RootOf(
_Z^4+2*_Z^3+2*_Z^2-2*_Z+1))+1/24*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2
*x)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))-1/24*2^(1/2)*ln((tanh(1/2*x)^
2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))+1/6*sum((-_
R^2+4*_R+1)/(2*_R^3-3*_R^2+2*_R+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^4-2*_Z^3
+2*_Z^2+2*_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)-\frac{4}{3}\int -\frac{\left(6e^{(-2x)}-e^{(-4x)}-1\right)e^{(-2x)}}{14e^{(-4x)}+e^{(-8x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^6),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
 4/3*integrate(-(6*e^(-2*x) - e^(-4*x) - 1)*e^(-2*x)/(14*e^(-4*x) + e^(-8*x
) + 1), x)
```

mupad [B] time = 2.55, size = 337, normalized size = 4.06

$$\frac{\sqrt{3}\,\ln\Big(\Big(6177144285775790080\,e^{2x}-2167269359741829120\,\sqrt{3}+3566375915854233600\,\sqrt{3}\,e^{2x}-37538206}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^6 + 1),x)
```

```
[Out] (log(3^(1/2)*(955607545932677120 - 2167269359741829120i) - exp(2*x)*(140094
49395540459520 + 6177144285775790080i) + 3^(1/2)*exp(2*x)*(8088359377641144
320 + 3566375915854233600i) - (1655160823988879360 - 3753820658157486080i))
*1i)/12 - (log(3^(1/2)*(955607545932677120 + 2167269359741829120i) - exp(2*
x)*(14009449395540459520 - 6177144285775790080i) + 3^(1/2)*exp(2*x)*(808835
9377641144320 - 3566375915854233600i) - (1655160823988879360 + 375382065815
7486080i))*1i)/12 - atan((6177144285775790080*exp(2*x) - 216726935974182912
0*3^(1/2) + 3566375915854233600*3^(1/2)*exp(2*x) - 3753820658157486080)/(14
009449395540459520*exp(2*x) + 955607545932677120*3^(1/2) + 8088359377641144
320*3^(1/2)*exp(2*x) + 1655160823988879360))/6 + (3^(1/2)*log((617714428577
5790080*exp(2*x) - 2167269359741829120*3^(1/2) + 3566375915854233600*3^(1/2
)*exp(2*x) - 3753820658157486080)^2 + (14009449395540459520*exp(2*x) + 9556
07545932677120*3^(1/2) + 8088359377641144320*3^(1/2)*exp(2*x) + 16551608239
88879360)^2))/12 - (3^(1/2)*log((6177144285775790080*exp(2*x) + 21672693597
41829120*3^(1/2) - 3566375915854233600*3^(1/2)*exp(2*x) - 37538206581574860
80)^2 + (14009449395540459520*exp(2*x) - 955607545932677120*3^(1/2) - 80883
59377641144320*3^(1/2)*exp(2*x) + 1655160823988879360)^2))/12 - (pi*sign(x
- log((24639*3^(1/2) + 42676)/(40545*3^(1/2) + 70226))/2))/6 + (pi*sign(617
7144285775790080*exp(2*x) - 2167269359741829120*3^(1/2) + 35663759158542336
00*3^(1/2)*exp(2*x) - 3753820658157486080))/6 - (2^(1/2)*log(21443225520701
44000*2^(1/2) - 17674880313941032960*exp(2*x) + 12498027726650736640*2^(1/2
)*exp(2*x) - 3032530035220152320))/12 + (2^(1/2)*log(17674880313941032960*e
xp(2*x) + 2144322552070144000*2^(1/2) + 12498027726650736640*2^(1/2)*exp(2*
x) + 3032530035220152320))/12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**6),x)
```

```
[Out] Timed out
```

3.72 $\int \frac{1}{1+\cosh^8(x)}\,dx$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[4]{-1}}}\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[4]{-1}}}\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{3/4}}}\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{3/4}}}\right)}{4\sqrt{1+(-1)^{3/4}}}$$

```
[Out] 1/4*arctanh(tanh(x)/(1-(-1)^(1/4))^(1/2))/(1-(-1)^(1/4))^(1/2)+1/4*arctanh(
tanh(x)/(1+(-1)^(1/4))^(1/2))/(1+(-1)^(1/4))^(1/2)+1/4*arctanh(tanh(x)/(1-(
-1)^(3/4))^(1/2))/(1-(-1)^(3/4))^(1/2)+1/4*arctanh(tanh(x)/(1+(-1)^(3/4))^(
1/2))/(1+(-1)^(3/4))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[4]{-1}}}\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[4]{-1}}}\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{3/4}}}\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{3/4}}}\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[1 - (-1)^(1/4)]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[Ta
nh[x]/Sqrt[1 + (-1)^(1/4)]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Tanh[x]/Sqrt
[1 - (-1)^(3/4)]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Tanh[x]/Sqrt[1 + (-1)^
(3/4)]]/(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\cosh^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\cosh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\cosh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\cosh^2(x)}\,dx$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \coth(x)\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[4]{-1}\right)x^2}\,dx, x, \text{cot}\right.$$

$$= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[4]{-1}}}\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[4]{-1}}}\right)}{4\sqrt{1+\sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{3/4}}}\right)}{4\sqrt{1-(-1)^{3/4}}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{3/4}}}\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Mathematica [C] time = 0.14, size = 127, normalized size = 0.98

$$16\text{RootSum}\left[\#1^8+8\#1^7+28\#1^6+56\#1^5+326\#1^4+56\#1^3+28\#1^2+8\#1+1\&, \frac{\#1^3x+\#1^3\log(-\#1\sinh(}{\#1^7+7\#1^6+21\#1^5+3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[1 + 8*#1 + 28*#1^2 + 56*#1^3 + 326*#1^4 + 56*#1^5 + 28*#1^6 + 8*
#1^7 + #1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]*#1]
*#1^3)/(1 + 7*#1 + 21*#1^2 + 163*#1^3 + 35*#1^4 + 21*#1^5 + 7*#1^6 + #1^7)
& ]
```

fricas [B] time = 1.28, size = 3773, normalized size = 29.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^8),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2
) + 4)^(3/4)*sqrt(2*sqrt(2) + 3)*(sqrt(2) - 1)*arctan(1/31*(2*(13*sqrt(2) -
 20)*e^(2*x) - 23*sqrt(2) + 33)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 1
/496*(32*(10*sqrt(2) - 13)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (((355
*sqrt(2) - 508)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 8
6)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(
2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3)
)*(2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqr
t(2) + 8) + 4*((76*sqrt(2) - 105)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
 2*(53*sqrt(2) - 72)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt
(2) - 33)*sqrt(2*sqrt(2) + 3))*sqrt(-4*(sqrt(2) - 1)*e^(2*x) + (2*(sqrt(2)
- 1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) + 5*sqrt(2) - 6)*sqrt(2*sqrt(2) + 4)
+ 6*sqrt(2) - 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8
)*(2*sqrt(2) + 4)^(1/4) - 4*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) +
 2*e^(4*x) + 10) + 1/248*((((254*sqrt(2) - 355)*e^(2*x) + 102*sqrt(2) - 145
)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x)
+ 23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sq
rt(2) - 164)*e^(2*x) + 39*sqrt(2) - 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2)
+ 3) + 2*((63*sqrt(2) - 85)*e^(2*x) + 17*sqrt(2) - 19)*sqrt(2*sqrt(2) + 3))
*(2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt
(2) + 8) + 1/124*(((105*sqrt(2) - 152)*e^(2*x) - 13*sqrt(2) + 20)*sqrt(2*sq
rt(2) + 4)*sqrt(2*sqrt(2) + 3) + 4*((36*sqrt(2) - 53)*e^(2*x) + 23*sqrt(2)
- 33)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 1/31*((33*sqrt(2) - 46)*e^
(2*x) + 3*sqrt(2) - 7)*sqrt(2*sqrt(2) + 3)) - 1/16*sqrt(2*sqrt(2*sqrt(2) +
4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(3/4)*sqrt(2*sqrt(2) +
```

```
3)*(sqrt(2) - 1)*arctan(-1/31*(2*(13*sqrt(2) - 20)*e^(2*x) - 23*sqrt(2) + 3
3)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) - 13)*sq
rt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - (((355*sqrt(2) - 508)*sqrt(2*sqrt(2
) + 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86)*sqrt(2*sqrt(2) + 3))*(2*sq
rt(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2)
 + 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt
(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) + 4*((76*sqrt(2) -
105)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(53*sqrt(2) - 72)*sqrt(2*s
qrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 3)
)*sqrt(-4*(sqrt(2) - 1)*e^(2*x) - (2*(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)
*e^(2*x) + 5*sqrt(2) - 6)*sqrt(2*sqrt(2) + 4) + 6*sqrt(2) - 6)*sqrt(2*sqrt(
2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 4*s
qrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) + 2*e^(4*x) + 10) + 1/248*((((
254*sqrt(2) - 355)*e^(2*x) + 102*sqrt(2) - 145)*sqrt(2*sqrt(2) + 4)*sqrt(2*
sqrt(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x) + 23*sqrt(2) - 33)*sqrt(2*sqr
t(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) - 164)*e^(2*x) + 39*sqr
t(2) - 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*((63*sqrt(2) - 85)*e
^(2*x) + 17*sqrt(2) - 19)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt(
2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) - 1/124*(((105*sqrt(
2) - 152)*e^(2*x) - 13*sqrt(2) + 20)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3
) + 4*((36*sqrt(2) - 53)*e^(2*x) + 23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 3))*sq
rt(2*sqrt(2) + 4) - 1/31*((33*sqrt(2) - 46)*e^(2*x) + 3*sqrt(2) - 7)*sqrt(2
*sqrt(2) + 3)) + 1/16*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt
(2) + 8)*(sqrt(2) + 1)*(-2*sqrt(2) + 4)^(3/4)*sqrt(-2*sqrt(2) + 3)*arctan(1
/31*(2*(13*sqrt(2) + 20)*e^(2*x) - 23*sqrt(2) - 33)*sqrt(-2*sqrt(2) + 4)*sq
rt(-2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) + 13)*sqrt(-2*sqrt(2) + 4)*sqrt(
-2*sqrt(2) + 3) - (((355*sqrt(2) + 508)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2
) + 3) + 6*(59*sqrt(2) + 86)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(3/4) +
 4*((82*sqrt(2) + 119)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + (85*sqrt
(2) + 126)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sqrt(-2*(2*sqrt(2)
 + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 4*((76*sqrt(2) + 105)*sqrt(-2
*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*(53*sqrt(2) + 72)*sqrt(-2*sqrt(2) +
3))*sqrt(-2*sqrt(2) + 4) + 16*(23*sqrt(2) + 33)*sqrt(-2*sqrt(2) + 3))*sqrt(
4*(sqrt(2) + 1)*e^(2*x) + (2*(sqrt(2) + 1)*e^(2*x) + ((sqrt(2) + 2)*e^(2*x)
 + 5*sqrt(2) + 6)*sqrt(-2*sqrt(2) + 4) + 6*sqrt(2) + 6)*sqrt(-2*(2*sqrt(2)
+ 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2) + 4)^(1/4) + 4*(sqrt
(2) + 2)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 2*e^(4*x) + 10) - 1/248*((((254
*sqrt(2) + 355)*e^(2*x) + 102*sqrt(2) + 145)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*s
qrt(2) + 3) + 2*(3*(43*sqrt(2) + 59)*e^(2*x) + 23*sqrt(2) + 33)*sqrt(-2*sqr
t(2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) + 164)*e^(2*x) + 39*sq
rt(2) + 60)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*((63*sqrt(2) + 85
)*e^(2*x) + 17*sqrt(2) + 19)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*
sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 1/124*(((10
5*sqrt(2) + 152)*e^(2*x) - 13*sqrt(2) - 20)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sq
rt(2) + 3) + 4*((36*sqrt(2) + 53)*e^(2*x) + 23*sqrt(2) + 33)*sqrt(-2*sqrt(2
) + 3))*sqrt(-2*sqrt(2) + 4) + 1/31*((33*sqrt(2) + 46)*e^(2*x) + 3*sqrt(2)
+ 7)*sqrt(-2*sqrt(2) + 3)) + 1/16*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) +
 4) + 4*sqrt(2) + 8)*(sqrt(2) + 1)*(-2*sqrt(2) + 4)^(3/4)*sqrt(-2*sqrt(2) +
 3)*arctan(-1/31*(2*(13*sqrt(2) + 20)*e^(2*x) - 23*sqrt(2) - 33)*sqrt(-2*sq
rt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 1/496*(32*(10*sqrt(2) + 13)*sqrt(-2*sqrt(
2) + 4)*sqrt(-2*sqrt(2) + 3) + (((355*sqrt(2) + 508)*sqrt(-2*sqrt(2) + 4)*s
qrt(-2*sqrt(2) + 3) + 6*(59*sqrt(2) + 86)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2)
 + 4)^(3/4) + 4*((82*sqrt(2) + 119)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) +
3) + (85*sqrt(2) + 126)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sqrt(
-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 4*((76*sqrt(2) +
 105)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*(53*sqrt(2) + 72)*sqrt(
-2*sqrt(2) + 3))*sqrt(-2*sqrt(2) + 4) + 16*(23*sqrt(2) + 33)*sqrt(-2*sqrt(2
) + 3))*sqrt(4*(sqrt(2) + 1)*e^(2*x) - (2*(sqrt(2) + 1)*e^(2*x) + ((sqrt(2)
 + 2)*e^(2*x) + 5*sqrt(2) + 6)*sqrt(-2*sqrt(2) + 4) + 6*sqrt(2) + 6)*sqrt(-
```

```
2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2) + 4)^(1
/4) + 4*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 2*e^(4*x) + 10) -
1/248*((((254*sqrt(2) + 355)*e^(2*x) + 102*sqrt(2) + 145)*sqrt(-2*sqrt(2) +
 4)*sqrt(-2*sqrt(2) + 3) + 2*(3*(43*sqrt(2) + 59)*e^(2*x) + 23*sqrt(2) + 33
)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) + 164)*e^
(2*x) + 39*sqrt(2) + 60)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*((63
*sqrt(2) + 85)*e^(2*x) + 17*sqrt(2) + 19)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2)
 + 4)^(1/4))*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)
- 1/124*(((105*sqrt(2) + 152)*e^(2*x) - 13*sqrt(2) - 20)*sqrt(-2*sqrt(2) +
4)*sqrt(-2*sqrt(2) + 3) + 4*((36*sqrt(2) + 53)*e^(2*x) + 23*sqrt(2) + 33)*s
qrt(-2*sqrt(2) + 3))*sqrt(-2*sqrt(2) + 4) - 1/31*((33*sqrt(2) + 46)*e^(2*x)
 + 3*sqrt(2) + 7)*sqrt(-2*sqrt(2) + 3)) - 1/64*sqrt(-2*(2*sqrt(2) + 3)*sqrt
(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + 2*s
qrt(2))*(-2*sqrt(2) + 4)^(1/4)*log(2*(sqrt(2) + 1)*e^(2*x) + 1/2*(2*(sqrt(2
) + 1)*e^(2*x) + ((sqrt(2) + 2)*e^(2*x) + 5*sqrt(2) + 6)*sqrt(-2*sqrt(2) +
4) + 6*sqrt(2) + 6)*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2
) + 8)*(-2*sqrt(2) + 4)^(1/4) + 2*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 2*sq
rt(2) + e^(4*x) + 5) + 1/64*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) +
4*sqrt(2) + 8)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + 2*sqrt(2))*(-2*sqrt(2)
 + 4)^(1/4)*log(2*(sqrt(2) + 1)*e^(2*x) - 1/2*(2*(sqrt(2) + 1)*e^(2*x) + ((
sqrt(2) + 2)*e^(2*x) + 5*sqrt(2) + 6)*sqrt(-2*sqrt(2) + 4) + 6*sqrt(2) + 6)
*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2)
+ 4)^(1/4) + 2*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 2*sqrt(2) + e^(4*x) + 5
) - 1/64*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(sqrt(
2*sqrt(2) + 4)*(sqrt(2) - 1) + 2*sqrt(2))*(2*sqrt(2) + 4)^(1/4)*log(-2*(sqr
t(2) - 1)*e^(2*x) + 1/2*(2*(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) +
 5*sqrt(2) - 6)*sqrt(2*sqrt(2) + 4) + 6*sqrt(2) - 6)*sqrt(2*sqrt(2*sqrt(2)
+ 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 2*sqrt(2*sqrt
(2) + 4)*(sqrt(2) - 2) - 2*sqrt(2) + e^(4*x) + 5) + 1/64*sqrt(2*sqrt(2*sqrt
(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1
) + 2*sqrt(2))*(2*sqrt(2) + 4)^(1/4)*log(-2*(sqrt(2) - 1)*e^(2*x) - 1/2*(2*
(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) + 5*sqrt(2) - 6)*sqrt(2*sqrt
(2) + 4) + 6*sqrt(2) - 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sq
rt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 2*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 2*
sqrt(2) + e^(4*x) + 5)
```

giac [A] time = 0.14, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.09, size = 47, normalized size = 0.36

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+1\right)}\_R\ln\left(2\tanh\left(\frac{x}{2}\right)\_R+\tanh^2\left(\frac{x}{2}\right)+1\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x)^8),x)
```

```
[Out] 1/8*sum(_R*ln(2*tanh(1/2*x)*_R+tanh(1/2*x)^2+1),_R=RootOf(2*_Z^8-4*_Z^6+6*_
Z^4-4*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh(x)^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(cosh(x)^8 + 1), x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^8 + 1),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x)**8),x)
```

```
[Out] Timed out
```

3.73 $\int \frac{1}{1-\cosh^5(x)}\,dx$

Optimal. Leaf size=205

$$\frac{2\tanh^{-1}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tanh^{-1}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\tan^{-1}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\right.}{5\sqrt{-1-(-1)^{3}}}$$

```
[Out] -1/5*sinh(x)/(1-cosh(x))+2/5*arctanh(((1-(-1)^(3/5))/(1+(-1)^(3/5)))^(1/2)*
tanh(1/2*x))/(1+(-1)^(1/5))^(1/2)+2/5*arctanh(((1-(-1)^(1/5))/(1+(-1)^(1/5)
))^(1/2)*tanh(1/2*x))/(1-(-1)^(2/5))^(1/2)+2/5*arctan(((-1-(-1)^(4/5))/(1-(
-1)^(4/5)))^(1/2)*tanh(1/2*x))/(-1-(-1)^(3/5))^(1/2)-2/5*arctan(tanh(1/2*x)
/((-1+(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2))/(-1+(-1)^(4/5))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3213, 2648, 2659, 208, 205}

$$\frac{2\tanh^{-1}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tanh^{-1}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\tan^{-1}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\right.}{5\sqrt{-1-(-1)^{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[Tanh[x/2]/Sqrt[-((1 - (-1)^(2/5))/(1 + (-1)^(2/5)))]])/(5*Sqrt[-
1 + (-1)^(4/5)]) + (2*ArcTan[Sqrt[-((1 + (-1)^(4/5))/(1 - (-1)^(4/5)))]*Tan
h[x/2]])/(5*Sqrt[-1 - (-1)^(3/5)]) + (2*ArcTanh[Sqrt[(1 - (-1)^(1/5))/(1 +
(-1)^(1/5))]*Tanh[x/2]])/(5*Sqrt[1 - (-1)^(2/5)]) + (2*ArcTanh[Sqrt[(1 - (-
1)^(3/5))/(1 + (-1)^(3/5))]*Tanh[x/2]])/(5*Sqrt[1 + (-1)^(1/5)]) - Sinh[x]/
(5*(1 - Cosh[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1-\cosh^5(x)}\,dx = \int \left(\frac{1}{5(1-\cosh(x))} + \frac{1}{5\left(1+\sqrt[5]{-1}\cosh(x)\right)} + \frac{1}{5\left(1-(-1)^{2/5}\cosh(x)\right)} + \frac{1}{5\left(1+(-1)^{3/5}\cosh(\right.}$$

$$= \frac{1}{5}\int \frac{1}{1-\cosh(x)}\,dx + \frac{1}{5}\int \frac{1}{1+\sqrt[5]{-1}\cosh(x)}\,dx + \frac{1}{5}\int \frac{1}{1-(-1)^{2/5}\cosh(x)}\,dx + \frac{1}{5}\int \frac{}{1+(}$$

$$= -\frac{\sinh(x)}{5(1-\cosh(x))} + \frac{2}{5}\text{Subst}\left(\int \frac{1}{1+\sqrt[5]{-1}-\left(1-\sqrt[5]{-1}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) + \frac{2}{5}\text{Subst}\left(\int \frac{}{1}\right.$$

$$= -\frac{2\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{-1+(-1)^{4/5}}} + \frac{2\tan^{-1}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}} + \frac{2\tanh^{-1}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}$$

Mathematica [C] time = 0.10, size = 445, normalized size = 2.17

$$\frac{1}{10}\text{RootSum}\left[\#1^8+2\#1^7+8\#1^6+14\#1^5+30\#1^4+14\#1^3+8\#1^2+2\#1+1\&, \frac{\#1^6 x+2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^5)^(-1),x]
```

```
[Out] Coth[x/2]/5 + RootSum[1 + 2*#1 + 8*#1^2 + 14*#1^3 + 30*#1^4 + 14*#1^5 + 8*#
1^6 + 2*#1^7 + #1^8 & , (x + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 -
Sinh[x/2]*#1] + 4*x*#1 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh
[x/2]*#1]*#1 + 15*x*#1^2 + 30*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - S
inh[x/2]*#1]*#1^2 + 40*x*#1^3 + 80*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#
1 - Sinh[x/2]*#1]*#1^3 + 15*x*#1^4 + 30*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x
/2]*#1 - Sinh[x/2]*#1]*#1^4 + 4*x*#1^5 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cos
h[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Co
sh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(1 + 8*#1 + 21*#1^2 + 60*#1^3 + 35*#1^4 +
24*#1^5 + 7*#1^6 + 4*#1^7) & ]/10
```

fricas [B] time = 2.12, size = 3260, normalized size = 15.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^5),x, algorithm="fricas")
```

```
[Out] 1/8000*(8*sqrt(10)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*s
qrt(5) + 60)*((sqrt(5) - 1)*e^x - sqrt(5) + 1)*(40*sqrt(5) + 200)^(1/4)*sqr
t(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*arctan(1/40*sqrt(10)*((sqrt(5) - 5)*e^x
- sqrt(5) - 1)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 1/400*sqrt(10)*(sqrt
(10)*((sqrt(5) - 5)*e^x - 2*sqrt(5))*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)
+ 10*((sqrt(5) - 1)*e^x + sqrt(5) + 1)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) +
5) - 1/64000*(80*sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) -
```

```
5) + 8*sqrt(10)*(sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) -
5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 1))*sqrt(sqrt(5) + 5) - sqrt(2*sqrt(
10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*((sqrt(10)*sqrt(2*
sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) - 3) + 2*(3*sqrt(5) - 7)*sqrt(2*sqr
t(5) + 5))*(40*sqrt(5) + 200)^(3/4) + 4*(sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(
sqrt(5) + 5)*(sqrt(5) - 5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 1))*(40*sqrt
(5) + 200)^(1/4)) + 1600*sqrt(2*sqrt(5) + 5))*sqrt(-20*sqrt(10)*sqrt(sqrt(5
) + 5)*(sqrt(5) - 5) + 200*(sqrt(5) + 1)*e^x + 2*(sqrt(10)*(sqrt(5)*e^x - s
qrt(5) + 5)*sqrt(sqrt(5) + 5) - 5*sqrt(5) + 25)*sqrt(2*sqrt(10)*(2*sqrt(5)
 - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5) + 200)^(1/4) + 400*e^
(2*x) + 400) - 1/16000*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) -
20*sqrt(5) + 60)*((sqrt(10)*(5*(sqrt(5) - 3)*e^x - 2*sqrt(5))*sqrt(2*sqrt(5
) + 5)*sqrt(sqrt(5) + 5) + 10*((3*sqrt(5) - 7)*e^x + 2)*sqrt(2*sqrt(5) + 5)
)*(40*sqrt(5) + 200)^(3/4) + 20*(sqrt(10)*((sqrt(5) - 5)*e^x - 2*sqrt(5))*s
qrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 10*((sqrt(5) - 1)*e^x + sqrt(5) + 1)
*sqrt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(1/4)) + 1/4*sqrt(2*sqrt(5) + 5)*(
sqrt(5) + 2*e^x + 1)) + 8*sqrt(10)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqr
t(5) + 5) - 20*sqrt(5) + 60)*((sqrt(5) - 1)*e^x - sqrt(5) + 1)*(40*sqrt(5)
+ 200)^(1/4)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*arctan(-1/40*sqrt(10)*((
sqrt(5) - 5)*e^x - sqrt(5) - 1)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) - 1/4
00*sqrt(10)*(sqrt(10)*((sqrt(5) - 5)*e^x - 2*sqrt(5))*sqrt(2*sqrt(5) + 5)*s
qrt(sqrt(5) + 5) + 10*((sqrt(5) - 1)*e^x + sqrt(5) + 1)*sqrt(2*sqrt(5) + 5)
)*sqrt(sqrt(5) + 5) + 1/64000*(80*sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5)
 + 5)*(sqrt(5) - 5) + 8*sqrt(10)*(sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5)
 + 5)*(sqrt(5) - 5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 1))*sqrt(sqrt(5) +
5) + sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(
(sqrt(10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) - 3) + 2*(3*sqrt(5
) - 7)*sqrt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(3/4) + 4*(sqrt(10)*sqrt(2*s
qrt(5) + 5)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) + 10*sqrt(2*sqrt(5) + 5)*(sqrt(
5) - 1))*(40*sqrt(5) + 200)^(1/4)) + 1600*sqrt(2*sqrt(5) + 5))*sqrt(-20*sqr
t(10)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) + 200*(sqrt(5) + 1)*e^x - 2*(sqrt(10)
*(sqrt(5)*e^x - sqrt(5) + 5)*sqrt(sqrt(5) + 5) - 5*sqrt(5) + 25)*sqrt(2*sqr
t(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5) + 20
0)^(1/4) + 400*e^(2*x) + 400) - 1/16000*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqr
t(sqrt(5) + 5) - 20*sqrt(5) + 60)*((sqrt(10)*(5*(sqrt(5) - 3)*e^x - 2*sqrt(
5))*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 10*((3*sqrt(5) - 7)*e^x + 2)*sq
rt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(3/4) + 20*(sqrt(10)*((sqrt(5) - 5)*e
^x - 2*sqrt(5))*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 5) + 10*((sqrt(5) - 1)*e
^x + sqrt(5) + 1)*sqrt(2*sqrt(5) + 5))*(40*sqrt(5) + 200)^(1/4)) - 1/4*sqrt
(2*sqrt(5) + 5)*(sqrt(5) + 2*e^x + 1)) + 4*((sqrt(5) + 1)*e^x - sqrt(5) - 1
)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-2*
sqrt(5) + 5)*(-40*sqrt(5) + 200)^(3/4)*arctan(-1/32000*((20*(3*sqrt(5) + 7)
*e^x + (5*(sqrt(5) + 3)*e^x - 2*sqrt(5))*sqrt(-40*sqrt(5) + 200) - 40)*(-40
*sqrt(5) + 200)^(3/4) + 20*(20*(sqrt(5) + 1)*e^x + ((sqrt(5) + 5)*e^x - 2*s
qrt(5))*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) - 20)*(-40*sqrt(5) + 200)^(1/4
))*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-2
*sqrt(5) + 5) + 1/128000*((((sqrt(5) + 3)*sqrt(-40*sqrt(5) + 200) + 12*sqrt
(5) + 28)*(-40*sqrt(5) + 200)^(3/4) + 4*((sqrt(5) + 5)*sqrt(-40*sqrt(5) + 2
00) + 20*sqrt(5) + 20)*(-40*sqrt(5) + 200)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqr
t(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-2*sqrt(5) + 5) + 4*(((sqrt(5)
 + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 20)*sqrt(-40*sqrt(5) + 200) +
20*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) - 800)*sqrt(-2*sqrt(5) + 5))*sqrt(
-200*(sqrt(5) - 1)*e^x + ((sqrt(5)*e^x - sqrt(5) - 5)*sqrt(-40*sqrt(5) + 20
0) - 10*sqrt(5) - 50)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sq
rt(5) + 60)*(-40*sqrt(5) + 200)^(1/4) + 10*(sqrt(5) + 5)*sqrt(-40*sqrt(5) +
 200) + 400*e^(2*x) + 400) - 1/1600*(20*((sqrt(5) + 5)*e^x - sqrt(5) + 1)*s
qrt(-40*sqrt(5) + 200) + (20*(sqrt(5) + 1)*e^x + ((sqrt(5) + 5)*e^x - 2*sqr
t(5))*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) - 20)*sqrt(-40*sqrt(5) + 200) +
400*sqrt(5) - 800*e^x - 400)*sqrt(-2*sqrt(5) + 5)) + 4*((sqrt(5) + 1)*e^x -
```

```
 sqrt(5) - 1)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) +
60)*sqrt(-2*sqrt(5) + 5)*(-40*sqrt(5) + 200)^(3/4)*arctan(-1/32000*((20*(3*
sqrt(5) + 7)*e^x + (5*(sqrt(5) + 3)*e^x - 2*sqrt(5))*sqrt(-40*sqrt(5) + 200
) - 40)*(-40*sqrt(5) + 200)^(3/4) + 20*(20*(sqrt(5) + 1)*e^x + ((sqrt(5) +
5)*e^x - 2*sqrt(5))*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) - 20)*(-40*sqrt(5)
 + 200)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) +
 60)*sqrt(-2*sqrt(5) + 5) + 1/128000*((((sqrt(5) + 3)*sqrt(-40*sqrt(5) + 20
0) + 12*sqrt(5) + 28)*(-40*sqrt(5) + 200)^(3/4) + 4*((sqrt(5) + 5)*sqrt(-40
*sqrt(5) + 200) + 20*sqrt(5) + 20)*(-40*sqrt(5) + 200)^(1/4))*sqrt(-(2*sqrt
(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*sqrt(-2*sqrt(5) + 5) -
4*(((sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 20)*sqrt(-40*sqrt(
5) + 200) + 20*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) - 800)*sqrt(-2*sqrt(5)
 + 5))*sqrt(-200*(sqrt(5) - 1)*e^x - ((sqrt(5)*e^x - sqrt(5) - 5)*sqrt(-40*
sqrt(5) + 200) - 10*sqrt(5) - 50)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) +
200) + 20*sqrt(5) + 60)*(-40*sqrt(5) + 200)^(1/4) + 10*(sqrt(5) + 5)*sqrt(-
40*sqrt(5) + 200) + 400*e^(2*x) + 400) + 1/1600*(20*((sqrt(5) + 5)*e^x - sq
rt(5) + 1)*sqrt(-40*sqrt(5) + 200) + (20*(sqrt(5) + 1)*e^x + ((sqrt(5) + 5)
*e^x - 2*sqrt(5))*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) - 20)*sqrt(-40*sqrt(
5) + 200) + 400*sqrt(5) - 800*e^x - 400)*sqrt(-2*sqrt(5) + 5)) + 2*(sqrt(10
)*((sqrt(5) - 5)*e^x - sqrt(5) + 5)*sqrt(sqrt(5) + 5) - 40*e^x + 40)*sqrt(2
*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5)
+ 200)^(1/4)*log(-20*sqrt(10)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) + 200*(sqrt(5
) + 1)*e^x + 2*(sqrt(10)*(sqrt(5)*e^x - sqrt(5) + 5)*sqrt(sqrt(5) + 5) - 5*
sqrt(5) + 25)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5
) + 60)*(40*sqrt(5) + 200)^(1/4) + 400*e^(2*x) + 400) - 2*(sqrt(10)*((sqrt(
5) - 5)*e^x - sqrt(5) + 5)*sqrt(sqrt(5) + 5) - 40*e^x + 40)*sqrt(2*sqrt(10)
*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(40*sqrt(5) + 200)^(1
/4)*log(-20*sqrt(10)*sqrt(sqrt(5) + 5)*(sqrt(5) - 5) + 200*(sqrt(5) + 1)*e^
x - 2*(sqrt(10)*(sqrt(5)*e^x - sqrt(5) + 5)*sqrt(sqrt(5) + 5) - 5*sqrt(5) +
 25)*sqrt(2*sqrt(10)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 5) - 20*sqrt(5) + 60)*(
40*sqrt(5) + 200)^(1/4) + 400*e^(2*x) + 400) + (((sqrt(5) + 5)*e^x - sqrt(5
) - 5)*sqrt(-40*sqrt(5) + 200) + 80*e^x - 80)*sqrt(-(2*sqrt(5) + 5)*sqrt(-4
0*sqrt(5) + 200) + 20*sqrt(5) + 60)*(-40*sqrt(5) + 200)^(1/4)*log(-200*(sqr
t(5) - 1)*e^x + ((sqrt(5)*e^x - sqrt(5) - 5)*sqrt(-40*sqrt(5) + 200) - 10*s
qrt(5) - 50)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 6
0)*(-40*sqrt(5) + 200)^(1/4) + 10*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 4
00*e^(2*x) + 400) - (((sqrt(5) + 5)*e^x - sqrt(5) - 5)*sqrt(-40*sqrt(5) + 2
00) + 80*e^x - 80)*sqrt(-(2*sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(
5) + 60)*(-40*sqrt(5) + 200)^(1/4)*log(-200*(sqrt(5) - 1)*e^x - ((sqrt(5)*e
^x - sqrt(5) - 5)*sqrt(-40*sqrt(5) + 200) - 10*sqrt(5) - 50)*sqrt(-(2*sqrt(
5) + 5)*sqrt(-40*sqrt(5) + 200) + 20*sqrt(5) + 60)*(-40*sqrt(5) + 200)^(1/4
) + 10*(sqrt(5) + 5)*sqrt(-40*sqrt(5) + 200) + 400*e^(2*x) + 400) + 3200)/(
e^x - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.07, size = 64, normalized size = 0.31

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+10\_Z^{4}+5\right)}\frac{\left(-\_R^{6}+5\_R^{4}-5\_R^{2}+5\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^{7}+5\_R^{3}}\right)}{10}+\frac{1}{5\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^5),x)
```

```
[Out] 1/10*sum((-_R^6+5*_R^4-5*_R^2+5)/(_R^7+5*_R^3)*ln(tanh(1/2*x)-_R),_R=RootOf
(_Z^8+10*_Z^4+5))+1/5/tanh(1/2*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{5\left(e^{x}-1\right)}+\int \frac{2\left(e^{(7x)}+4e^{(6x)}+15e^{(5x)}+40e^{(4x)}+15e^{(3x)}+4e^{(2x)}+e^{x}\right)}{5\left(e^{(8x)}+2e^{(7x)}+8e^{(6x)}+14e^{(5x)}+30e^{(4x)}+14e^{(3x)}+8e^{(2x)}+2e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^5),x, algorithm="maxima")
```

```
[Out] 2/5/(e^x - 1) + integrate(2/5*(e^(7*x) + 4*e^(6*x) + 15*e^(5*x) + 40*e^(4*x
) + 15*e^(3*x) + 4*e^(2*x) + e^x)/(e^(8*x) + 2*e^(7*x) + 8*e^(6*x) + 14*e^(
5*x) + 30*e^(4*x) + 14*e^(3*x) + 8*e^(2*x) + 2*e^x + 1), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^5 - 1),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**5),x)
```

```
[Out] Timed out
```

3.74 $\int \frac{1}{1-\cosh^6(x)}\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\coth(x)}{3}$$

```
[Out] 1/3*coth(x)+1/3*arctanh(tanh(x)/(1+(-1)^(1/3))^(1/2))/(1+(-1)^(1/3))^(1/2)+
1/3*arctanh(tanh(x)/(1-(-1)^(2/3))^(1/2))/(1-(-1)^(2/3))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\coth(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[1 + (-1)^(1/3)]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[Ta
nh[x]/Sqrt[1 - (-1)^(2/3)]]/(3*Sqrt[1 - (-1)^(2/3)]) + Coth[x]/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cosh^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1-\cosh^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\cosh^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\cosh^2(x)}\,dx$$

$$= -\left(\frac{1}{3}\int \operatorname{csch}^2(x)\,dx\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[3]{-1}\right)x^2}\,dx, x, \coth(x)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}} + \frac{1}{3}i\,\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)$$

$$= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}} + \frac{\coth(x)}{3}$$

Mathematica [C] time = 0.23, size = 111, normalized size = 1.56

$$-\frac{\sinh(x)(8\cosh(2x)+\cosh(4x)+15)\left(-6\cosh(x)+\sqrt[4]{-3}\sinh(x)\left(\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{(-1)^{3/4}\left(\sqrt{3}-i\right)\tanh(x)}{2\sqrt[4]{3}}\right)+(3\right.\right.}{144\left(\cosh^6(x)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^6)^(-1),x]
```

```
[Out] -1/144*((15 + 8*Cosh[2*x] + Cosh[4*x])*Sinh[x]*(-6*Cosh[x] + (-3)^(1/4)*((3
*I + Sqrt[3])*ArcTan[((-1)^(3/4)*(-I + Sqrt[3])*Tanh[x])/(2*3^(1/4))] + (3
+ I*Sqrt[3])*ArcTan[((-1/3)^(1/4)*(I + Sqrt[3])*Tanh[x])/2])*Sinh[x]))/(-1
+ Cosh[x]^6)
```

fricas [B] time = 1.67, size = 694, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^6),x, algorithm="fricas")
```

```
[Out] 1/144*(4*(12^(1/4)*sqrt(6)*e^(2*x) - 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8)
*arctan((sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3
)*e^(2*x) + 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sq
rt(3) + 36*e^(4*x) + 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3) +
3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) - 36*sqrt(3) - 72) +
 2/3*sqrt(3)*(2*sqrt(3) - 3) + 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) - (12^(
3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) + 3*
12^(1/4)*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) + 2*sqrt(3) - 4) + 4*(
12^(1/4)*sqrt(6)*e^(2*x) - 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8)*arctan(-(
sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(-6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*x
) + 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) +
36*e^(4*x) + 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1/
4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) + 36*sqrt(3) + 72) - 2/3*sqr
t(3)*(2*sqrt(3) - 3) + 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) - (12^(3/4)*sqr
t(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) + 3*12^(1/4)
```

```
*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) - 2*sqrt(3) + 4) - (12^(1/4)*s
qrt(6)*(sqrt(3) + 2)*e^(2*x) - 12^(1/4)*sqrt(6)*(sqrt(3) + 2))*sqrt(-4*sqrt
(3) + 8)*log(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*x) + 12^(1/4)*sqrt(6)*(
5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) + 36*e^(4*x) + 144*e^(2*
x) + 252) + (12^(1/4)*sqrt(6)*(sqrt(3) + 2)*e^(2*x) - 12^(1/4)*sqrt(6)*(sqr
t(3) + 2))*sqrt(-4*sqrt(3) + 8)*log(-6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2
*x) + 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3)
+ 36*e^(4*x) + 144*e^(2*x) + 252) + 96)/(e^(2*x) - 1)
```

giac [A] time = 0.14, size = 10, normalized size = 0.14

$$\frac{2}{3\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^6),x, algorithm="giac")
```

```
[Out] 2/3/(e^(2*x) - 1)
```

maple [B] time = 0.09, size = 426, normalized size = 6.00

$$\frac{\tanh\left(\frac{x}{2}\right)}{6}+\frac{3^{\frac{1}{4}}\sqrt{2}\;\arctan\left(\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}+1\right)}{12}+\frac{3^{\frac{1}{4}}\sqrt{2}\;\arctan\left(\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}-1\right)}{12}+\frac{3^{\frac{1}{4}}\sqrt{2}\;\ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)+}{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)+}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^6),x)
```

```
[Out] 1/6*tanh(1/2*x)+1/12*3^(1/4)*2^(1/2)*arctan(1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)
+1)+1/12*3^(1/4)*2^(1/2)*arctan(1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)-1)+1/24*3^(
1/4)*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*3^(1/4)*tanh(1/2*x)+3^(1/2))/(tanh(1
/2*x)^2-2^(1/2)*3^(1/4)*tanh(1/2*x)+3^(1/2)))-1/72*3^(3/4)*2^(1/2)*ln((tanh
(1/2*x)^2-2^(1/2)*3^(1/4)*tanh(1/2*x)+3^(1/2))/(tanh(1/2*x)^2+2^(1/2)*3^(1/
4)*tanh(1/2*x)+3^(1/2)))-1/36*3^(3/4)*2^(1/2)*arctan(1/3*3^(3/4)*tanh(1/2*x
)*2^(1/2)+1)-1/36*3^(3/4)*2^(1/2)*arctan(1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)-1)
+1/6/tanh(1/2*x)+1/36*3^(3/4)*2^(1/2)*arctan(2^(1/2)*3^(1/4)*tanh(1/2*x)-1)
+1/72*3^(3/4)*2^(1/2)*ln((tanh(1/2*x)^2+1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1/3
*3^(1/2))/(tanh(1/2*x)^2-1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1/3*3^(1/2)))+1/36
*3^(3/4)*2^(1/2)*arctan(2^(1/2)*3^(1/4)*tanh(1/2*x)+1)-1/12*2^(1/2)*3^(1/4)
*arctan(2^(1/2)*3^(1/4)*tanh(1/2*x)-1)-1/24*2^(1/2)*3^(1/4)*ln((tanh(1/2*x)
^2-1/3*3^(3/4)*tanh(1/2*x)*2^(1/2)+1/3*3^(1/2))/(tanh(1/2*x)^2+1/3*3^(3/4)*
tanh(1/2*x)*2^(1/2)+1/3*3^(1/2)))-1/12*2^(1/2)*3^(1/4)*arctan(2^(1/2)*3^(1/
4)*tanh(1/2*x)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3\left(e^{(2x)}-1\right)}+\int\frac{e^{(3x)}+4\,e^{(2x)}+e^{x}}{3\left(e^{(4x)}+2\,e^{(3x)}+6\,e^{(2x)}+2\,e^{x}+1\right)}\,dx-\int\frac{e^{(3x)}-4\,e^{(2x)}+e^{x}}{3\left(e^{(4x)}-2\,e^{(3x)}+6\,e^{(2x)}-2\,e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^6),x, algorithm="maxima")
```

```
[Out] 2/3/(e^(2*x) - 1) + integrate(1/3*(e^(3*x) + 4*e^(2*x) + e^x)/(e^(4*x) + 2*
e^(3*x) + 6*e^(2*x) + 2*e^x + 1), x) - integrate(1/3*(e^(3*x) - 4*e^(2*x) +
 e^x)/(e^(4*x) - 2*e^(3*x) + 6*e^(2*x) - 2*e^x + 1), x)
```

mupad [B] time = 4.52, size = 329, normalized size = 4.63

$$\ln\left(\frac{1061158912\,e^{2x}}{27}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{2539651072\,e^{2x}}{9}-\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{21515730944\,e^{2x}}{9}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^6 - 1),x)
```

```
[Out] log((1061158912*exp(2*x))/27 + (1/72 - (3^(1/2)*1i)/216)^(1/2)*((2539651072
*exp(2*x))/9 - (1/72 - (3^(1/2)*1i)/216)^(1/2)*((21515730944*exp(2*x))/9 +
(1/72 - (3^(1/2)*1i)/216)^(1/2)*(19788726272*exp(2*x) + 2864709632) + 38702
94016/9) + 548405248/27) + 351797248/81)*(1/72 - (3^(1/2)*1i)/216)^(1/2) +
log((1061158912*exp(2*x))/27 + ((3^(1/2)*1i)/216 + 1/72)^(1/2)*((2539651072
*exp(2*x))/9 - ((3^(1/2)*1i)/216 + 1/72)^(1/2)*((21515730944*exp(2*x))/9 +
((3^(1/2)*1i)/216 + 1/72)^(1/2)*(19788726272*exp(2*x) + 2864709632) + 38702
94016/9) + 548405248/27) + 351797248/81)*((3^(1/2)*1i)/216 + 1/72)^(1/2) -
log((1061158912*exp(2*x))/27 - (1/72 - (3^(1/2)*1i)/216)^(1/2)*((2539651072
*exp(2*x))/9 + (1/72 - (3^(1/2)*1i)/216)^(1/2)*((21515730944*exp(2*x))/9 -
(1/72 - (3^(1/2)*1i)/216)^(1/2)*(19788726272*exp(2*x) + 2864709632) + 38702
94016/9) + 548405248/27) + 351797248/81)*(1/72 - (3^(1/2)*1i)/216)^(1/2) -
log((1061158912*exp(2*x))/27 - ((3^(1/2)*1i)/216 + 1/72)^(1/2)*((2539651072
*exp(2*x))/9 + ((3^(1/2)*1i)/216 + 1/72)^(1/2)*((21515730944*exp(2*x))/9 -
((3^(1/2)*1i)/216 + 1/72)^(1/2)*(19788726272*exp(2*x) + 2864709632) + 38702
94016/9) + 548405248/27) + 351797248/81)*((3^(1/2)*1i)/216 + 1/72)^(1/2) +
2/(3*(exp(2*x) - 1))
```

sympy [B] time = 22.05, size = 632, normalized size = 8.90

$$-\frac{\sqrt{2}\,\sqrt[4]{3}\,\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\,\sqrt[4]{3}\,\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{24}-\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\,\sqrt[4]{3}\,\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**6),x)
```

```
[Out] -sqrt(2)*3**(1/4)*log(4*tanh(x/2)**2 - 4*sqrt(2)*3**(1/4)*tanh(x/2) + 4*sqr
t(3))/24 - sqrt(2)*3**(3/4)*log(4*tanh(x/2)**2 - 4*sqrt(2)*3**(1/4)*tanh(x/
2) + 4*sqrt(3))/72 + sqrt(2)*3**(3/4)*log(4*tanh(x/2)**2 + 4*sqrt(2)*3**(1/
4)*tanh(x/2) + 4*sqrt(3))/72 + sqrt(2)*3**(1/4)*log(4*tanh(x/2)**2 + 4*sqrt
(2)*3**(1/4)*tanh(x/2) + 4*sqrt(3))/24 - sqrt(2)*3**(1/4)*log(36*tanh(x/2)*
*2 - 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/24 - sqrt(2)*3**(3/4)*log(
36*tanh(x/2)**2 - 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/72 + sqrt(2)*
3**(3/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/
72 + sqrt(2)*3**(1/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3**(3/4)*tanh(x/2) +
 12*sqrt(3))/24 + tanh(x/2)/6 - sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(1/4)*tanh
(x/2) - 1)/12 + sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(1/4)*tanh(x/2) - 1)/36 -
sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(1/4)*tanh(x/2) + 1)/12 + sqrt(2)*3**(3/4)
*atan(sqrt(2)*3**(1/4)*tanh(x/2) + 1)/36 - sqrt(2)*3**(3/4)*atan(sqrt(2)*3*
*(3/4)*tanh(x/2)/3 - 1)/36 + sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(3/4)*tanh(x/
2)/3 - 1)/12 - sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*tanh(x/2)/3 + 1)/36 +
 sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(3/4)*tanh(x/2)/3 + 1)/12 + 1/(6*tanh(x/2
))
```

3.75 $\int \frac{1}{1-\cosh^8(x)}\,dx$

Optimal. Leaf size=69

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{\coth(x)}{4}$$

```
[Out] 1/4*coth(x)+1/4*arctanh(tanh(x)/(1-I)^(1/2))/(1-I)^(1/2)+1/4*arctanh(tanh(x
)/(1+I)^(1/2))/(1+I)^(1/2)+1/8*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{\coth(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[1 - I]]/(4*Sqrt[1 - I]) + ArcTanh[Tanh[x]/Sqrt[1 + I]]
/(4*Sqrt[1 + I]) + ArcTanh[Tanh[x]/Sqrt[2]]/(4*Sqrt[2]) + Coth[x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cosh^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\cosh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-i\cosh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+i\cosh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\cosh}$$

$$= -\left(\frac{1}{4}\int \operatorname{csch}^2(x)\,dx\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(x)\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-(1+i)x^2}\right.$$

$$= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{4\sqrt{1-i}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{4\sqrt{1+i}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{4}i\,\text{Subst}(\int 1\,dx, x, -i\coth(x)$$

$$= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{4\sqrt{1-i}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{4\sqrt{1+i}} + \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{\coth(x)}{4}$$

Mathematica [A] time = 0.46, size = 64, normalized size = 0.93

$$\frac{1}{8}\left(\frac{2\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{\sqrt{1-i}} + \frac{2\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + \sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right) + 2\coth(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTanh[Tanh[x]/Sqrt[1 - I]])/Sqrt[1 - I] + (2*ArcTanh[Tanh[x]/Sqrt[1 +
 I]])/Sqrt[1 + I] + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]] + 2*Coth[x])/8
```

fricas [B] time = 1.87, size = 713, normalized size = 10.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^8),x, algorithm="fricas")
```

```
[Out] 1/32*(4*(2^(1/4)*e^(2*x) - 2^(1/4))*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt(
2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) +
6) - (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2)
 + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x) + 2^(1/4)*(3*sqrt(2) + 4))*sq
rt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) + 2*e^(2*x) + 5) + 1/14*sqrt(2)*(3
*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6)
)*e^(2*x) + 2^(3/4)*(2*sqrt(2) + 1) + 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*sq
rt(2) + 4) + 1/7*sqrt(2) - 3/7) + 4*(2^(1/4)*e^(2*x) - 2^(1/4))*sqrt(-2*sqr
t(2) + 4)*arctan(-1/14*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) +
1/28*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*
sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) + 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x)
 + 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) + 2*
e^(2*x) + 5) - 1/14*sqrt(2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 1
1) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2*x) + 2^(3/4)*(2*sqrt(2) + 1) + 2*2^(1/
4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2) + 4) - 1/7*sqrt(2) + 3/7) - (2^(1/4)*(s
qrt(2) + 1)*e^(2*x) - 2^(1/4)*(sqrt(2) + 1))*sqrt(-2*sqrt(2) + 4)*log((2^(3
/4)*e^(2*x) + 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e
^(4*x) + 2*e^(2*x) + 5) + (2^(1/4)*(sqrt(2) + 1)*e^(2*x) - 2^(1/4)*(sqrt(2)
 + 1))*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*x) + 2^(1/4)*(3*sqrt(2) + 4)
)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) + 2*e^(2*x) + 5) + 2*(sqrt(2)*
e^(2*x) - sqrt(2))*log(-(2*(2*sqrt(2) - 3)*e^(2*x) + 12*sqrt(2) - e^(4*x) -
 17)/(e^(4*x) + 6*e^(2*x) + 1)) + 16)/(e^(2*x) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^8),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.12, size = 136, normalized size = 1.97

$$\frac{\tanh\left(\frac{x}{2}\right)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2+1\right)}\_R\ln\left(2\tanh\left(\frac{x}{2}\right)\_R+\tanh^2\left(\frac{x}{2}\right)+1\right)\right)}{8}+\frac{1}{8\tanh\left(\frac{x}{2}\right)}+\frac{\sqrt{2}\,\ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\,\tanh(}{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\,\tanh(}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(x)^8),x)
```

```
[Out] 1/8*tanh(1/2*x)+1/8*sum(_R*ln(2*tanh(1/2*x)*_R+tanh(1/2*x)^2+1),_R=RootOf(2
*_Z^4-2*_Z^2+1))+1/8/tanh(1/2*x)+1/32*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tan
h(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))-1/32*2^(1/2)*ln((tanh(1/
2*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+\frac{1}{2\left(e^{(2x)}-1\right)}+8\int\frac{e^{(4x)}}{e^{(8x)}+4e^{(6x)}+22e^{(4x)}+4e^{(2x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)^8),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1/
2/(e^(2*x) - 1) + 8*integrate(e^(4*x)/(e^(8*x) + 4*e^(6*x) + 22*e^(4*x) + 4
*e^(2*x) + 1), x)
```

mupad [B] time = 2.63, size = 271, normalized size = 3.93

$$\frac{\sqrt{2}\,\ln\left(582732658686033920\,e^{2x}+70697326355677184\sqrt{2}+412054214575915008\sqrt{2}\,e^{2x}+999811177544\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^8 - 1),x)
```

```
[Out] (2^(1/2)*log(582732658686033920*exp(2*x) + 70697326355677184*2^(1/2) + 4120
54214575915008*2^(1/2)*exp(2*x) + 99981117754441728))/16 - (2^(1/2)*log(706
97326355677184*2^(1/2) - 582732658686033920*exp(2*x) + 412054214575915008*2
^(1/2)*exp(2*x) - 99981117754441728))/16 + 1/(2*(exp(2*x) - 1)) - (2^(1/2)*
(1 - 1i)^(1/2)*log((70836483296067584 - 69311013991743488i) - 2^(1/2)*(1 -
1i)^(1/2)*(54684829282729984 - 21956972328779776i) - 2^(1/2)*(1 - 1i)^(1/2)
*exp(2*x)*(12296353929494528 - 271474128182050816i) - exp(2*x)*(15561343400
2538496 + 429723297714798592i)))/16 + (2^(1/2)*(1 - 1i)^(1/2)*log(2^(1/2)*(
1 - 1i)^(1/2)*(54684829282729984 - 21956972328779776i) - exp(2*x)*(15561343
4002538496 + 429723297714798592i) + 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(122963
53929494528 - 271474128182050816i) + (70836483296067584 - 69311013991743488
i)))/16 - (2^(1/2)*(1 + 1i)^(1/2)*log((70836483296067584 + 693110139917434
8i) - 2^(1/2)*(1 + 1i)^(1/2)*(54684829282729984 + 21956972328779776i) - 2^(
```

```
1/2)*(1 + 1i)^(1/2)*exp(2*x)*(12296353929494528 + 271474128182050816i) - ex
p(2*x)*(155613434002538496 - 429723297714798592i)))/16 + (2^(1/2)*(1 + 1i)^
(1/2)*log(2^(1/2)*(1 + 1i)^(1/2)*(54684829282729984 + 219569723287797776i) -
 exp(2*x)*(155613434002538496 - 429723297714798592i) + 2^(1/2)*(1 + 1i)^(1/
2)*exp(2*x)*(12296353929494528 + 271474128182050816i) + (70836483296067584
+ 69311013991743488i)))/16
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(x)**8),x)
```

```
[Out] Timed out
```

3.76 $\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx$

Optimal. Leaf size=15

$$\log(\cosh(x))-\frac{1}{2}\log\left(\cosh^2(x)+1\right)$$

```
[Out] ln(cosh(x))-1/2*ln(1+cosh(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3194, 36, 29, 31}

$$\log(\cosh(x))-\frac{1}{2}\log\left(\cosh^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(1 + Cosh[x]^2),x]
```

```
[Out] Log[Cosh[x]] - Log[1 + Cosh[x]^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,\cosh^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,\cosh^2(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\cosh^2(x)\right)\\ &= \log(\cosh(x))-\frac{1}{2}\log\left(1+\cosh^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\log(\cosh(x))-\frac{1}{2}\log\left(\cosh^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(1 + Cosh[x]^2),x]
[Out] Log[Cosh[x]] - Log[1 + Cosh[x]^2]/2
```

fricas [B] time = 1.14, size = 47, normalized size = 3.13

$$-\frac{1}{2}\log\left(\frac{2\left(\cosh(x)^2+\sinh(x)^2+3\right)}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)+\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)^2),x, algorithm="fricas")
[Out] -1/2*log(2*(cosh(x)^2 + sinh(x)^2 + 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [A] time = 1.00, size = 23, normalized size = 1.53

$$-\frac{1}{2}\log\left(e^{(4x)}+6e^{(2x)}+1\right)+\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)^2),x, algorithm="giac")
[Out] -1/2*log(e^(4*x) + 6*e^(2*x) + 1) + log(e^(2*x) + 1)
```

maple [A] time = 0.09, size = 14, normalized size = 0.93

$$\ln(\cosh(x))-\frac{\ln\left(1+\cosh^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+cosh(x)^2),x)
[Out] ln(cosh(x))-1/2*ln(1+cosh(x)^2)
```

maxima [A] time = 0.30, size = 23, normalized size = 1.53

$$-\frac{1}{2}\log\left(6e^{(-2x)}+e^{(-4x)}+1\right)+\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)^2),x, algorithm="maxima")
[Out] -1/2*log(6*e^(-2*x) + e^(-4*x) + 1) + log(e^(-2*x) + 1)
```

mupad [B] time = 1.11, size = 27, normalized size = 1.80

$$\ln\left(-5184\,e^{2x}-5184\right)-\frac{\ln\left(54\,e^{2x}+9\,e^{4x}+9\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(cosh(x)^2 + 1),x)
[Out] log(- 5184*exp(2*x) - 5184) - log(54*exp(2*x) + 9*exp(4*x) + 9)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh(x)}{\cosh^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)**2),x)
```

```
[Out] Integral(tanh(x)/(cosh(x)**2 + 1), x)
```

3.77 $\int \sqrt{a+b\cosh^2(x)}\,\tanh(x)\,dx$

Optimal. Leaf size=39

$$\sqrt{a+b\cosh^2(x)}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)$$

```
[Out] -arctanh((a+b*cosh(x)^2)^(1/2)/a^(1/2))*a^(1/2)+(a+b*cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3194, 50, 63, 208}

$$\sqrt{a+b\cosh^2(x)}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]^2]*Tanh[x],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]]) + Sqrt[a + b*Cosh[x]^2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\cosh^2(x)}\,\tanh(x)\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cosh^2(x)\right)$$
$$= \sqrt{a+b\cosh^2(x)} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cosh^2(x)\right)$$
$$= \sqrt{a+b\cosh^2(x)} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^2(x)}\right)}{b}$$
$$= -\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b\cosh^2(x)}$$

Mathematica [**A**] time = 0.03, size = 39, normalized size = 1.00

$$\sqrt{a+b\cosh^2(x)} - \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]^2]*Tanh[x],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]]) + Sqrt[a + b*Cosh[x]^2]
```

fricas [**B**] time = 1.21, size = 357, normalized size = 9.15

$$\left[\frac{\sqrt{a}\,(\cosh(x)+\sinh(x))\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(4\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+4\,a+b\right)\sinh(x)^2-4\,\sqrt{2}\,\sqrt{a}\,\sqrt{\cdots}}{\cosh(x)^4+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\cosh(x)^2+1\right)\sinh(x)\cdots}\right)}{2\,(\cosh(x)+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cosh(x) + sinh(x))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
+ b*sinh(x)^4 + 2*(4*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 4*a + b)*sinh(x)
^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2))*(cosh(x) + sinh(x)) + 4*(b*cosh(x)^3 +
(4*a + b)*cosh(x))*sinh(x) + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*s
inh(x) + 1)) + sqrt(2)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x) + sinh(x)), 1/2*(2*sqrt(-a)*(
cosh(x) + sinh(x))*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(x)^2 + b*sinh(x
)^2 + 2*a + b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x) + a*
sinh(x))) + sqrt(2)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x) + sinh(x))]
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x)^2+a}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(x)^2 + a)*tanh(x), x)
```

maple [A] time = 0.08, size = 42, normalized size = 1.08

$$\sqrt{a+b\left(\cosh^{2}(x)\right)}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cosh^{2}(x)\right)}}{\cosh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)^2)^(1/2)*tanh(x),x)
```

```
[Out] (a+b*cosh(x)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cosh(x)^2)^(1/2))/cosh
(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cosh(x)^{2}+a}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(x)^2 + a)*tanh(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\tanh(x)\,\sqrt{b\cosh(x)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)*(a + b*cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\cosh^{2}(x)}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)**2)**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*cosh(x)**2)*tanh(x), x)
```

3.78 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx$

Optimal. Leaf size=26

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] -arctanh((a+b*cosh(x)^2)^(1/2)/a^(1/2))/a^(1/2)

Rubi [A] time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[a + b*Cosh[x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]]/Sqrt[a])

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cosh^2(x)\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^2(x)}\right)}{b}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]]/Sqrt[a])
```

fricas [B] time = 4.00, size = 248, normalized size = 9.54

$$\left[\frac{\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(4\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+4\,a+b\right)\sinh(x)^2-4\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{b\cosh(x)^2+b\sinh(x)^2+2\,a+b}{\cosh(x)^2-2\,\cosh(x)\sinh(x)+\sinh(}}}{\cosh(x)^4+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\cosh(x)^2+1\right)\sinh(x)^2+2\,\cosh(x)^2+4\left(\cosh(x)^3+\right.}\right)}{2\,\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(4*a + b)*c
osh(x)^2 + 2*(3*b*cosh(x)^2 + 4*a + b)*sinh(x)^2 - 4*sqrt(2)*sqrt(a)*sqrt((
b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))*(cosh(x) + sinh(x)) + 4*(b*cosh(x)^3 + (4*a + b)*cosh(x))*sinh(x) +
b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(
x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))/sqrt(a), sqrt(-a
)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x) + a*sinh(x)))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x)^2 + a), x)
```

maple [A] time = 0.10, size = 31, normalized size = 1.19

$$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cosh^2(x)\right)}}{\cosh(x)}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x)^2)^(1/2),x)
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cosh(x)^2)^(1/2))/cosh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(b*cosh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*cosh(x)^2)^(1/2),x)
[Out] int(tanh(x)/(a + b*cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)**2)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*cosh(x)**2), x)
```

3.79 $\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}} \, dx$

Optimal. Leaf size=13

$$-\tanh^{-1}\left(\sqrt{\cosh^2(x)+1}\right)$$

```
[Out] -arctanh((1+cosh(x)^2)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3194, 63, 207}

$$-\tanh^{-1}\left(\sqrt{\cosh^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[1 + Cosh[x]^2],x]
```

```
[Out] -ArcTanh[Sqrt[1 + Cosh[x]^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} \, dx, x, \cosh^2(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, \sqrt{1+\cosh^2(x)}\right) \\
&= -\tanh^{-1}\left(\sqrt{1+\cosh^2(x)}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\tanh^{-1}\left(\sqrt{\cosh^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[1 + Cosh[x]^2],x]
```

```
[Out] -ArcTanh[Sqrt[1 + Cosh[x]^2]]
```

fricas [B] time = 0.49, size = 63, normalized size = 4.85

$$\log\left(\frac{\sqrt{2}\sqrt{\frac{\cosh(x)^2+\sinh(x)^2+3}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}}-2\cosh(x)-2\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log((sqrt(2)*sqrt((cosh(x)^2 + sinh(x)^2 + 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{\cosh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(cosh(x)^2 + 1), x)
```

maple [A] time = 0.09, size = 12, normalized size = 0.92

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\cosh^2(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+cosh(x)^2)^(1/2),x)
```

```
[Out] -arctanh(1/(1+cosh(x)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{\cosh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(cosh(x)^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\tanh(x)}{\sqrt{{\cosh(x)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(cosh(x)^2 + 1)^(1/2),x)
```

```
[Out] int(tanh(x)/(cosh(x)^2 + 1)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{\cosh^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(cosh(x)**2 + 1), x)
```

3.80 $\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx$

Optimal. Leaf size=13

$$-\tanh^{-1}\left(\sqrt{-\sinh^2(x)}\right)$$

```
[Out] -arctanh((-sinh(x)^2)^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3176, 3205, 63, 206}

$$-\tanh^{-1}\left(\sqrt{-\sinh^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[1 - Cosh[x]^2],x]
```

```
[Out] -ArcTanh[Sqrt[-Sinh[x]^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{-\sinh^2(x)}}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-x}\,(1+x)}\,dx, x, \sinh^2(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sinh^2(x)}\right)$$
$$= -\tanh^{-1}\left(\sqrt{-\sinh^2(x)}\right)$$

Mathematica **[A]** time = 0.01, size = 21, normalized size = 1.62

$$\frac{2\sinh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[1 - Cosh[x]^2],x]
[Out] (2*ArcTan[Tanh[x/2]]*Sinh[x])/Sqrt[-Sinh[x]^2]
```

fricas **[A]** time = 0.46, size = 1, normalized size = 0.08

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1-cosh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac **[C]** time = 0.72, size = 38, normalized size = 2.92

$$-\frac{\log\left(e^x+i\right)}{\text{sgn}\left(-e^{(3x)}+e^x\right)} + \frac{\log\left(e^x-i\right)}{\text{sgn}\left(-e^{(3x)}+e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1-cosh(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(e^x + I)/sgn(-e^(3*x) + e^x) + log(e^x - I)/sgn(-e^(3*x) + e^x)
```

maple **[A]** time = 0.17, size = 12, normalized size = 0.92

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-\left(\sinh^2(x)\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1-cosh(x)^2)^(1/2),x)
[Out] -arctanh(1/(-sinh(x)^2)^(1/2))
```

maxima **[C]** time = 0.43, size = 7, normalized size = 0.54

$$-2i\,\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*arctan(e^(-x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1 - cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)/(1 - cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{-(\cosh(x)-1)(\cosh(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1-cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(-(cosh(x) - 1)*(cosh(x) + 1)), x)
```

3.81 $\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx$

Optimal. Leaf size=153

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cosh(x)+b^{2/3}\cosh^2(x)\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cosh(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cosh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-$$

```
[Out] ln(cosh(x))/a+1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*cosh(x))/a^(5/3)-1/6*b^(2/3)*l
n(a^(2/3)-a^(1/3)*b^(1/3)*cosh(x)+b^(2/3)*cosh(x)^2)/a^(5/3)-1/3*ln(a+b*cos
h(x)^3)/a+1/2*sech(x)^2/a-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*cosh(x)
)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3230, 1834, 1871, 200, 31, 634, 617, 204, 628, 260}

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cosh(x)+b^{2/3}\cosh^2(x)\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cosh(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cosh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Cosh[x]^3),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Cosh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(5/3))) + Log[Cosh[x]]/a + (b^(2/3)*Log[a^(1/3) + b^(1/3)*Cosh[x]])/(3*
a^(5/3)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Cosh[x] + b^(2/3)*Cosh[x]
^2])/(6*a^(5/3)) - Log[a + b*Cosh[x]^3]/(3*a) + Sech[x]^2/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx = -\text{Subst}\left(\int \frac{1-x^2}{x^3\left(a+bx^3\right)}\,dx, x, \cosh(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{ax}+\frac{b\left(-1+x^2\right)}{a\left(a+bx^3\right)}\right)dx, x, \cosh(x)\right)$$
$$= \frac{\log(\cosh(x))}{a}+\frac{\text{sech}^2(x)}{2a}-\frac{b\,\text{Subst}\left(\int \frac{-1+x^2}{a+bx^3}\,dx, x, \cosh(x)\right)}{a}$$
$$= \frac{\log(\cosh(x))}{a}+\frac{\text{sech}^2(x)}{2a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \cosh(x)\right)}{a}-\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x,\right.}{a}$$
$$= \frac{\log(\cosh(x))}{a}-\frac{\log\left(a+b\cosh^3(x)\right)}{3a}+\frac{\text{sech}^2(x)}{2a}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \cosh(x)\right.}{3a^{5/3}}$$
$$= \frac{\log(\cosh(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cosh(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\cosh^3(x)\right)}{3a}+\frac{\text{sech}^2(x)}{2a}-\frac{b^{2/}}{}$$
$$= \frac{\log(\cosh(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cosh(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cosh(x)+b^{2/3}\,\text{c}\right.}{6a^{5/3}}$$
$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\cosh(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{\log(\cosh(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cosh(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\log\left(a\right.}{}$$

Mathematica [C] time = 1.42, size = 145, normalized size = 0.95

$$\frac{-2\text{RootSum}\left[\#1^6b+3\#1^4b+8\#1^3a+3\#1^2b+b\&, \frac{-3\#1^4bx+3\#1^4b\log(e^x-\#1)-4\#1^3ax+4\#1^3a\log(e^x-\#1)+b\log(e^x-\#1)-bx}{\#1^4b+4\#1^3a+2\#1^2b+b}\&\right]}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Cosh[x]^3),x]
```

```
[Out] (-6*x + 6*Log[Cosh[x]] - 2*RootSum[b + 3*b*#1^2 + 8*a*#1^3 + 3*b*#1^4 + b*#
1^6 & , (-(b*x) + b*Log[E^x - #1] - 4*a*x*#1^3 + 4*a*Log[E^x - #1]*#1^3 - 3
*b*x*#1^4 + 3*b*Log[E^x - #1]*#1^4)/(b + 2*b*#1^2 + 4*a*#1^3 + b*#1^4) & ]
+ 3*Sech[x]^2)/(6*a)
```

fricas [C] time = 1.44, size = 1138, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(1/3)*(a*e^(4*x) + 2*a*e^(2*x) + a)*sqrt((((1/2)^(1/3)*(I*sqr
t(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a +
 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) - 2*a
*b*e^(3*x) - 2*a*b*e^x + (a^2*b*e^(3*x) - 4*a^3*e^(2*x) + a^2*b*e^x)*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a) + b
^2 + 2*(2*a^2 + b^2)*e^(2*x))*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^
```

```
5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3
) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1
/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)
/a^2) - sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b
^2)/a^5)^(1/3) + 2/a)^2*a^5*e^x - 4*a^2*b*e^(2*x) + 4*a^3*e^x - 4*a^2*b + 2
*(a^3*b*e^(2*x) - 2*a^4*e^x + a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2))
*e^(-x)/b^2) + 2*(a*e^(4*x) + 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*log(-((1/2)^(1/3)*(I*sqrt
(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a^2*e^x + b*e^(2*
x) + 2*a*e^x + b) - ((a*e^(4*x) + 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a) - 6*e^(4*x) - 12*e^(2
*x) - 6)*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^
5)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) - 2*a*b*e^(3*x) - 2*a*b*e^x + (
a^2*b*e^(3*x) - 4*a^3*e^(2*x) + a^2*b*e^x)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/
a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a) + b^2 + 2*(2*a^2 + b^2)*e^(2*
x)) - 12*(e^(4*x) + 2*e^(2*x) + 1)*log(e^(2*x) + 1) - 24*e^(2*x))/(a*e^(4*x
) + 2*a*e^(2*x) + a)
```

giac [A] time = 8.02, size = 191, normalized size = 1.25

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+e^{(-x)}+e^{x}\right|\right)}{3a^2}+\frac{\log\left(e^{(-x)}+e^{x}\right)}{a}-\frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)^3+8a\right|\right)}{3a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\cdots\right.\right.}{\cdots}\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)^3),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(-2*(-a/b)^(1/3) + e^(-x) + e^x))/a^2 + log(e^(-
x) + e^x)/a - 1/3*log(abs(b*(e^(-x) + e^x)^3 + 8*a))/a + 1/3*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + e^(-x) + e^x)/(-a/b)^(1/3))/a^2
 + 1/6*(-a*b^2)^(1/3)*log((e^(-x) + e^x)^2 + 2*(-a/b)^(1/3)*(e^(-x) + e^x)
+ 4*(-a/b)^(2/3))/a^2 - 1/2*(3*(e^(-x) + e^x)^2 - 4)/(a*(e^(-x) + e^x)^2)
```

maple [C] time = 0.14, size = 150, normalized size = 0.98

$$-\frac{\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^3+(-3a-3b)\_Z^2+(3a-3b)\_Z-a-b\right)}\frac{\left(\_R^2a-\_R^2b-2\_Ra-4\_Rb+a+b\right)\ln\left(\tanh^2\left(\frac{x}{2}\right)-\_R\right)}{\_R^2a-\_R^2b-2\_Ra-2\_Rb+a-b}}{3a}-\frac{2}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}+\frac{2}{a\left(\tanh^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*cosh(x)^3),x)
```

```
[Out] -1/3/a*sum((_R^2*a-_R^2*b-2*_R*a-4*_R*b+a+b)/(_R^2*a-_R^2*b-2*_R*a-2*_R*b+a
-b)*ln(tanh(1/2*x)^2-_R),_R=RootOf((a-b)*_Z^3+(-3*a-3*b)*_Z^2+(3*a-3*b)*_Z-
a-b))-2/a/(tanh(1/2*x)^2+1)+2/a/(tanh(1/2*x)^2+1)^2+1/a*ln(tanh(1/2*x)^2+1)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)^3),x, algorithm="maxima")
```

[Out] Timed out

mupad [B]   time = 0.92, size = 1173, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*cosh(x)^3),x)
```

```
[Out] 2/(a + a*exp(2*x)) - 2/(a + 2*a*exp(2*x) + a*exp(4*x)) + symsum(log(-(50331
648*a^6*exp(2*x) - 786432*b^6*exp(2*x) + 452984832*root(27*a^5*z^3 + 27*a^4
*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^7 + 50331648*a^6 - 786432*b^6 + 1358954
496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^2*a^8 + 13589
54496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*a^9 + 505
93792*a^2*b^4 - 102498304*a^4*b^2 + 1358954496*root(27*a^5*z^3 + 27*a^4*z^2
 + 9*a^3*z + a^2 - b^2, z, k)^2*a^8*exp(2*x) + 1358954496*root(27*a^5*z^3 +
 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*a^9*exp(2*x) + 50593792*a^2*b^4*
exp(2*x) - 102498304*a^4*b^2*exp(2*x) + 7602176*root(27*a^5*z^3 + 27*a^4*z^
2 + 9*a^3*z + a^2 - b^2, z, k)*a^3*b^4 - 465305600*root(27*a^5*z^3 + 27*a^4
*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^5*b^2 + 524288*a*b^5*exp(x) + 24379392*
root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^2*a^4*b^4 - 13833
33888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^2*a^6*b^2 +
 18874368*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*a^5*b
^4 - 1370750976*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3
*a^7*b^2 + 452984832*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z,
 k)*a^7*exp(2*x) - 5242880*a^3*b^3*exp(x) - 524288*root(27*a^5*z^3 + 27*a^4
*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^2*b^5*exp(x) - 8912896*root(27*a^5*z^3
+ 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^4*b^3*exp(x) + 7602176*root(27*
a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^3*b^4*exp(2*x) - 465305
600*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^5*b^2*exp(2
*x) + 14155776*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*
a^6*b^3*exp(x) + 24379392*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^
2, z, k)^2*a^4*b^4*exp(2*x) - 1383333888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a
^3*z + a^2 - b^2, z, k)^2*a^6*b^2*exp(2*x) + 18874368*root(27*a^5*z^3 + 27*
a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*a^5*b^4*exp(2*x) - 1370750976*root(2
7*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*a^7*b^2*exp(2*x))/(3*
a^6*b^6))*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k), k, 1,
3) + log(3221225472*a^6*exp(2*x) - 786432*b^6*exp(2*x) + 3221225472*a^6 - 7
86432*b^6 + 101449728*a^2*b^4 - 3321888768*a^4*b^2 + 101449728*a^2*b^4*exp(
2*x) - 3321888768*a^4*b^2*exp(2*x))/a
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\cosh^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*cosh(x)**3),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*cosh(x)**3), x)
```

3.82 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

[Out] -2/3*arctanh((a+b*cosh(x)^3)^(1/2)/a^(1/2))/a^(1/2)

Rubi [A] time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[a + b*Cosh[x]^3],x]

[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]])/(3*Sqrt[a])

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \cosh(x)\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cosh^3(x)\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^3(x)}\right)}{3b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]^3],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(Expression(Integer)),failed) can
not be coerced to mode SparseUnivariatePolynomial(Expression(Integer))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(tanh(x)/sqrt(b*cosh(x)^3 + a), x)
```

maple [A] time = 0.11, size = 21, normalized size = 0.75

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\left(\cosh^3(x)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*cosh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(b*cosh(x)^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*cosh(x)^3)^(1/2),x)
[Out] int(tanh(x)/(a + b*cosh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)**3)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*cosh(x)**3), x)
```

3.83 $\int \sqrt{a+b\cosh^3(x)}\, \tanh(x)\, dx$

Optimal. Leaf size=45

$$\frac{2}{3}\sqrt{a+b\cosh^3(x)}-\frac{2}{3}\sqrt{a}\, \tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)$$

```
[Out] -2/3*arctanh((a+b*cosh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*cosh(x)^3)^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a+b\cosh^3(x)}-\frac{2}{3}\sqrt{a}\, \tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]^3]*Tanh[x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Cosh[
x]^3])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
```

```
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \cosh(x)\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cosh^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\cosh^3(x)} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cosh^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\cosh^3(x)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^3(x)}\right)}{3b}\\ &= -\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{a+b\cosh^3(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{2}{3}\sqrt{a+b\cosh^3(x)} - \frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]^3]*Tanh[x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Cosh[
x]^3])/3
```

fricas [B] time = 1.43, size = 1648, normalized size = 36.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^3)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*(cosh(x) + sinh(x))*log(-(b^2*cosh(x)^12 + 12*b^2*cosh(x)*sin
h(x)^11 + b^2*sinh(x)^12 + 6*b^2*cosh(x)^10 + 64*a*b*cosh(x)^9 + 6*(11*b^2*
cosh(x)^2 + b^2)*sinh(x)^10 + 15*b^2*cosh(x)^8 + 4*(55*b^2*cosh(x)^3 + 15*b
^2*cosh(x) + 16*a*b)*sinh(x)^9 + 192*a*b*cosh(x)^7 + 3*(165*b^2*cosh(x)^4 +
 90*b^2*cosh(x)^2 + 192*a*b*cosh(x) + 5*b^2)*sinh(x)^8 + 24*(33*b^2*cosh(x)
^5 + 30*b^2*cosh(x)^3 + 96*a*b*cosh(x)^2 + 5*b^2*cosh(x) + 8*a*b)*sinh(x)^7
 + 192*a*b*cosh(x)^5 + 4*(128*a^2 + 5*b^2)*cosh(x)^6 + 4*(231*b^2*cosh(x)^6
 + 315*b^2*cosh(x)^4 + 1344*a*b*cosh(x)^3 + 105*b^2*cosh(x)^2 + 336*a*b*cos
h(x) + 128*a^2 + 5*b^2)*sinh(x)^6 + 15*b^2*cosh(x)^4 + 24*(33*b^2*cosh(x)^7
 + 63*b^2*cosh(x)^5 + 336*a*b*cosh(x)^4 + 35*b^2*cosh(x)^3 + 168*a*b*cosh(x
)^2 + 8*a*b + (128*a^2 + 5*b^2)*cosh(x))*sinh(x)^5 + 64*a*b*cosh(x)^3 + 3*(
165*b^2*cosh(x)^8 + 420*b^2*cosh(x)^6 + 2688*a*b*cosh(x)^5 + 350*b^2*cosh(x
)^4 + 2240*a*b*cosh(x)^3 + 320*a*b*cosh(x) + 20*(128*a^2 + 5*b^2)*cosh(x)^2
 + 5*b^2)*sinh(x)^4 + 6*b^2*cosh(x)^2 + 4*(55*b^2*cosh(x)^9 + 180*b^2*cosh(
x)^7 + 1344*a*b*cosh(x)^6 + 210*b^2*cosh(x)^5 + 1680*a*b*cosh(x)^4 + 480*a*
```

```
b*cosh(x)^2 + 20*(128*a^2 + 5*b^2)*cosh(x)^3 + 15*b^2*cosh(x) + 16*a*b)*sin
h(x)^3 + 6*(11*b^2*cosh(x)^10 + 45*b^2*cosh(x)^8 + 384*a*b*cosh(x)^7 + 70*b
^2*cosh(x)^6 + 672*a*b*cosh(x)^5 + 320*a*b*cosh(x)^3 + 10*(128*a^2 + 5*b^2)
*cosh(x)^4 + 15*b^2*cosh(x)^2 + 32*a*b*cosh(x) + b^2)*sinh(x)^2 + b^2 - 16*
(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 3*b*cosh(x)^6 + (28*b*
cosh(x)^2 + 3*b)*sinh(x)^6 + 16*a*cosh(x)^5 + 2*(28*b*cosh(x)^3 + 9*b*cosh(
x) + 8*a)*sinh(x)^5 + 3*b*cosh(x)^4 + (70*b*cosh(x)^4 + 45*b*cosh(x)^2 + 80
*a*cosh(x) + 3*b)*sinh(x)^4 + 4*(14*b*cosh(x)^5 + 15*b*cosh(x)^3 + 40*a*cos
h(x)^2 + 3*b*cosh(x))*sinh(x)^3 + b*cosh(x)^2 + (28*b*cosh(x)^6 + 45*b*cosh
(x)^4 + 160*a*cosh(x)^3 + 18*b*cosh(x)^2 + b)*sinh(x)^2 + 2*(4*b*cosh(x)^7
+ 9*b*cosh(x)^5 + 40*a*cosh(x)^4 + 6*b*cosh(x)^3 + b*cosh(x))*sinh(x))*sqrt
(a)*sqrt((b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + 3*b*cosh(x) + 4*a)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 12*(b^2*cosh(x)^11 + 5*b^2*cosh(x)^9
 + 48*a*b*cosh(x)^8 + 10*b^2*cosh(x)^7 + 112*a*b*cosh(x)^6 + 80*a*b*cosh(x)
^4 + 2*(128*a^2 + 5*b^2)*cosh(x)^5 + 5*b^2*cosh(x)^3 + 16*a*b*cosh(x)^2 + b
^2*cosh(x))*sinh(x))/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(
11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))
*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8
+ 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6
 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cos
h(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x
)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x
)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(
x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(
x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 2*sqrt((b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + 3*b*cosh(x) + 4*a)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x) + sinh(x)), 1/3*(sqrt(-a)*(co
sh(x) + sinh(x))*arctan(8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(
-a)*sqrt((b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + 3*b*cosh(x) + 4*a)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 +
 b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 16*a*cosh(
x)^3 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x) + 4*a)*sinh(x)^3 + 3*b*cosh(x)^2 + 3*
(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + 16*a*cosh(x) + b)*sinh(x)^2 + 6*(b*cosh(x)
^5 + 2*b*cosh(x)^3 + 8*a*cosh(x)^2 + b*cosh(x))*sinh(x) + b)) + sqrt((b*cos
h(x)^3 + 3*b*cosh(x)*sinh(x)^2 + 3*b*cosh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(cosh(x) + sinh(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x)^3 + a}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^3)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(x)^3 + a)*tanh(x), x)
```

maple [A] time = 0.09, size = 34, normalized size = 0.76

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\cosh^3(x)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{a+b\left(\cosh^3(x)\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)^3)^(1/2)*tanh(x),x)
```

```
[Out] -2/3*arctanh((a+b*cosh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*cosh(x)^3)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cosh(x)^3 + a}\, \tanh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^3)^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(x)^3 + a)*tanh(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(x)\, \sqrt{b \cosh(x)^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*cosh(x)^3)^(1/2),x)
```

```
[Out] int(tanh(x)*(a + b*cosh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cosh^3(x)}\, \tanh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)**3)**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*cosh(x)**3)*tanh(x), x)
```

3.84 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] -2*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Cosh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \cosh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cosh^n(x)\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^n(x)}\right)}{bn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 0.48, size = 113, normalized size = 3.90

$$\left[\frac{\log\left(\frac{b\cosh\left(n\log(\cosh(x))\right)+b\sinh\left(n\log(\cosh(x))\right)-2\sqrt{b\cosh\left(n\log(\cosh(x))\right)+b\sinh\left(n\log(\cosh(x))\right)+a}\,\sqrt{a}+2\,a}{\cosh\left(n\log(\cosh(x))\right)+\sinh\left(n\log(\cosh(x))\right)}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*cosh(n*log(cosh(x))) + b*sinh(n*log(cosh(x))) - 2*sqrt(b*cosh(n*log
(cosh(x))) + b*sinh(n*log(cosh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(cosh(x)
)) + sinh(n*log(cosh(x)))))/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt(b*cosh(n*lo
g(cosh(x))) + b*sinh(n*log(cosh(x))) + a)*sqrt(-a)/a)/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x)^n + a), x)
```

maple [A] time = 0.07, size = 24, normalized size = 0.83

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\left(\cosh^n(x)\right)}}{\sqrt{a}}\right)}{n\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x)^n)^(1/2),x)
[Out] -2*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b \cosh(x)^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(b*cosh(x)^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tanh(x)}{\sqrt{a + b \cosh(x)^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*cosh(x)^n)^(1/2),x)
[Out] int(tanh(x)/(a + b*cosh(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b \cosh^{n}(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)**n)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*cosh(x)**n), x)
```

3.85 $\int \sqrt{a+b\cosh^n(x)}\,\tanh(x)\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a+b\cosh^n(x)}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*cosh(x)^n)^(1/2)
/n
```

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b\cosh^n(x)}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]^n]*Tanh[x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Cosh[
x]^n])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
```

```
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cosh^n(x)}\,\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, \cosh(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cosh^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\cosh^n(x)}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cosh^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\cosh^n(x)}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^n(x)}\right)}{bn}\\
&= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{a+b\cosh^n(x)}}{n}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.96

$$
\frac{2\sqrt{a+b\cosh^n(x)} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]^n]*Tanh[x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]^n
])/n
```

fricas [A] time = 0.47, size = 156, normalized size = 3.32

$$
\left[\frac{\sqrt{a}\log\left(\frac{b\cosh\left(n\log(\cosh(x))\right)+b\sinh\left(n\log(\cosh(x))\right)-2\sqrt{b\cosh\left(n\log(\cosh(x))\right)+b\sinh\left(n\log(\cosh(x))\right)+a}\,\sqrt{a}+2a}{\cosh\left(n\log(\cosh(x))\right)+\sinh\left(n\log(\cosh(x))\right)}\right)+2\sqrt{b\cosh\left(n\right.}}{n}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^n)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*cosh(n*log(cosh(x))) + b*sinh(n*log(cosh(x))) - 2*sqrt(b*c
osh(n*log(cosh(x))) + b*sinh(n*log(cosh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*lo
g(cosh(x))) + sinh(n*log(cosh(x))))) + 2*sqrt(b*cosh(n*log(cosh(x))) + b*si
nh(n*log(cosh(x))) + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*cosh(n*log(cosh(x)))
+ b*sinh(n*log(cosh(x))) + a)*sqrt(-a)/a) + sqrt(b*cosh(n*log(cosh(x))) + b
*sinh(n*log(cosh(x))) + a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{b\cosh(x)^n + a}\,\tanh(x)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^n)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(x)^n + a)*tanh(x), x)
```

maple [A] time = 0.05, size = 38, normalized size = 0.81

$$\frac{2\sqrt{a+b\left(\cosh^{n}(x)\right)}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\cosh^{n}(x)\right)}}{\sqrt{a}}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)^n)^(1/2)*tanh(x),x)
```

```
[Out] 1/n*(2*(a+b*cosh(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x)^{n}+a}\ \tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)^n)^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(x)^n + a)*tanh(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(x)\,\sqrt{a+b\cosh(x)^{n}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*cosh(x)^n)^(1/2),x)
```

```
[Out] int(tanh(x)*(a + b*cosh(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cosh^{n}(x)}\ \tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)**n)**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*cosh(x)**n)*tanh(x), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```